Fizika Nizkikh Temperatur, 2006, v. 32, No. 10, p. 1282–1285

Resonant effects in the strongly driven phase-biased Cooper-pair box

S.N. Shevchenko and A.N. Omelyanchouk

B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Lenin Ave., Kharkov 61103, Ukraine
E-mail: sshevchenko@ilt.kharkov.ua

Received April 4, 2006

We study the time-averaged upper level occupation probability in a strongly driven two-level system, particularly its dependence on the driving amplitude x_0 and frequency ω and the energy level separation ΔE. In contrast to the case of weak driving ($x_0 << \Delta E$), when the positions of the resonances almost do not depend on x_0, in the case of the strong driving ($x_0 \sim \Delta E$) their positions are strongly amplitude dependent. We study these resonances in the concrete system — the strongly driven phase-biased Cooper-pair box, which is considered to be weakly coupled to the tank circuit.

PACS: 03.67.Lx, 03.75.Lm, **74.50.+r**, 85.25.Am

Keywords: two-level systems, Landau—Zener transition, Josephson function.

Several mesoscopic superconducting devices, which behave as quantum-mechanical two-level systems (TLSs), were proposed and studied recently (see reviews [1,2]). And although these devices are formally analogous to microscopic TLSs (such as electrons, atoms, photons, etc. [3]), they differ in that the coupling to controlling gates, and the environment must be taken into account (this makes the numerical analysis of a mesoscopic TLS necessary). The study of the dynamic behaviour of the mesoscopic superconducting structures is interesting because they are suitable for observation of the quantum-mechanical features by measuring macroscopic values and because of their relevance for engineering on the mesoscopic scale, e.g., for potentially realizable quantum computers based on superconducting Josephson qubits. The following non-stationary effects were studied in the superconducting effectively TLSs: Rabi oscillations [4–7], multiphoton excitations [8–11], Landau—Zener transition [12,13], nonlinear excitations [14]. In this work we study the strongly driven superconducting TLS. Namely, we study the phase-biased Cooper-pair box (PBCPB) (also called the Cooper-pair transistor) [15–19] strongly driven via the gate electrode and probed by the classical resonant contour (tank circuit). The particular interest in this problem is because due to the interference between the Landau—Zener tunneling events, the system can be resonantly excited and the probability of the excitation oscillatory depends on the amplitude of the driving parameter [20–23]. That is why we are interested in the dynamics of the strongly driven superconducting TLS — to clarify this problem and to relate it to the experimental results [24].

The rest of the paper is organized as following. First we analyse the resonant excitations of a TLS, particularly, the difference between the weakly and strongly driven regimes. Then we study concrete situation of the strongly driven PBCPB, which is probed by the tank circuit. The paper ends with the conclusions.

We consider a TLS described by the Hamiltonian

$$\hat{H}(t) = \Delta\hat{\sigma}_x + (x_{\rm off} + x_0 \sin\omega t)\hat{\sigma}_z . \qquad (1)$$

Here $\hat{\sigma}_{x,z}$ are the Pauli matrices. We are interested in the time-averaged upper level occupation probability, which is assumed to be related with the observable values. A driven TLS can be resonantly excited from the ground state to the upper state [25]. When the driving amplitude x_0 is small compared to the energy level separation $\Delta E = 2\sqrt{\Delta^2 + x_{\rm off}^2}$, the positions of the resonances in the time-averaged upper level occupation probability is determined by the multiphoton relation, $\Delta E = K\hbar\omega$. Here ω is the driving frequency and K is an integer. If the amplitude x_0 is increased, the position of the resonances is shifted (the Bloch–Siegert shift) [14]. Thus, at fixing ω and ΔE and with increasing amplitude x_0 one should expect the (quasi-) periodic behaviour due to the shift of the multiphoton resonances. Below we analyse this issue in terms of the shift of the multiphoton resonances following Ref. 26. Alternatively the quasi-periodic behaviour of the probability can be described in terms of the sequential Landau–Zener transitions with the quantum-mechanical interference between the transition events taken into account as in Ref. 21.

Consider first, for simplicity, the case of the zero offset, $x_{\rm off} = 0$. In this case the position of the resonances in the dependence of the occupation probability on the system's parameters is determined by the following equation [26]:

$$\frac{2\Delta}{\hbar\omega}\sqrt{1+q^2}\,E\left(\frac{q}{\sqrt{1+q^2}}\right) = \frac{\pi}{2}K, \quad K = 1, 3, 5, \ldots, \qquad (2)$$

where $E(k) = \int_0^1 dx \frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}}$ is the complete elliptic integral of the second kind and $q = x_0/\Delta$. The parameter q is the key parameter of the problem; consider two limiting cases: that of weak driving, $q \ll 1$, and that of very strong driving, $q \gg 1$ (the term «strong driving» we reserve for the case $q \sim 1$); from Eq. (2) it follows that

$$\Delta E = K\hbar\omega, \quad q \ll 1 \qquad (3)$$

$$\frac{4x_0}{\hbar\omega} = \pi K, \quad q \gg 1. \qquad (4)$$

The first relation defines the multiphoton resonances, when the energy level separation, $\Delta E = 2\Delta$, is a multiple of a photon energy $\hbar\omega$. The resonances determined by Eq. (3) can be observed in the dependence of the occupation probability on ω or Δ, but not in the dependence on x_0. In the second case, the resonances, determined by Eq. (4) can be observed in the dependence on ω or x_0, but not in the dependence on Δ; in this case equation (4) also implies periodic (or quasi-periodic) dependence on the parameter $\varphi = 4x_0/\hbar\omega$, which was studied in Refs. 21 and 23. For the strong driving, $q \sim 1$, the resonances are expected in dependencies on each of the three parameters: ω, x_0, and Δ. Thus, we expect to find in the regime of the strong driving features typical for the two limiting cases: (i) quasi-periodic resonant dependence on x_0 and (ii) the resonances to appear in the dependence on Δ (with their positions being dependent on x_0).

Consider the PBCPB [15–19,23] excited through the gate electrode. The PBCPB is the small superconducting island, which is connected via two Josephson junctions (characterized by energies $E_{J1,2}$ and phase differences $\delta_{1,2}$) to the ring with low inductance L (which is pierced by the magnetic flux Φ_e) and via the capacitance C_g to the gate with voltage V_g. The PBCPB is described by the Hamiltonian:

$$\hat{H} = \frac{\varepsilon_J}{2}\hat{\sigma}_x - 2E_C(1 - n_g^{(0)} - n_g^{(1)}\sin\omega t)\hat{\sigma}_z, \qquad (5)$$

where the Coulomb energy of the island with the total capacitance $C_{\rm tot}$ is $E_C = e^2/2C_{\rm tot}$; the effective Josephson energy is

$$\varepsilon_J = (E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\delta)^{1/2};$$

the total phase difference, $\delta = \delta_1 + \delta_2$, is approximately equal to $2\pi\Phi_e/\Phi_0$; and the dimensionless gate voltage is $n_g(t) = n_g^{(0)} + n_g^{(1)}\sin\omega t = C_gV_g(t)/e$. The Hamiltonian of the PBCPB (5) coincides with the above Hamiltonian (1) introduced, with the substitutions: $\Delta = \varepsilon_J(\delta)/2$, $x_{\rm off} = -2E_C(1 - n_g^{(0)})$, and $x_0 = 2E_Cn_g^{(1)}$.

Now the parameter q is given by $q = 4E_Cn_g^{(1)}/\varepsilon_J$. Thus both limiting cases — of weak and of very strong driving — described above, can in principle be realized in the PBCPB [23], where the domination of the Coulomb energy of a Cooper pair $4E_C$ over the coupling energy ε_J is assumed, $4E_C/\varepsilon_J > 1$. In Ref. 11 we have studied the case of weak driving, and here we study the case of strong driving, $q \sim 1$, in detail.

We will study the dependencies on $n_g^{(1)}$ and on δ to demonstrate features (i) and (ii). The occupation probabilities of the PBCPB are assumed to be probed by the tank circuit, which is weakly coupled through the mutual inductance M to the PBCPB [27,28]. The average current $\langle\hat{I}\rangle$ through the PBCPB is related to the phase shift between the voltage and current α, when the tank circuit with the capacity C_T, the resistance R_T, and the inductance L_T is driven at the resonant frequency $\omega_T = 1/\sqrt{L_TC_T}$, as follows [11]:

$$\tan\alpha \simeq k^2QL\frac{2e}{\hbar}\frac{\partial\langle\hat{I}\rangle}{\partial\delta}, \qquad (6)$$

where $Q^{-1} = \omega_TC_TR_T$, $k^2 = M^2/(L\cdot L_T)$. To obtain the expectation value for the current in the qubit's ring, $\langle\hat{I}\rangle = {\rm Tr}(\hat{\rho}\hat{I})$, we solve numerically the Bloch equations for the reduced density matrix $\hat{\rho}$, as we did in Ref. 11. These equations describe the relaxation and dephasing processes by including phenomenologically the corresponding rates $\Gamma_{\rm relax}$ and Γ_φ.

In Fig. 1 we plot the time-averaged upper level occupation probability $\overline{P}$ as a function of the amplitude $n_g^{(1)}$ at $\delta = \pi$ by making use of the solution of the Bloch equations. The case of $n_g^{(0)} \neq 1$ (that is $x_{\rm off} \neq 0$) differs from the case of $n_g^{(0)} = 1$ ($x_{\rm off} = 0$) by the appearance of the additional peaks, which was discussed in Refs. 21 and 23. We point out that similar dependence, which illustrates the feature (i), in the case $x_{\rm off} = 0$ can be calculated alternatively by making use of other approaches, namely with Eq. (13) from Ref. 26 and with Eq. (17) from Ref. 21. The numerical solution of the Bloch equations allows us to overcome the restrictions of the analytical works: in Ref. 26 neither decoherence nor $x_{\rm off} \neq 0$ were taken into account while in Ref. 21 the assumption of very strong driving was done, which, for example, excludes the feature (ii), as it was explained above.

Since at $\delta = \pi$ the phase shift α is proportional to the time-averaged difference between the ground and excited state occupation probabilities [11], $1 - 2\overline{P}$, Fig. 1 presents also the dependence of α on $n_g^{(1)}$. In Fig. 2 the dependence of the phase shift α on the total phase difference δ is plotted for different amplitudes $n_g^{(1)}$. Note that, as it was explained in Ref. 11, the dependence of the phase shift α on δ has hyperbolic-like character in the vicinity of the resonances. The parameters of the system taken for the Figs. 1 and 2 are the following: $E_{J1}/E_C = 4.5$, $E_{J2}/E_C = 4$, $\hbar\omega/E_C = 0.25$, $k^2 Q 2e^2 L E_C/\hbar^2 = 0.01$; the temperature was considered to be zero (i.e. much less than $E_{J1} - E_{J2}$); the relaxation and dephasing rates we considered to be the functions of the energy level separation: $\Gamma_{\rm relax}, \Gamma_\varphi \propto \Delta E(\delta)$ [1] (we have taken $\Gamma_{\rm relax} \sim \Gamma_\varphi \sim 0.01 E_C$).

In conclusion, we have clarified from analytical consideration the qualitative difference between the weak driving of a TLS and very strong driving. Then the strongly driven PBCPB was studied. The numerical results (Figs. 1 and 2) demonstrated that (i) the dependence of the tank phase shift α on the amplitude $n_g^{(1)}$ at $\delta = \pi$ has resonant quasi-periodic character and (ii) the resonances appear in the dependence on the phase difference δ as the amplitude-dependent hyperbolic-like structures. We point out that the dependencies, characterized by the features (i) and (ii), similar to Figs. 1 and 2, were observed experimentally [24]. And also similar to Fig. 1 quasi-periodic dependence of the upper level occupation probability on the driving (microwave) amplitude was observed in the superconducting TLS based on a large Josephson-junction in Fig. 6 of Ref. 5.

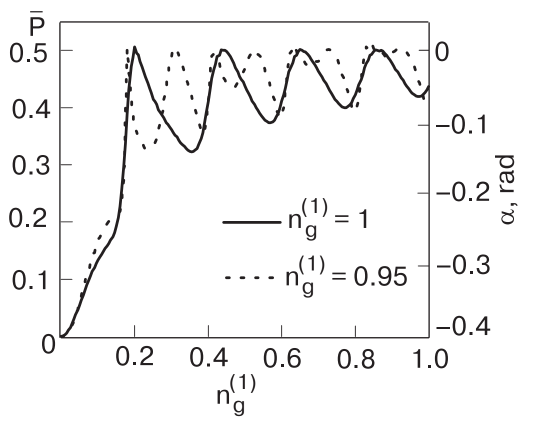

Fig. 1. Dependence of the time-averaged upper level occupation probability $\overline{P}$ (left) and phase shift α (right) on the amplitude $n_g^{(1)}$ at $\delta = \pi$.

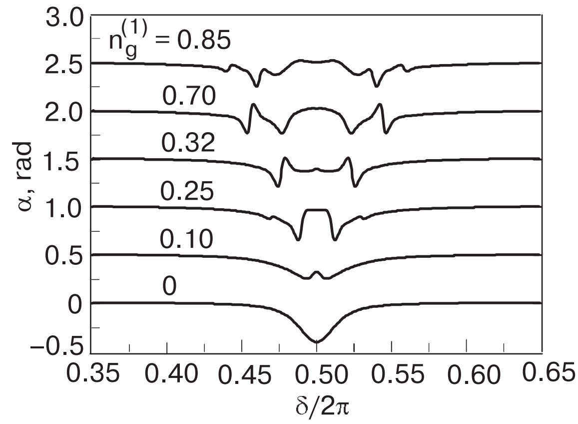

Fig. 2. The dependence of the phase shift α on the total phase difference δ for different amplitudes $n_g^{(1)}$. Upper curves are shifted vertically for clarity.

We thank the authors of Ref. 24 for communication of their experimental results prior to the publication and E. Il'ichev, W. Krech, and V.I. Shnyrkov for fruitful discussions. The authors acknowledge the grant «Nanosystems, nanomaterials, and nanotechnology» of the National Academy of Sciences of Ukraine. The work of S.N.S. was partly supported by grant of President of Ukraine (No. GP/P11/13).

Note added. During the preparation of the manuscript we became aware that similar works on strongly driven superconducting systems have appeared [29,30]. Those articles are devoted to the experimental and theoretical study of the interference fringes in the strongly driven Cooper-pair transistor [29] and the flux qubit [30].

1. Yu. Makhlin, G. Schön, and A. Shnirman, *Rev. Mod. Phys*. **73**, 357 (2001).
2. G. Wendin and V.S. Shumeiko, *arXiv*: *cond-mat/0508729*.
3. C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions*, Wiley, New York (1992).
4. Y. Nakamura, Yu.A. Pashkin, and J.S. Tsai, *Nature (London)* **398**, 786 (1999).

5. J. Martinis, S. Nam, J. Aumentado, and C. Urbina, *Phys. Rev. Lett*. **89**, 117901 (2002).
6. E. Il'ichev, N. Oukhanski, A. Izmalkov, Th. Wagner, M. Grajcar, H.-G. Meyer, A.Yu. Smirnov, A. Maassen van den Brink, M.H.S. Amin, and A.M. Zagoskin, *Phys. Rev. Lett*. **91**, 097906 (2003).
7. J. Claudon, F. Balestro, F.W.J. Hekking and O. Buisson, *Phys. Rev. Lett*. **93**, 187003 (2004).
8. Y. Nakamura, Yu.A. Pashkin, and J.S. Tsai, *Phys. Rev. Lett*. **87**, 246601 (2001).
9. A. Wallraff, T. Duty, A. Lukashenko, A.V. Ustinov, *Phys. Rev. Lett*. **90**, 037003 (2003).
10. S. Saito, M. Thorwart, H. Tanaka, M. Ueda, H. Nakano, K. Semba, and H. Takayanagi, *Phys. Rev. Lett*. **93**, 037001 (2004).
11. V.I. Shnyrkov, Th. Wagner, D. Born, S.N. Shevchenko, W. Krech, A.N. Omelyanchouk, E. Il'ichev, and H.-G. Meyer, *Phys. Rev*. **B73**, 024506 (2006).
12. A. Izmalkov, M. Grajcar, E. Il'ichev, N. Oukhanski, Th. Wagner, H.-G. Meyer, W. Krech, M.H.S. Amin, A. Maassen van den Brink, and A.M. Zagoskin, *Europhys. Lett*. **65**, 844 (2004).
13. G. Ithier, E. Collin, P. Oyez, D. Vion, D. Esteve, J. Ankerhold, and H. Grabert, *Phys. Rev. Lett*. **94**, 057004 (2005).
14. M.C. Goorden and F.K. Wilhelm, *Phys. Rev*. **B68**, 012508 (2003).
15. M. Tinkham, *Introduction to Superconductivity, 2nd edition*, McGraw-Hill, New York (1996), chap. 7.
16. G. Falci, R. Fazio, G.M. Palma, J. Siewert, and V. Vedral, *Nature (London)* **407**, 355 (2000).
17. A.B. Zorin, *Physica* **C368**, 284 (2002).
18. J.R. Friedman and D.V. Averin, *Phys. Rev. Lett*. **88**, 050403 (2002).
19. W. Krech, M. Grajcar, D. Born, I. Zhilyaev, Th. Wagner, E. Il'ichev, and Ya. Greenberg, *Phys. Lett*. **A303**, 352 (2002).
20. Y. Teranishi and H. Nakamura, *Phys. Rev. Lett*. **81**, 2032 (1998).
21. A.V. Shytov, D.A. Ivanov, and M.V. Feigel'man, *Eur. Phys. J*. **B36**, 263 (2003).
22. K. Saito and Y. Kayanuma, *Phys. Rev*. **B70**, 201304 (2004).
23. S.N. Shevchenko, A.S. Kiyko, A.N. Omelyanchouk, and W. Krech, *Fiz. Nizk. Temp*. **31**, 752 (2005) [*Low Temp. Phys*. **31**, 569 (2005).]
24. V.I. Shnyrkov et al., *unpublished*.
25. M. Grifoni and P. Hänggi, *Phys. Rep*. **304**, 229 (1998).
26. V.P. Krainov and V.P. Yakovlev, *Sov. Zh. Exp. Theor. Fiz*. **78**, 2204 (1980).
27. R. Rifkin and B.S. Deaver, Jr., *Phys. Rev*. **B13**, 3894 (1976).
28. E. Il'ichev, V. Zakosarenko, L. Fritzsch, R. Stolz, H.E. Hoenig, H.-G. Meyer, M. Götz, A.B. Zorin, V.V. Khanin, A.B. Pavolotsky, and J. Niemeyer, *Rev. Sci. Instr*. **72**, 1882 (2001).
29. M. Sillanpää, T. Lehtinen, A. Paila, Yu. Makhlin, and P. Hakonen, *arXiv: cond-mat/0510559*.
30. W.D. Oliver, Ya.Yu, J.C. Lee, K.K. Berggren, L.S. Levitov, and T.P. Orlando, *Science* **310**, 1653 (2005).