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Physica E 10 (2001) 549–552

PHYSICA E

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Conductivity of a two-dimensional curved microconstriction

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Received 28 November 2000; accepted 5 December 2000

Abstract

The quantum conductance of a long two-dimensional curved microconstriction has been considered theoretically. It is shown that over-the-barrier-reflection of electrons at points in which the curvature of the surface is changed, results in fine structure of quantum steps of the conductance. The observation of this structure would be a demonstration of the influence of the curvature on the quantum properties of two-dimensional electrons.

Keywords: Conductivity; Two-dimensional curved microconstriction

The two-dimensional electron gas (2DEG) has been a favorite model system for investigation of a variety of interesting quantum effects (for review, see Ref. [1]). It is practically realized, for example, in the inversion layers of GaAs heterostructures and Si metal-oxide–semiconductor field-effect transistors. As model systems, these structures possess desirable properties. The low electron density ($n_s = 10^{11}$–10^{12} cm^{-2}) implies a large Fermi wavelength λ_F (typically 40 nm). The electron mean free path, l, in the above systems is quite large, exceeding 10 μm in GaAs heterostructures. That is why some peculiar quantum effects have been observed in the 2DEG [1].

Over the last 10 years, experimentalists have been able to fabricate different conducting devices in 2DEG, among which the microconstrictions with the dimensions comparable with the Fermi wavelength λ_F are of particular interest. An interesting phenomenon observed in such microconstrictions is conductance quantization: the electrical conductance admits only discrete values nG_0, where n is an integer and $G_0 = 2e^2/h$ is the unit of conductance [2,3].

A further experimental development is the fabrication of non-planar two-dimensional electron systems [4–6]. A non-planar 2DEG can be produced using regrowth technology on previously patterned substrates. Using this method, it is possible to produce steps, and virtually any other feature in the 2DEG, rendering the investigation of electron motion on shaped surfaces possible [5,7,8]. Theoretical studies of various aspects of these shaped systems, such as their transport properties, have been reported [9–11].

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1386-9477/01/$ - see front matter
PII: S1386-9477(00)00298-8

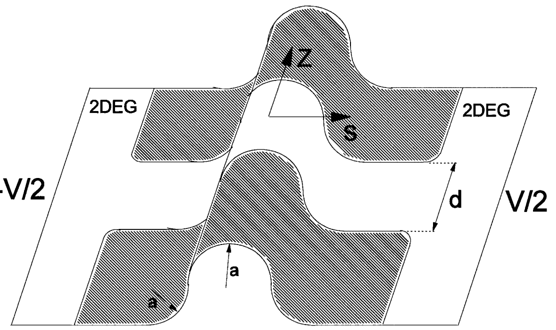

Fig. 1. Schematic of the experimental geometry, in which the effect of the surface curvature may be investigated.

The 2DEG in foregoing structures is not an ideal two-dimensional system. Electrons in them are confined to a potential well. As a result of confinement, the conduction band splits into a series of two-dimensional subbands. At low electron density only the lowest subband is partially occupied and the electrons freely move in a plane. The extension of the wavefunction in the direction perpendicular to the interface is of the order of 3–30 nm depending on the width of the well and the electron density. Hence, the electrons are "aware" of properties of surface. The Schrodinger equation for a quantum mechanical particle, constrained to a curved surface includes a geometrical potential term, which depends on the geometry of the surface [12]. For a flat sheet, the geometrical potential vanishes, but all kinetic and thermodynamic properties of a curved 2DEG will depend on this potential.

In this note we propose the basic idea of a novel experiment, in which the effect of "the geometrical potential" on electrical conductance of a curved 2DEG constriction can be observed and measured. Let us consider a clean, narrow, ribbon-shaped microconstriction between two large conducting banks best as shown in Fig. 1. It is adiabatically smooth on the scale of Fermi wavelength λ_F. In the zeroth order of reflection (flat surface) the $2e^2/h$ conductance quantization is obtained [13]. The conductance G of quantum contacts can be described by the Landauer formula [14–16]. At zero temperature, but finite voltages ($eV \ll \varepsilon_F$), this formula is written as

$$G = \frac{e^2}{h}\sum_{n=1}^{N}\left[T_n\left(\varepsilon_F + \frac{eV}{2}\right) + T_n\left(\varepsilon_F - \frac{eV}{2}\right)\right], \quad (1)$$

where the sum runs over the total number of propagating transversal quantum modes. These modes arise from the finite width of the constriction which discretizes the transversal momentum of an electron in the constriction. When a bias voltage eV is applied, the net current inside a long constriction is determined by the contributions of two electronic beams, differing in bias energy and moving in opposite directions [17]. The two terms in Eq. (1) correspond to these two beams. When contact width is increased, the energy of each transversal mode decreases and at a certain value of the contact diameter a new quantum mode opens up but not simultaneously for the two beams. Whenever a quantum mode becomes available to any of the two directions of the electron wave vector along the constriction, the conductance jumps by $G_0/2$ [18]. If the bias eV is larger than the distances between the minimal energies of quantum modes, it is possible to change the number of opened modes by changing the voltage V. Hence, in this case the conductance jumps in sequence of steps of height $G_0/2$, as a function of the voltage V [19]. The transmission probability of the nth mode, T_n depends on the maximum energy of electrons, and must be found by solving the Schrodinger equation. It can be shown that for a 2DEG rigidly bound to the curved surface $\boldsymbol{r} = \boldsymbol{r}(q_1, q_2)$, the Schrodinger equation is given by [12]

$$\frac{\hbar^2}{2m}\sum_{i,j=1}^{2}\frac{1}{\sqrt{g}}\frac{\partial}{\partial q_i}\left(\sqrt{g}(g^{-1})_{ij}\frac{\partial\psi}{\partial q_j}\right) + V_S(q_1,q_2)\psi = \varepsilon\psi, \quad (2)$$

where $g_{ij} = (\partial\boldsymbol{r}/\partial q_i)(\partial\boldsymbol{r}/\partial q_j)$ denote covariant components of the metric tensor of the surface and $g = \det(g_{ij})$, m is the effective mass of electrons, ε is the electron energy and $V_S(q_1, q_2)$ is the "geometrical potential",

$$V_S(q_1,q_2) = -\frac{\hbar^2}{8m}(\rho_1 - \rho_2)^2, \quad (3)$$

where ρ_1 and ρ_2 are the principal curvatures of the surface at the point q_1, q_2.

We assume a ribbon-shaped microconstriction, in which the non-planar parts are portions of a cylinder with radius a and width d. For this geometry one of the curvatures is equal to zero and the second is constant

(Fig. 1). In terms of the coordinates s along the contact axis and z along the cylindrical surface, we find

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial s^2}+\frac{\partial^2}{\partial z^2}+\frac{\rho^2(s)}{4}\right)\psi=\varepsilon\psi, \tag{4}$$

where

$$\rho(s)=\begin{cases}0, & |s|>\pi a,\\ \dfrac{1}{a}, & |s|<\pi a.\end{cases} \tag{5}$$

In fact there are two turning points at $s=\pm\pi a/2$, where the potential vanishes. But the width of these flat sections is essentially zero. So they have no effect on the results. The wavefunction ψ vanishes at the boundary of the constriction $z=\pm d/2$. Eqs. (4) and (5) correspond to the motion of a quantum particle over potential well, with the energy larger than the depth of the well. The transmission coefficient of this system is well known [20],

$$T_n(\varepsilon)=\left[1+\frac{1}{4}\left(\frac{k_n'}{k_n}-\frac{k_n}{k_n'}\right)^2\sin^2(2\pi k_n' a)\right]^{-1}, \tag{6}$$

where

$$k_n=\frac{1}{\hbar}\sqrt{2m(\varepsilon-\varepsilon_n)}, \tag{7}$$

$$k_n'=\sqrt{k_n^2+\frac{1}{4a^2}}. \tag{8}$$

$\varepsilon_n=(\hbar^2/2m)(\pi n/d)^2$ is the energy spectrum of the motion transverse to the contact axis. As a result of the electron reflection at the points $s=\pm\pi a$, where the potential energy is changed, the transmission coefficient oscillates as a function of energy and contact width. Changing the contact size, the maximum value of the electron wave vector k_n along the contact axis is changed. Since the transmission T_n depends on k_n, a fine structure appears in the quantum steps of the conductance (Fig. 2). A similar behavior is obtained, as a function of the applied bias eV (Fig. 3).

For the experimental realization and the observation of the above effects one needs a long and perfectly clean constriction in a 2DEG with smooth boundaries and narrow width $d\sim\lambda_F$, which is adiabatically connected to massive banks. The radius of the curved part, a, is of the order of the Fermi wavelength λ_F at liquid-helium temperatures ($T=10^{-3}$–$10^{-2}\varepsilon_F$). In

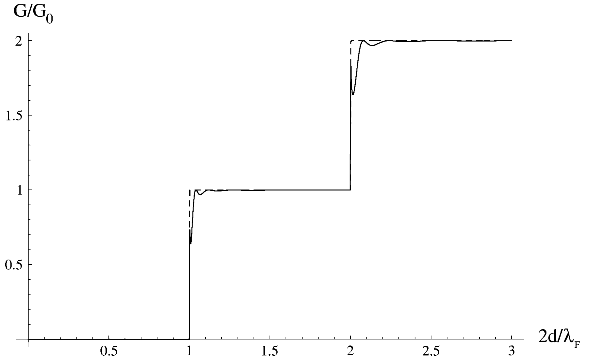

Fig. 2. The normalized conductance G as a function of the normalized constriction width d, $a=0.5\lambda_F$ for solid line, and $a=5\lambda_F$ for dashed line.

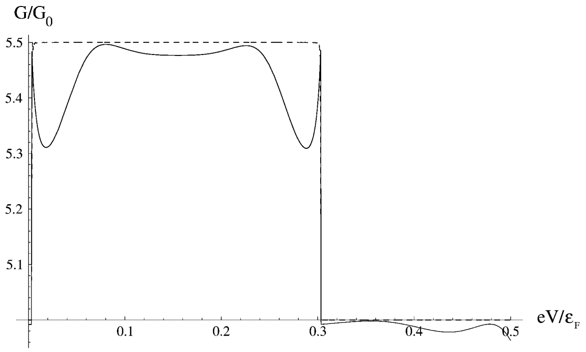

Fig. 3. The voltage dependence of the conductance for $d=3\lambda_F$, $a=0.5\lambda_F$ for solid line and $a=5\lambda_F$ for dashed line.

order to observe the effect of the curvature on the voltage dependence we should have the contact width near but smaller than $n\lambda_F/2$ (n is an integer). Then the new conducting mode can be switched on by a small bias. All these conditions can be realized experimentally.

A similar non-monotonic dependence of conductance steps can be caused by the electron reflection from the ends of the constrictions [21] and by quantum interference processes involving impurity scattering inside the constriction [1], both of which make the interpretation of experimental results difficult. Hence, for the observation of the effect of curvature on the electron reflection, adiabatic connection of the constriction to the banks and absence of electron scattering from impurities are very important. The effect of quantum interference depends on the positions of impurities [22] and will change or disappear after thermal cycling of the sample.

The positive result of the proposed experiment will be the demonstration of the existence of the “geometrical potential”, and the influence of the curved surface on the two-dimensional motion on the electron. The positions of the minima in $G(V,d)$ depend on a and this makes it possible to estimate the radius of curvature of the curved constriction.

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