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A new method of investigating the quantum channel surface

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Abstract
A new method of investigating the quantum channel surface was used to study the germanium quantum well in a SiGe/Ge/SiGe p-type heterostructure, with a hole concentration $p_H = 5.68 \times 10^{11}$ cm^{-2} and mobility $\mu = 4.68 \times 10^4$ cm^2 V^{-1} s^{-1} in magnetic fields up to 15 T and in the temperature range 50 mK–3 K using Shubnikov–de Haas (SdH) oscillations. The method is based on the deviation from theory describing the SdH-related conductivity oscillations. This deviation appears due to extra broadening of the Landau levels, which is attributed to the existence of an inhomogeneous distribution of the carrier concentration in the two-dimensional hole gas layer and, hence, a spread of energy. It is assumed that extra broadening is due to the natural variation of the well width equal to the interatomic distance. The effective hole mass ($m^* = 0.112m_0$) was found from the temperature dependence of the SdH oscillation amplitudes. The quantum scattering time, fluctuations of the carrier concentration and quantum well roughness were estimated from the magnetic field dependence of these oscillation amplitudes.

1. Introduction

There are a number of techniques for studying solid interfaces [1] including modern methods for studying solid–vacuum interfaces, such as x-ray photoelectron spectroscopy, Auger electron spectroscopy, low energy electron diffraction, electron energy loss spectroscopy, ion scattering spectroscopy, secondary ion mass spectrometry, and other surface analysis methods (see for example [2–4]). Optical techniques can be use to study interfaces under a wide variety of conditions: reflection-absorption infrared, surface-enhanced Raman and sum frequency generation [5] spectroscopies can be used to probe solid–vacuum as well as solid–gas, solid–liquid, and liquid–gas surfaces. Recently, scanning-tunneling microscopy and a family of methods descended from it have also become very popular. Two of these are atomic force microscopy [6] and scanning probe microscopy (SPM) [7]. These microscopies have considerably increased the ability and desire of surface scientists to measure the physical structure of many surfaces. However, investigations of the solid–solid interface surface, especially the quantum well (QW) surface in semiconductor heterostructures still have problems. In this paper we propose a new method to investigate the QW structure based on the peculiarities of the Shubnikov–de Haas oscillations (SdHO) arising from high mobility charge carriers in two-dimensional (2D) systems.

2. Sample description

Magnetoquantum effects have been studied for the 2D hole gas formed in the germanium quantum well of a SiGe/Ge/SiGe

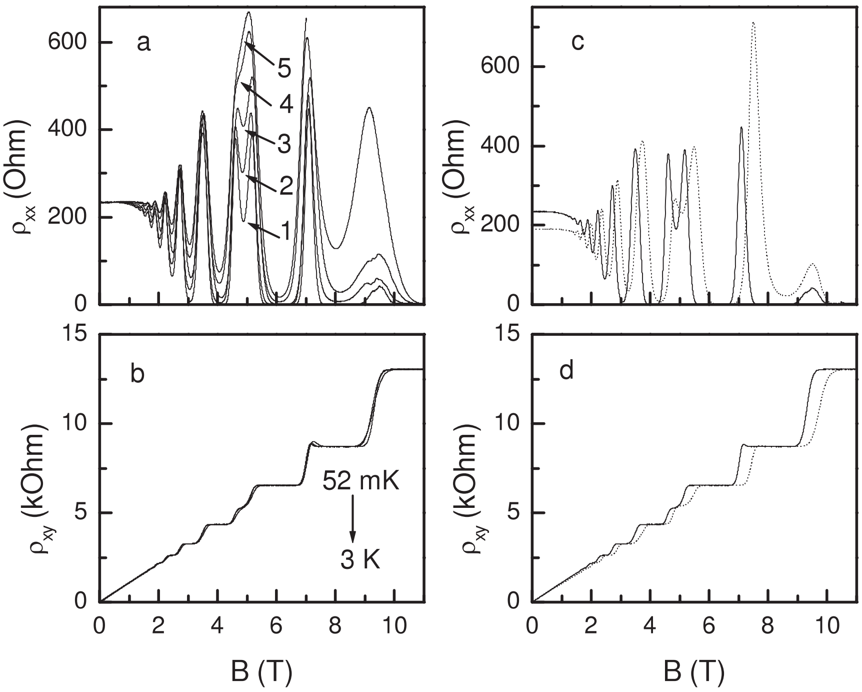

Figure 1. Magnetoresistance components ρ_{xx} and ρ_{xy}, showing the variation with temperature in (a) and (b) for $T = 52$ mK (1), $T = 0.5$ K (2), $T = 0.9$ (3), $T = 2$ K (4), $T = 3$ K (5); and a comparison in (c) and (d) before (solid line) and after LED irradiation (dotted line).

Table 1. Characteristic parameters of the sample.

Property	ρ_{xx} (ohm)	p_{Hall} (cm^{-2}, 10^{11})	p_{ShH} (cm^{-2}, 10^{11})	μ_{Hall} (cm^2 V^{-1} s^{-1})	τ (s, 10^{-12})	ℓ (cm, 10^{-5})	ε_F (meV)
Dark	234	5.81	5.68	4.68×10^4	3.0	5.85	12.52
Illuminated	191	6.17	6.13	5.41×10^4	3.4	6.99	13.95

heterostructure prepared by low energy plasma-enhanced chemical vapor deposition [8]. The germanium quantum well is bordered by $Si_{0.3}Ge_{0.7}$ layers on each side, with 10 nm thick undoped spacers separating the well from regions of boron-doped $Si_{0.3}Ge_{0.7}$. The diagonal and off-diagonal components of the magnetoresistance were measured as functions of the magnetic field strength up to 15 T in the temperature range 50 mK–3 K at the Grenoble High Magnetic Field Laboratory (CNRS), France. The dependences $\rho_{xx}(B)$ and $\rho_{xy}(B)$ taken at the different temperatures are shown in figure 1 (ρ is the resistance per square of a 2D system). The curves display distinct Shubnikov–de Haas oscillations (at $B \geqslant 1$ T) and the quantum Hall effect (at $B \geqslant 2.5$ T).

After a brief illumination of the sample by a red LED, the SdHOs moved to higher magnetic field due to the increasing concentration of carriers in the potential well. The sample parameters found from measuring the resistance (zero magnetic field ρ_{xx}), Hall effect (hole concentration p_{Hall}, mobility μ_{Hall}, transport relaxation time τ, mean free path ℓ, Fermi energy ε_F) and Shubnikov–de-Haas oscillations (hole concentration p_{ShH}) at a temperature of 52 mK before and after illumination are listed in table 1.

3. Investigation of magnetoquantum oscillations

The effective mass m^* and quantum scattering time τ_q can be estimated from the temperature and magnetic field dependences of the SdH oscillation amplitudes ΔR. (ΔR is the deviation of the adjacent resistance maximum, or minimum, from the background mean resistance R_0). The monotonic part of the magnetoresistance (figures 1(a) and (c)) also shows a negative, quadratic field dependence that arises form a quantum correction due to hole–hole interactions [11], but by looking at deviations from this slowly varying background the effect will not disturb the analysis of SdHOs. The change in conductivity of the 2D gas in the regime where quantum effects are important has been considered theoretically in [9, 10] for a single isotropic band. Although the valance band of bulk Ge is isotropic it is doubly degenerate at zone center, with heavy and light holes of effective mass $m^*_{\mathrm{hh}} = 0.28m_0$ and $m^*_{\mathrm{lh}} = 0.044m_0$, respectively, where m_0 is the free electron mass. However, in this 2D system, confinement and strain lift the degeneracy and conduction is entirely by one type of carriers whose effective mass is different from that in the bulk crystal and will be determined below. According to [10] the resistance variation is described as:

$$\rho_{xx} = \frac{1}{\sigma_0}\left[1 + 4\sum_{s=1}^{\infty}\frac{\Psi s}{\sinh \Psi s}\exp\left(\frac{\pi s}{\omega_c \tau_q}\right) \times \cos\left(\frac{2\pi s \varepsilon_F}{\hbar\omega_c} - \Phi\right)\right] \quad (1)$$

where $\Psi = (2\pi^2 k_B T)/\hbar\omega_c$ determines the temperature and magnetic field dependence of the oscillation amplitude, $\omega_c = eB/m^*$ is the cyclotron frequency, τ_q is the quantum (single particle) scattering time of a charge particle that characterizes the collision-induced broadening of the Landau levels, and Φ is the phase. The Fermi energy in the 2D case is $\varepsilon_F = \pi\hbar^2 p/m^*$. The charge carrier concentration p can be found from the oscillatory period in inverse magnetic field when the effective mass m^* is known. Analysis of SdHOs using equation (1) is usually performed by just considering the first harmonic ($s = 1$) which then describes the oscillations as a damped cosine. In practice this gives good results provide we restrict the analysis to oscillations that are not too large, when the minima approach zero and appreciable higher harmonics lead to peak sharpening.

The effective mass m^* and the quantum relaxation time τ_q can be determined from the temperature and magnetic field variation of the SdHO amplitude, by using equation (1) to construct the dependence of $\ln[(\Delta R/R_0)(\sinh\Psi/\Psi)]$ on $1/\omega_c\tau$ or $1/\mu B$ (the exponent in the oscillating term of equation (1) is first transformed to $-\pi\alpha/\omega_c\tau$, where $\alpha = \tau/\tau_q$). According to equation (1), points corresponding to extrema with different quantum numbers ν must fall onto a single straight line. In this case, m^* is a fitting parameter bringing the points taken at different temperatures into coincidence on a single curve. Such coincidence was achieved with $m^* = 0.112m_0$. However, the dependence obtained in figure 2 is, as often found, not a straight line. This deviation from equation (1) means that the parameter α cannot be estimated reliably. Figure 2 illustrates how different lines could be drawn through the experimental points in different magnetic field regions, but we are constrained by the requirement of equation (1) that as $1/\omega_c\tau \to 0$ the curve should approach the value $\ln 4 = 1.386$ and this line would only fit a few points taken at high field.

The nonlinearity of the dependence shown in figure 2 implies a deviation of the magnetic field dependence of the SdHO amplitude from equation (1). It is an indication that some factor changes the behavior of the Landau level broadening and causes an additional magnetic field variation in the oscillation amplitude. Attention has previously been drawn to this nonlinearity in high mobility systems, for instance in [12, 13]. In [13] it has been suggested that the nonlinearity is associated with spatial variations of charge carrier concentration in the plane of the two-dimensional system and, hence, variations in the Fermi energy. As a result, the oscillation extrema occur at different magnetic fields in different areas of the sample. In this case the observed amplitude of the oscillations decreases compared to that in a homogeneous sample—this effect is equivalent to an additional effective broadening of the Landau levels, called

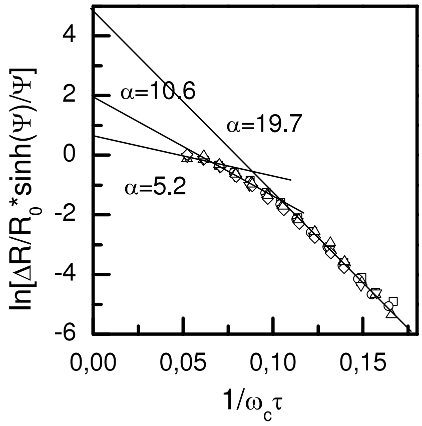

Figure 2. Analysis of experimental data according to equation (1) for temperatures: 52 mK (□), 0.2 K (○), 0.5 K (▽), 1.1 K (△), 2 K (◇) and 3.55 K (◁).

'inhomogeneous broadening'. The formation of SdHOs in the case of such long-range fluctuations of the potential, carrier concentration and Fermi energy was theoretically analyzed by one of the authors of [13] (Shik), assuming a Gaussian distribution of fluctuations. It was shown that an additional exponential factor with the exponent $-(\pi\delta\varepsilon_F/\hbar\omega_c)^2$ (Shik term) appears in the oscillation amplitude of equation (1). The exponential factor in equation (1) then becomes:

$$\exp\left[-\frac{\pi}{\omega_c\tau_q} - \left(\frac{\pi^2\hbar\delta p}{m^*\omega_c}\right)^2\right] \quad (2)$$

where δp is a value describing the concentration fluctuation of the charge carriers. The first term in the exponent of equation (2) describes the collision broadening of the Landau levels and is inversely proportional to the magnetic field; the second term allows for the 'inhomogeneous broadening' of the Landau levels and is inversely proportional to the squared field. Hence, the effect of the second term decreases in very strong magnetic fields. On the other hand, the relative effect of this Shik term increases with the carrier mobility.

Figure 3 illustrates that our experimental data can be very well described by the theory of [13]. Thus we obtain $\alpha = 5.34$ from the linear term (dashed line in figure 3 that is constrained to intercept at $\ln 4$) and $\delta p = 3.8 \times 10^{10}$ cm^{-2} from the quadratic term (straight line in figure 3), which is 6.69% of the average hole concentration $p_H = 5.68 \times 10^{11}$ cm^{-2} obtained from the Hall coefficient. Illumination of the sample by a red LED changes the period and amplitudes of the SdH, but the points corresponding to the extrema with different quantum numbers in coordinates $\ln[\Delta R/R_0 \cdot \sinh\Psi/\Psi]$ on $1/1\omega_c\tau$ fall onto the same line as in dark on figure 3. Hence, the extracted values of $m^* = 0.112m_0$ and $\alpha = 5.34$ remain unchanged and the fluctuation in hole concentration of $\delta p = 4.1 \times 10^{10}$ cm^{-2} is again 6.64% of the, now increased, hole concentration $p_H = 6.13 \times 10^{11}$ cm^{-2}.

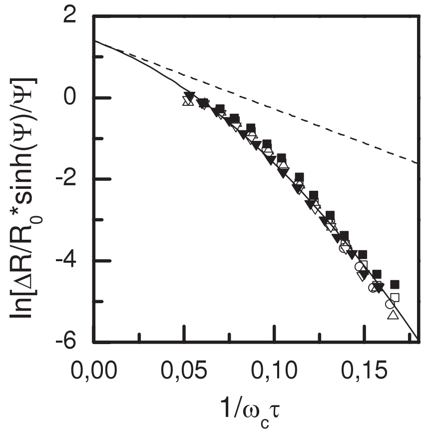

Figure 3. Procedure of numerical calculation of m^* and α, for temperatures: 52 mK(□), 0.2 K (○), 0.5 K (▽), 1.1 K (△), before LED illumination and for temperatures: 0.57 K (■), 1.1 K (▲) after illumination. Dotted line is calculated according to theory [10], straight line according to theory [13].

4. Discussion

When the dominant scattering of holes is short-range, by impurity ions actually in the QW, the quantum lifetime τ_q (time between scattering events) and the transport scattering time τ, which accounts for the angle of scattering in relaxing momentum, are similar and their ratio $\alpha = 1$. If the dominant interaction is long-range, between the holes and the potential of remote ionized impurities, τ is appreciably greater than τ_q and α takes a value of about 10. Our intermediate value of $\alpha = 5.34$ suggests there are few impurities in the Ge layer, but in addition to remote ionized impurities there is another scattering mechanism, which brings the times τ_q and τ closer. We believe that this comes from the holes interacting with the roughness of the well boundaries.

We will now discuss how the carrier concentration fluctuations can also be related to the structure of the quantum well. Along the $Si_{0.3}Ge_{0.7}$–Ge interface that forms the QW boundaries there will be irregularities in the form of monoatomic steps that vary the thickness, L, of the Ge layer in which the holes reside. Since the layer thickness varies *discretely*, namely, by the height of an atomic step, the corresponding quantum confined levels are separated in energy from the ground quantum level of the average layer thickness. We can make an appropriate estimation assuming a rectangular QW for simplicity. The energy levels are [14].

$$E_n = \frac{\pi^2\hbar^2}{2m^*L^2}N^2 \tag{3}$$

where N is the level number. For the intended well width of $L = 150$ Å, the energy of the first level is $E_1 = 13.44$ meV, the second level is $E_2 = 53.77$ meV, and so on. The Fermi energy corresponding to the measured hole concentration is $\varepsilon_F = 12.52$ meV, so only the first quantum level is occupied.

The height of an atomic ledge on the Ge(100) face is 2.8 Å. In regions of the QW where the thickness is smaller by just this value, i.e. $L = 147.2$ Å, the energy of the first quantum level is $E_1 = 13.96$ meV which is 0.52 meV above the energy level for the average thickness. Similarly, QW regions one step wider at $L = 152.8$ Å have an energy $E_1 = 12.9$ meV and the quantum level lies 0.5 meV lower. In each case, the relative variation in energy, and hence hole concentration, is 3.8%, making a total variation in these parameters of 7%, which is close to the value found above from the analysis of SdHO amplitudes. Key support for this conclusion comes from the fact that a similar ratio of $\delta p/p_H$ was found before and after illumination, when filling of the quantum states was different but the structure of the quantum well remain unchanged.

In summary, the analysis outlined here allows one to calculate the carrier concentration fluctuations in the plane of the quantum well and from this information to find the interface roughness that brings about the above-mentioned 'inhomogeneous broadening' of the Landau levels.

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