In memory of Vadim Tkachenko

1937–2023

Vadim was born in Moscow on December 19, 1937. He did not know his father. His mother, Mira Lyubich, died in 1939 at the age of only 22, and Vadim was taken in by his grandfather's family, which soon moved to Kharkiv. His grandfather died in 1941, when Vadim was four. At that point, Vadim's aunt Genya Lyubich, whose husband had been executed in Stalin's purges in 1938, took in her little nephew. In 1949, Genya herself was arrested and soon died in prison. Twelve year old Vadim was left alone with his cousin, his aunt's son, Yuri Lyubich, at that time an eighteen year old undergraduate student at Kharkiv State University.

In 1955, Vadim started his undergraduate studies at the Hydraulic Engineering Department of the Kharkiv Institute of Civil Engineering. In 1956, due to the closure of this faculty, he transferred to the Engineering and Physics Department of the Kharkiv Polytechnic Institute (KhPI), which he graduated in 1961 with honors. Vadim Tkachenko stayed to work at KhPI. At that time, mathematics at KhPI was taught by legendary mathematicians and lecturers Naum Akhiezer, Israel Glazman, Yuri Lyubich, and Vladimir Marchenko. Not surprisingly Vadim soon started working on his graduate degree in mathematics under the guidance of Prof. Glazman.

In 1966, he defended his PhD thesis on "Spectral theorems for some classes of non- self-adjoint operators". In 1969, Vadim joined the Function Theory Department of the Institute for Low Temperature Physics and Engineering of the Academy of Sciences of UkSSR. In parallel, he taught at Kharkiv Polytechnic Institute and Kharkiv State University. In 1980, he defended his doctoral thesis on the topic of "Spectral theory in spaces of analytical functions and functionals". In 1995, Vadim moved to Israel and soon afterward became a professor at Ben-Gurion University of the Negev in Beer Sheva. In his later years, following his retirement and the death of his wife, Renata, Vadim divided his time between Beer Sheva and the San Francisco Bay area.

We briefly highlight some of Vadim's works. In graduate school, Vadim wrote his first paper - a joint work with Ilya Sukharevskii on the asymptotic solution of the Poisson equation in the cylindrical domains whose cross-sections are thin topological annuli with smooth boundary.

In 1964 he published in "Proceedings of the USSR Academy of Sciences" short note on the spectral analysis of non-selfadjoint Hill operators, that is, operators $Ly = -y'' + qy$ with a complex-valued periodic potential q, acting on $L^2(\mathbb{R})$. The specral theory of these operators became Vadim's lifelong passion. He revisited this topic many times both as a sole author and in papers co-authored with Fritz Gesztesy, Leonid Pastur, and Jean-Jacques Sansuc. The culmination of this series of works is the complete description of the Lyapunov function and the spectral data of such operators, discovered by him in the 1990s.

In a series of joint works with Yuri Lyubich published between 1965–1972, Vadim developed a theory of local Laplace transform that had been introduced a few years earlier by Lyubich in his studies of the abstract Cauchy problem

$$\frac{dx(t)}{dt} = Ax(t), \ t \in [0, T], x(0) = x_0,$$

where A is a linear, closed, densely defined operator in a Banach space. The the- ory

developed by Lyubich and Tkachenko, yields, in particular, the completeness theorem for exponential solutions of the abstract Cauchy problem, and a new proof of the Laurent Schwartz fundamental theorem of mean-periodic functions. Closely related intricate approximation and uniqueness problems for solutions of convolution equations in spaces of analytic functions became one of the main areas of Vadim's interest in the 1970-80s.

In the beginning of 1970s Lyubich and Tkachenko introduced and studied counterparts of the classical quasi-analyticity for abstract operators in Banach spaces, replacing differentiation with an unbounded operator in a Banach space with empty spectrum.

During the 1980s, Pastur and Tkachenko developed the spectral theory for Schr¨odinger operators with real-valued limit-periodic potential, which can be significantly fast approximated by periodic ones. The starting point of their work was the spectral theory of Schrödinger operators with periodic potential, developed by Vladimir Marchenko and Iossif Ostrovskii. The "closure" of this theory enabled Pastur and Tkachenko to obtain a complete description of the independent spectral data that uniquely determines the potential and to establish the basic facts of the spectral theory. Some of us still remember the weekly meeting of the Kharkiv complex analysis seminar where Tkachenko spoke about their work. The meeting so well attended that the talk had to be moved to a much larger lecture hall. This work had a significant impact on the further development of the theory of almost periodic operators with Cantor-type spectra.

In a series of joint works with Genrich Belitskii, started in the 1990s, Tkachenko considered various classes of functional equations including the classical equation on implicit functions, Abel, Schr¨oder, cohomological, and their wide generalizations. The questions they investigated included the existence of local and global solutions, obstacles to solvability, and a description of the set of solutions in various classes of functions (such as C^k, quasianalytic, analytic). Some of their work was summa- rized in the book "One-dimensional functional equations," published in 2003. They were planning to write its continuation, "Multi-dimensional functional equations." Unfortunately, the plan for this follow-on publication was never realized.

It is impossible to write about Tkachenko without mentioning his wife, Renata — a philologist, a brilliant storyteller, and the author of wonderful children's verses, which she published under her maiden name, Muha. Renata was first diagnosed with cancer in 1984, and it is to a large extent thanks to Vadim's dedicated support that she lived and continued to work for another 25 years. After her death in 2009, Vadim did much to preserve her memory, creating the website renatamuha.com, which collects her poetry, transcripts and recordings of her performances, and memories from the friends.

Vadim Tkachenko passed away in the San Francisco Bay area on the 25th of July, 2023. Vadim is survived by his two sons, Dmitry, who lives in Kharkiv, and Aleksei, who resides in the San Francisco Bay area with his wife Anna and daughter Rachel.