On the Replica Symmetric Solution for the Sherrington-Kirkpatrick Model

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Abstract

We prove that replica symmetric equations for the free energy and Edvards-Anderson order parameter for the Sherrington-Kirkpatrick model with Gaussian magnetic field hold above some line on the $T-h$ plane. This line coincides with AT-line at the point $h=0$ and behave similarly as $T \to 0$.

1 Introduction

Many interesting models in modern physics admit generalizations in which some parameter, whose value in the initial model is, by its nature fixed, is regarded as a free and is allowed, in particular, to take large values. It was found rather useful to study the behaviour of the model in the asymptotic regime when the value of such a parameter tends to infinity and to construct the limiting model or even the corresponding asymptotic expansion.

The oldest and the best known example of such a parameter is the interaction radius R. It was understood in 1950s and proved in 1970s (see [1]), that many realistic models of statistical physics in the limit of large R are equivalent to the Curie-Weiss model, which can be solved exactly. Hence it was naturally to expect that realistic models of the spin glass theory can be studied in the limit $R \to \infty$ by using so-called Sherrington-Kirkpatrick (SK) model, introduced by Sherrington and Kirkpatrick in 1975 ([2]) as a mean field model of spin glass.

$$H = -\frac{1}{\sqrt{N}} \sum_{1=i<j}^{N} J_{ij}\sigma_i\sigma_j - \sum_{i=1}^{N} h_i\sigma_i \tag{1}$$

By using so-called replica trick, Sherrington and Kirkpatrick [2] found the following expression for the mean free energy in the thermodynamic limit:

$$\beta f_{SK} = -(\beta J)^2/4(1-q)^2 - \frac{1}{\sqrt{2\pi}} \int \log 2\cosh(\beta J q^{1/2} u + \beta h_1) e^{-u^2/2} du d\mu(h_1), \tag{2}$$

$$q = \frac{1}{\sqrt{2\pi}} \int \tanh^2(\beta J q^{1/2} u + \beta h_1) e^{-u^2/2} du d\mu(h_1), \tag{3}$$

where β is the inverse temperature. However this "SK solution" cannot be correct in the most interesting low temperature region, since it does not satisfy general and important requirements such as nonnegativity of the entropy and magnetic susceptibility, some stability conditions etc.

The SK model has been considered in numerous physical papers (see e.g. book [3] and references therein), in which the rich and complex structure of this model was discovered and studied. The physical theory developed contains a number of new fundamental concepts and facts, which have no analogs in nonrandom systems and can be applied to a wide range of complex systems. According to the Parisi theory [3], the SK model has some new type phase transition which occurs when we cross so-called Almeida- Touless (AT) line $T_c(h) = \beta_c^{-1}(h)$ at the $T-h$ -plane (here and below T is the temperature and h is the variance of the external magnetic field).

$$\frac{(\beta_c J)^2}{\sqrt{2\pi}} \int \cosh^{-4}(\beta_c J q^{1/2} u + \beta_c h_1) e^{-u^2/2} du d\mu(h_1) = 1 \tag{4}$$

Above this line the free energy of the SK model has replica symmetric form (2), the Edvards-Anderson parameter

$$q_N = \frac{1}{N} \sum_i < \sigma >_i^2 \tag{5}$$

becomes nonrandom in the thermodynamic limit and its limiting value q is a solution of equation (3). But below the AT line the Edwards-Anderson order parameter is random and its distribution is a solution of rather complicated variational problem which includes the nonlinear partial differential equation.

Unfortunately, all these results have been obtained by using so-called replica trick, which is not rigorous from mathematical point of view. The problem of rigorous justification of the Parisi theory is still open.

Let us mention some mathematical results known in this field. One of the first results has been obtained in the paper [5]. It was shown that for $T > J$ and zero external field ($h = 0$) the partition function Z_N of the SK model has the "strong selfaveraging property": $E(N^{-1} \log Z_N) = N^{-1} \log E(Z_N) + o(1)$ where N (the number of spins) tends to infinity. Thus there is no phase transition in the high temperature region $T \geq J$. The main disadvantage of the method of this paper is that it is not applicable to the model with external magnetic field and moreover cannot be extended to low temperatures $T < J$. Similar result was obtained in [6] for the case $T << J$. The selfaveraging property of the free energy was proved in [7]. Here the idea to use the martingale differences method was proposed. The same idea has been used later to prove the selfaveraging of the free energies of a number of others mean-field type models (see e.g.[16], [8]). In the paper [9] similar method was used to obtain the large deviation type bounds for the free energy of the SK and the Hopfield models.

Interesting rigorous results were obtained in the papers [11]-[13]. In these papers it was proved that there exists some nonempty set of functions $0 \leq x(q) \leq 1$ such that the SK free energy can be expressed in terms of the solution of a non linear partial differential equation, which is the same as that found by Parisi by means of the replica trick.

Some rigorous results about validity of the replica symmetric solution (2), (3) in the high temperature field were obtained resently in [14].

A method, relating the selfaveraging property of the Edwards-Anderson order parameter and the replica symmetry solution for this model was proposed in [7], [15]. Since this result is important for us we formulate it below

Theorem 1 *Consider the SK model with the Hamiltonian (1) where* J_{ij}, $1 \leq i < j \leq N$ *are independent identically distributed random variables with zero mean, variance* J^2 *and bounded third moments*

$$E(|J_{ij}|^3) \leq C < \infty \tag{6}$$

and h_i, $i = 1, \ldots, N$ *are independent Gaussian random variables with zero mean and variance* h^2.

If the Edwards-Anderson parameter of the model (5) is selfaveraging, i.e. it satisfies the condition

$$\Delta_N \equiv E\{(q_N - E\{q_N\})^2\} \to 0 \quad as \quad N \to \infty, \tag{7}$$

for values of J, β, h *belonging to some intervals* $J \in (J_0, J_0 + \epsilon)$, $\beta \in (\beta_0, \beta_0 + \epsilon)$ *and* $h \in (h_0, h_0 + \epsilon)$, $\epsilon > 0$, *then the mean free energy* $E\{f_N\}$ *of the model coincides in the thermodynamic limit* $N \to \infty$ *with SK ("replica symmetric") expression (2), (3).*

Let us remark, that the statement of Theorem 1 is that the selfaveraging of the Edwards-Anderson order parameter is a sufficient condition for the validity of the replica symmetry solution. Since we know (see [3]) that the SK expression for the free energy gives the negative entropy in the low temperature region and therefore cannot be valid in this region, then we can rigorously derive from this theorem the fact that the Edwards-Anderson order parameter is not selfaveraging in this region.

The main result of the present paper is

Theorem 2 *Consider the SK model of the form (1) under the conditions of Theorem 1. Let the following condition be fulfilled at some point* (J, β, h)

$$C(\beta, h) \equiv \frac{(\beta J)^2}{\sqrt{2\pi}} \int_0^1 d\xi \int\int du d\mu(h_1) e^{-u^2/2} \cosh^{-4}(\beta J (q\xi)^{1/2} u + \beta h_1 2) \leq 1 \tag{8}$$

where q *is the solution of the replica symmetric equation (3). Then the mean free energy* $E\{f_N\}$ *of the model coincides in the thermodynamic limit* $N \to \infty$ *with SK ("replica symmetric") expression (2), (3).*

Remarks. 1. Comparing our result with AT-equation (4), one can see that they coincide only if $q = 0$, i.e. if $h = 0$ and $\beta \leq J^{-1}$. But Theorem 2 implies also, that replica symmetric equations hold for any h if $\beta < J^{-1}$.
2. Another important corollary of Theorem 2 is that for any inverse temperature β the replica symmetric equations hold if the field is large enough $h > h^*(\beta)$, and the behaviour of $h^*(\beta)$ as $\beta \to \infty$ is similar to that for the AT-expression.
3. The method proposed in this paper is applicable also to the Hopfield model. By using this method, the following result has been obtained for the Hopfield model (similar results were obtained recently in [18], [19]).

Theorem 3 *Consider the Hopfield model of the form:*

$$H_1 = -\frac{1}{2N}\sum_{\mu=1}^{p}\sum_{i,j=1}^{N}\xi_i^\mu\xi_j^\mu\sigma_i\sigma_j - h^1\sum_{i=1}^{N}\xi_i^1\sigma_i - \varepsilon\sum_{\mu=1}^{p}\gamma^\mu N^{-1/2}\sum_{i=1}^{N}\xi_i^\mu\sigma_i,$$

where $\xi_i^\mu = \pm 1$, $i = 1,\dots,N$, $\mu = 1,\dots,p$ are independent random variables with zero mean, γ^μ are independent Gaussian variables with zero mean and variance 1, ε and h^1 are positive parameters, and $p \to \infty$ as $N \to \infty$ so that $p/N \to \alpha$. Define

$$q_N = \frac{1}{N}\sum_{i=1}^{N} < \sigma_i >^2, \qquad r_N = \frac{1}{pN}\sum_{\mu=1}^{p}\left(\sum_{i=1}\langle\xi_i^\mu\sigma_i\rangle\right)^2, \qquad m_N^1 = \frac{1}{N}\sum_{i=1}\xi_i^1\langle\sigma_i\rangle.$$

Let the following condition be fulfilled at some point $(\alpha,\beta,\varepsilon,h^1)$

$$\frac{(\alpha\beta)^2}{(1-\beta(1-q))^2}(1+8r)\int d\zeta\max_u E\{\int\frac{dve^{-v^2/2}}{\sqrt{2\pi}}\cosh^{-4}(\beta\sqrt{\alpha\zeta r(\varepsilon)}(v+u)+(m+h^1)\xi_1^1)\} \le 1,$$

where r, $r(\varepsilon)$, q and m^1 are solutions of the replica symmetric system of equations

$$\begin{aligned} m^1 &= E\left\{\int\frac{dv\exp(-\frac{v^2}{2})}{\sqrt{2\pi}}\,\xi_1^1\tanh\beta(\sqrt{\alpha r(\varepsilon)}v + (m^1+h^1)\xi_1^1)\right\}, \\ r &= \frac{q+\varepsilon^2\beta^2(1-q)^2}{(1-\beta(1-q))^2}, \\ q &= E\left\{\int\frac{dv\exp(-\frac{v^2}{2})}{\sqrt{2\pi}}\,\tanh^2\beta(\sqrt{\alpha r(\varepsilon)}v + (m^1+h^1)\xi_1^1)\right\}, \end{aligned} \tag{9}$$

with

$$r(\varepsilon) = r + \frac{2\beta\varepsilon^2}{1-\beta(1-q)} + \varepsilon^2.$$

Then the variances of q_N and r_N vanish as $N \to \infty$, there exist the limits as $N \to \infty$ for $E\{q_N\}$, $E\{r_N\}$ and $E\{m_N^1\}$, and these limits coincide with solutions of the replica symmetric system (9).

2 Proof of the main result.

An important property, which we use to prove Theorem 2, is given by the lemma:

Lemma 1 *Consider two sequences of convex random functions $\{f_n(t)\}_{n=1}^\infty$ and $\{g_n(t)\}_{n=1}^\infty$ ($g_n'' \le 0$, $f_n'' \le 0$), the mean values of which have common limit.*

$$\lim_{n\to\infty} E\{f_n(t)\} = \lim_{n\to\infty} E\{g_n(t)\} = f(t).$$

If functions f_n and g_n are selfaveraging, i.e.

$$\lim_{n\to\infty} E\{(f_n(t) - E\{f_n(t)\})^2\} = \lim_{n\to\infty} E\{(g_n(t) - E\{g_n(t)\})^2\} = 0,$$

then for all point t, where $f'(t)$ is continuous

$$\begin{aligned}
\lim_{n\to\infty} E\{f_n'(t)\} = \lim_{n\to\infty} E\{g_n'(t)\} = f'(t), \\
\lim_{N\to\infty} E\left\{\left(\tfrac{d}{dt}f_n(t) - f'(t)\right)^2\right\} = 0, \\
\lim_{N\to\infty} E\left\{\left(\tfrac{d}{dt}g_n(t) - f'(t)\right)^2\right\} = 0,
\end{aligned} \tag{10}$$

i.e. the derivatives $f_n'(t)$ and $g_n'(t)$ are also convergent, selfaveraging ones and have common limit $f'(t)$ for almost all t.

Proof. The first line of (10) follows from the Griffitz lemma [20], according to which the sequence of derivatives $E\{f_n'(t)\}$ and $E\{g_n'(t)\}$ of the convergent sequence of convex functions $E\{f_n(t)\}$ and $E\{g_n(t)\}$ converges to the derivative $f'(t)$ of the limiting function $f(t)$ for all points t of continuity of $f'(t)$. The proof of the selfaveraging properties (10) is based on the following inequalities resulting from the convexity of $f_n(t)$, $g_n(t)$:

$$\begin{aligned}
\frac{f_n(t) - f_n(t-\epsilon_1)}{\epsilon_1} \geq f_n'(t) \geq \frac{f_n(t+\epsilon_1) - f_n(t)}{\epsilon_1}, \\
\frac{g_n(t) - g_n(t-\epsilon_1)}{\epsilon_1} \geq g_n(t) \geq \frac{g_n(t+\epsilon_1) - g_n(t)}{\epsilon_1}.
\end{aligned}$$

By using these inequalities and the selfaveraging properties of the functions $f_n(t)$ and $g_n(t)$, one can easily prove (10).

Remark. We are going to apply this lemma to the sequences of free energies, which are evidently convex functions with respect to the parameter J and h. But since we cannot prove that the free energy of the SK model for any J, h has the limit when $N \to \infty$ we use the following trick. According to the Helly theorem, one can chose the subsequence N_n such that there exists $\lim_{n\to\infty} E\{f(H_{N_n}(J,h))\}$. We apply Lemma 1 to this subsequence to prove that its derivatives with respect to J and h are selfaveraging for almost all h, J. But finally we prove that the limit of this subsequence coincides with SK expression (2). And since it can be done for any convergent subsequence, one can conclude that $E\{f(H_N(J,h))\}$ (at least in the field of parameters, which we study) has the limit equal to the SK expression (2). However, to simplify notations everywhere below we omit the subindex n.

The other very important tool in our proof is the formula of integration by parts, which is valid for any differentiable function φ and Gaussian variable X with zero mean.

$$E\{X\varphi(X)\} = E\{X^2\}E\{\frac{d\varphi(X)}{dX}\}. \tag{11}$$

The analogue of this formula for nongaussian case, which allows us to operate with variables J_{ij} like with Gaussian ones, is the following estimate, valid for any differentiable functions $\varphi(N^{-1/2}\mathbf{J})$, with $\mathbf{J} = \{J_{ij}\}_{i<j}$ and different $J_{i_1j_1}, \ldots, J_{i_kj_k}$ which satisfy condition (6)

$$E\{J_{i_1j_1}\ldots J_{i_kj_k}\varphi(N^{-1/2}\mathbf{J})\} = J^{2k}E\{\frac{\partial}{\partial J_{i_1j_1}\ldots\partial J_{i_kj_k}}\varphi(\sigma, N^{-1/2}\mathbf{J})\} + O(N^{-(k+1)/2}). \tag{12}$$

To prove Theorem 2 we obtain the upper bound for Δ_N defined by formula (7). Due to the symmetry of the initial Hamiltonian (1) with respect to variables σ_i, one can see that

$$\Delta_N = E\{\langle\sigma_1\rangle^2 \cdot (q_N - \overline{q}_N)\} = E\{\langle\sigma_1\rangle^2 \cdot \dot{q}'_{N-1}\} + O(N^{-1}), \tag{13}$$

where

$$q'_{N-1} = N^{-1}\sum_{i=2}^{N}\langle\sigma_i\rangle^2, \qquad \dot{q}'_{N-1} = q'_{N-1} - \overline{q}_N, \qquad \overline{q}_N \equiv E\{q_N\}.$$

Consider a system of $N-1$ spins $\sigma_2, \ldots, \sigma_N$ with a Hamiltonian obtained from (1) by replacing the spin σ_1 with a continuously varying parameter $\pm\sqrt{\tau}$. We "forget" for a moment the term $h_1\sigma_1$ because it gives only some constant to be added to all our computations. Thus we introduce two Hamiltonians of $N-1$ spins:

$$\begin{aligned} H_+(\tau) &= -\tfrac{1}{2\sqrt{N}}\textstyle\sum_{i,j=2}^N J_{ij}\sigma_i\sigma_j - \sum_{i=2}^N h_i\sigma_i - \frac{\sqrt{\tau}}{\sqrt{N}}\sum_{i=2}^N J_{1i}\sigma_i, \\ H_-(\tau) &= -\tfrac{1}{2\sqrt{N}}\textstyle\sum_{i,j=2}^N J_{ij}\sigma_i\sigma_j - \sum_{i=2}^N h_i\sigma_i + \frac{\sqrt{\tau}}{\sqrt{N}}\sum_{i=2}^N J_{1i}\sigma_i. \end{aligned} \tag{14}$$

Let $Z_+(\tau)$, $Z_-(\tau)$ be partition functions and $\langle\ldots\rangle_{+\tau}$, $\langle\ldots\rangle_{-\tau}$ the Gibbs averages corresponding to the Hamiltonians $H_+(\tau)$ and $H_-(\tau)$ respectively. Let us introduce also

$$\begin{aligned} q_+(\tau) &= N^{-1}\textstyle\sum_{i=2}^N\langle\sigma_i\rangle_{+\tau}^2, \qquad \dot{q}_+(\tau) = q_+(\tau) - \overline{q}_N, \\ q_-(\tau) &= N^{-1}\textstyle\sum_{i=2}^N\langle\sigma_i\rangle_{-\tau}^2, \qquad \dot{q}_-(\tau) = q_-(\tau) - \overline{q}_N, \\ q_\pm(\tau) &= N^{-1}\textstyle\sum_{i=2}^N\langle\sigma_i\rangle_{+\tau}\langle\sigma_i\rangle_{-\tau}, \qquad \dot{q}_\pm(\tau) = q_\pm(\tau) - \overline{q}_N. \end{aligned} \tag{15}$$

The following lemma establishes the connections between the properties of H_N and $H_\pm(\tau)$.

Lemma 2 *For almost all h, J the following relations hold for any $0 \le \tau < 1$:*

$$E\{(q_+(\tau))^n\} = E\{q_N^n\} + o(1), \qquad (n = 1, 2), \tag{16}$$

$$\begin{aligned} 2E\{N^{-2}\textstyle\sum_{i,j=2}^N\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}\langle\sigma_i\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}\} &= \Delta_N + o(1), \\ E\{N^{-2}\textstyle\sum_{i,j=2}^N\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}^2\} &= \Delta_N + o(1), \end{aligned} \tag{17}$$

where Δ_N is defined by (7), and in addition

$$E\{N^{-2}\sum_{i,j=2}^N\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}h_ih_j\} = o(1) \tag{18}$$

with $\dot{\sigma}_i = \sigma_i - \langle\sigma_i\rangle_{+\tau}$.

Remarks. 1. Let us note that relation (18) means that for almost all J and h

$$N^{-1}\sum_{i=1}^{N}\dot{\sigma}_i h_i \to 0, \quad as \quad N\to\infty \tag{19}$$

in the Gibbs measure and in probability.
2. By changing $J_{1i}\to -J_{1i}$ one can easily derive all statement of Lemma 2 for the Hamiltonian $H_-(\tau)$.

Proof. To prove Lemma 2 we use Lemma 1 for the sequences $f_N(H_N(J,h))$ and $f_N(H_+(\tau;J,h))$. It is evident that any their subsequences have the same limit, therefore their derivatives are selfaveraging at the same h, J. Relations (10) imply that

$$E\{\frac{1}{N}\sum_{i=2}\langle\sigma_i\rangle_{+\tau}^2(\frac{1}{N}\sum_{j=2}h_j\langle\sigma_j\rangle_{+\tau} - E\{\frac{1}{N}\sum_{j=2}h_j\langle\sigma_j\rangle_{+\tau}\})\}\to 0.$$

Integrating by parts with respect to h_i, we obtain

$$2E\{N^{-2}\sum_{i,j=2}^{N}\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}\langle\sigma_i\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}\} = \Delta_N(\tau)+o(1), \tag{20}$$

where

$$\Delta_N(\tau) = E\{(q_+(\tau)-E\{q_+(\tau)\})^2\}.$$

Similarly

$$E\{N^{-2}\sum_{i,j=2}^{N}\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}^2\} = \Delta_N(\tau)+o(1). \tag{21}$$

Integrating by parts the l.h.s. of (18) and using (20) and (21), we obtain (18).
Moreover, (20) and (21) and their analogs for $H_N(J,h)$ imply that

$$\begin{gathered}\frac{d}{dJ}E\{f_N(H_+(\tau;J,h))\} = \beta JE\{N^{-2}\sum_{i,j=2}^{N}\langle\sigma_i\sigma_j\rangle_{+\tau}^2 - 1\} = \\ \beta JE\{(q_N(\tau))^2-1\} + \beta JE\{N^{-2}\textstyle\sum_{i,j=2}^{N}\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}^2\} + \\ 2\beta JE\{N^{-2}\textstyle\sum_{i,j=2}^{N}\langle\dot{\sigma}_i\dot{\sigma}_j\rangle_{+\tau}\langle\sigma_i\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}\} = \\ \beta JE\{(q_N(\tau))^2-1\} + 2\beta JE\{(q_N(\tau))^2\} - 2\beta J(\overline{q}_N(\tau))^2 + o(1).\end{gathered} \tag{22}$$

By the same way we obtain

$$\frac{d}{dJ}E\{f_N(H_N(J,t))\} = \beta JE\{(q_N^2-1)\} + 2\beta JE\{q_N^2\} - 2\beta J\overline{q}_N^2 + o(1) \tag{23}$$

Since, on the other hand, we have that

$$\frac{d}{dh}E\{f_N(H_N(J,h))\} = h\beta(\overline{q}_N-1), \qquad \frac{d}{dh}E\{f_N(H_+(J,h))\} = h\beta(\overline{q}_N(\tau)-1), \tag{24}$$

the first statement of Lemma 1 applied to the derivatives with respect to J and h gives us (16). Combining (16) with (20) and (21), we prove (17).

Lemma 2 is proved.

To proceed further we introduce the variable

$$u(\tau) = \frac{1}{2} \log \frac{Z_+(\tau)}{Z_-(\tau)}. \tag{25}$$

One can easily see that

$$\langle \sigma_1 \rangle = \frac{Z_+(1)e^{\beta h_1} - Z_-(1)e^{-\beta h_1}}{Z_+(1)e^{\beta h_1} + Z_-(1)e^{-\beta h_1}} = \tanh(u(1) + \beta h_1) \tag{26}$$

and similarly

$$\begin{gathered} \dot{q}'_{N-1} = \frac{\dot{q}_+(1)Z_+^2(1)e^{2\beta h_1} + \dot{q}_-(1)Z_-^2(1)e^{-2\beta h_1} + 2\dot{q}_\pm(1)Z_+(1)Z_-(1)}{(Z_+(1)e^{\beta h_1} + Z_-(1)e^{-\beta h_1})^2} = \\ \dot{q}_+(1)\frac{e^{4u(1)+2\beta h_1}}{(e^{2u(1)+\beta h_1} + e^{-\beta h_1})^2} + \\ \dot{q}_-(1)\frac{e^{-2\beta h_1}}{(e^{2u(1)+\beta h_1} + e^{-\beta h_1})^2} + 2\dot{q}_\pm(1)\frac{e^{2u(1)}}{(e^{2u(1)+\beta h_1} + e^{-\beta h_1})^2}. \end{gathered} \tag{27}$$

Hence to study the r.h.s. of (13) it would be very useful to study the behaviour of the functionals

$$\begin{aligned} \Phi_+(\phi_1, \tau) &= E\{\dot{q}_+(\tau)\phi_1(u(\tau))\}, \\ \Phi_-(\phi_2, \tau) &= E\{\dot{q}_-(\tau)\phi_2(u(\tau))\}, \\ \Phi_\pm(\phi_3, \tau) &= E\{\dot{q}_\pm(\tau)\phi_3(u(\tau))\}, \end{aligned} \tag{28}$$

which are defined for any smooth enough functions $\phi_1(u)$, $\phi_2(u)$, $\phi_3(u)$, satisfying the conditions:

$$\begin{gathered} ||\phi_{1,2,3}(u)|| \equiv E^{1/2}\{\phi_{1,2,3}^2(u)\} < \infty, \\ ||\phi'_{1,2,3}(u)|| < \infty, \qquad ||\phi''_{1,2,3}(u)|| < \infty. \end{gathered} \tag{29}$$

To this end we compute

$$\begin{gathered} \tfrac{d}{d\tau}\Phi_+(\phi_1, \tau) = E\{\tfrac{\beta}{\sqrt{\tau}}\textstyle\sum_{i,j=2}^N N^{-3/2} J_{1i}\langle \dot{\sigma}_i \dot{\sigma}_j\rangle_{+\tau}\langle \sigma_j\rangle_{+\tau}\phi_1(u)\} + \\ E\{\tfrac{\beta}{4\sqrt{\tau}}\textstyle\sum_{i=2}^N N^{-1/2} J_{1i}\dot{q}_+(\langle \sigma_i\rangle_{+\tau} + \langle \sigma_i\rangle_{-\tau})\phi_1'(u)\} \end{gathered} \tag{30}$$

Denote by $I_1^{(1)}$ and $I_1^{(2)}$ the first and the second terms in the r.h.s. of (30) respectively. Then, using the integration by parts with respect to J_{1i} (11) or its analogue (12) for nongaussian case, and the relations

$$\begin{aligned} \frac{d}{dJ_{1i}}\langle \dots \rangle_{+\tau} &= \frac{\sqrt{\tau}}{N^{1/2}}\frac{d}{dh_i}\langle \dots \rangle_{+\tau} \\ \frac{d}{dJ_{1i}}\langle \dots \rangle_{-\tau} &= -\frac{\sqrt{\tau}}{N^{1/2}}\frac{d}{dh_i}\langle \dots \rangle_{-\tau}, \end{aligned} \tag{31}$$

we obtain

$$I_1^{(1)} = \tfrac{\beta J^2}{N^2} E\{\phi_1(u) \textstyle\sum_{i,j=2}^N \tfrac{d}{dh_i}(\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau})\}+$$
$$\tfrac{(\beta J)^2}{2N^2} E\{\phi_1'(u) \textstyle\sum_{i,j=2}^N \langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}(\langle\sigma_i\rangle_{+\tau}+\langle\sigma_i\rangle_{-\tau})\}. \tag{32}$$

On the other hand, on the basis of Lemma 2, we conclude that

$$\frac{(\beta J)^2}{hN^2} E\{\sum_{i,j=2}^N h_i\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}\phi_1(u)\} = o(||\phi_1||)$$

Using integration by parts with respect to h_i, we have that

$$\tfrac{\beta J^2}{hN^2} E\{\phi_1(u) \textstyle\sum_{i,j=2}^N \tfrac{d}{dh_i}(\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_i\rangle_{+\tau})\}+$$
$$\tfrac{(\beta J)^2}{2N^2} E\{\phi_1'(u) \textstyle\sum_{i,j=2}^N \langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}(\langle\sigma_i\rangle_{+\tau}-\langle\sigma_i\rangle_{-\tau})\} = o(||\phi_1||). \tag{33}$$

Hence, subtracting (33) from (32), we obtain

$$I_1^{(1)} = \frac{(\beta J)^2}{N^2} E\{\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}\langle\sigma_i\rangle_{-\tau}\phi_1'(u)\} + o(||\phi_1||) \tag{34}$$

Using a similar technique, we derive that

$$I_1^{(2)} = \tfrac{(\beta J)^2}{2N^2} E\{\textstyle\sum_{i,j=2}^N \langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}(\langle\sigma_i\rangle_{+\tau}+\langle\sigma_i\rangle_{-\tau})\phi_1'(u)\}+$$
$$\tfrac{(\beta J)^2}{4} E\{\dot q_+(q_- - q_+)\phi_1'(u)\} + \tfrac{(\beta J)^2}{8N} E\{\textstyle\sum_{i=2}^N \dot q_+(\langle\sigma_i\rangle_{+\tau}+\langle\sigma_i\rangle_{-\tau})\phi_1''(u)\} \tag{35}$$

On the other hand, since according to Lemma 1

$$E\{(N^{-1}\textstyle\sum_{i=2}^N h_i\langle\sigma_i\rangle_{+\tau} - h\beta(1-\overline{q}_N))^2\} = o(1),$$
$$E\{(N^{-1}\textstyle\sum_{i=2}^N h_i\langle\sigma_i\rangle_{-\tau} - h\beta(1-\overline{q}_N))^2\} = o(1),$$

we have that

$$\frac{\beta J^2}{4t} E\{\dot q_+(N^{-1}\sum_{i=2}^N h_i(\langle\sigma_i\rangle_{+\tau}-\langle\sigma_i\rangle_{-\tau})\phi_1'(u)\} = o(||\phi_1'||).$$

Integrating with respect to h_i, we find

$$\tfrac{(\beta J)^2}{2N^2} E\{\phi_1'(u) \textstyle\sum_{i,j=2}^N \langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau}\langle\sigma_j\rangle_{+\tau}(\langle\sigma_i\rangle_{+\tau}-\langle\sigma_i\rangle_{-\tau})\}+$$
$$\tfrac{(\beta J)^2}{4} E\{\dot q_+(q_- - q_+)\phi_1'(u)\} + \tfrac{(\beta J)^2}{8N} E\{\textstyle\sum_{i=2}^N \dot q_+(\langle\sigma_i\rangle_{+\tau}+\langle\sigma_i\rangle_{-\tau})\phi_1''(u)\} = o(||\phi_1'||). \tag{36}$$

Subtracting (36) from (35), we obtain

$$I_1^{(2)} = \tfrac{(\beta J)^2}{N^2} E\{\phi_1'(u) \textstyle\sum_{i,j=2}^N \langle\sigma_j\rangle_{+\tau}\langle\sigma_i\rangle_{-\tau}(\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau})\} + \tfrac{(\beta J)^2}{2} E\{\dot q_+ q_\pm \phi_1''(u)\} + o(||\phi_1||) + o(||\phi_1'||). \tag{37}$$

Combining (34) and (37), we find

$$\tfrac{d}{d\tau}\Phi_+(\phi_1,\tau) = \tfrac{2(\beta J)^2}{N^2} E\{\textstyle\sum_{i,j=2}^N \langle\sigma_j\rangle_{+\tau}\langle\sigma_i\rangle_{-\tau}(\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau})\phi_1'(u)\} + \tfrac{(\beta J)^2}{2} E\{\dot q_+ q_\pm \phi_1''(u)\} + o(||\phi_1||) + o(||\phi_1'||). \tag{38}$$

Finally, using the relation

$$\tfrac{2}{N^2} E\{\textstyle\sum_{i,j=2}^N \langle\sigma_j\rangle_{+\tau}\langle\sigma_i\rangle_{-\tau}(\langle\dot\sigma_i\dot\sigma_j\rangle_{+\tau})\phi_1'(u)\} = E\{\dot q_+\dot q_-\phi_1'(u)\} + \tfrac{1}{2}E\{\dot q_+(\dot q_- - \dot q_\pm)\phi_1''(u)\} + o(||\phi_1'||), \tag{39}$$

which one can derive integrating by parts with respect to h_i the l.h.s. of the identity

$$E\{\dot q_+ N^{-1}\phi_1'(u)\sum_{i=2}^N h_i\langle\sigma_i\rangle_{-\tau}\} = E\{\dot q_+ N^{-1}\phi_1'(u)\}E\{\sum_{i=2}^N h_i\langle\sigma_i\rangle_{-\tau}\} + o(||\phi_1'||), \tag{40}$$

we obtain

$$\frac{d}{d\tau}\Phi_+(\phi_1,\tau) = \frac{(\beta J)^2\overline{q}_N}{2}\Phi_+(\phi_1'',\tau) + (\beta J)^2 E\{\dot q_+\dot q_-(\tfrac{1}{2}\phi_1''(u) + \phi_1'(u))\} + o(||\phi_1||) + o(||\phi_1'||). \tag{41}$$

By using a similar technique, one can find also that

$$\frac{d}{d\tau}\Phi_-(\phi_2,\tau) = \frac{(\beta J)^2\overline{q}_N}{2}\Phi_-(\phi_2'',\tau) + (\beta J)^2 E\{\dot q_+\dot q_-(\tfrac{1}{2}\phi_2''(u) - \phi_2'(u))\} + o(||\phi_2||) + o(||\phi_2'||), \tag{42}$$

and

$$\frac{d}{d\tau}\Phi_\pm(\phi_1,\tau) = \frac{(\beta J)^2\overline{q}_N}{2}\Phi_\pm(\phi_3'',\tau) + (\beta J)^2 E\{\dot q_+\dot q_-(\tfrac{1}{2}\phi_3''(u) - 2\phi_3(u))\} + o(||\phi_3||) + o(||\phi_3'||), \tag{43}$$

where the functionals $\Phi_-(\phi_2,\tau)$ and $\Phi_\pm(\phi_3,\tau)$ are defined by the relations (28).

Let us introduce notations:

$$\begin{aligned} p_+(\tau,u) &= E\{\dot q_+\delta(u(\tau)-u)\}, \\ p_-(\tau,u) &= E\{\dot q_-\delta(u(\tau)-u)\}, \\ p_\pm(\tau,u) &= E\{\dot q_\pm\delta(u(\tau)-u)\}, \\ p(\tau,u) &= E\{\dot q_+\dot q_-\delta(u(\tau)-u)\}. \end{aligned} \tag{44}$$

Then relations (41)-(43) can be rewritten in terms of these functions as follows

$$
\begin{aligned}
&\frac{d}{d\tau}\int \phi_1(u)p_+(\tau,u)du = \frac{(\beta J)^2\overline{q}_N}{2}\int \phi_1''(u)p_+(\tau,u)du+ \\
&(\beta J)^2\int p(\tau,u)(\frac{1}{2}\phi_1''(u)+\phi_1'(u))du + o(||\phi_1||)+o(||\phi_1'||), \\
&\frac{d}{d\tau}\int \phi_2(u)p_-(\tau,u)du = \frac{(\beta J)^2\overline{q}_N}{2}\int \phi_2''(u)p_-(\tau,u)du+ \\
&(\beta J)^2\int p(\tau,u)(\frac{1}{2}\phi_2''(u)-\phi_2'(u))du + o(||\phi_2||)+o(||\phi_2'||), \\
&\frac{d}{d\tau}\int \phi_3(u)p_\pm(\tau,u)du = \frac{(\beta J)^2\overline{q}_N}{2}\int \phi_3''(u)p_\pm(\tau,u)du+ \\
&(\beta J)^2\int p(\tau,u)(\frac{1}{2}\phi_3''(u)-2\phi_3(u))du + o(||\phi_3||)+o(||\phi_3'||).
\end{aligned}
\tag{45}
$$

Using the fact that the functions ϕ_1, ϕ_2 and ϕ_3 are chosen arbitrarily, we derive from (45) the partial differential equations

$$
\begin{aligned}
\frac{\partial}{\partial\tau}p_+(\tau,u) &= \frac{(\beta J)^2\overline{q}_N}{2}\frac{\partial}{\partial u^2}p_+(\tau,u) + (\beta J)^2(\frac{1}{2}\frac{\partial^2}{\partial u^2}p(\tau,u) - \frac{\partial}{\partial u}p(\tau,u)) + d_1(\tau,u), \\
\frac{\partial}{\partial\tau}p_-(\tau,u) &= \frac{(\beta J)^2\overline{q}_N}{2}\frac{\partial^2}{\partial u^2}p_-(\tau,u) + (\beta J)^2(\frac{1}{2}\frac{\partial^2}{\partial u^2}p(\tau,u) + \frac{\partial}{\partial u}p(\tau,u)) + d_2(\tau,u), \\
\frac{\partial}{\partial\tau}p_\pm(\tau,u) &= \frac{(\beta J)^2\overline{q}_N}{2}\frac{\partial^2}{\partial u^2}p_\pm(\tau,u) + (\beta J)^2(\frac{1}{2}\frac{\partial}{\partial u^2}p(\tau,u) - 2p(\tau,u)) + d_3(\tau,u),
\end{aligned}
\tag{46}
$$

where the remainder functions $d_{1,2,3}(\tau,u)$ admit the following bound, valid for any smooth function $\phi(u)$

$$
|\int d_{1,2,3}(\tau,u)\phi(u)du| \leq o(1)(||\phi||+||\phi'||)
\tag{47}
$$

By the virtue of Lemma 2,

$$
\Phi_+(\phi,0) = E\{\phi(0)\dot{q}_+(0)\} = \phi(0)(E\{q_+(0)\}-\overline{q}_N) = o(1)\phi(0).
$$

Similarly

$$
\Phi_-(\phi,0) = o(1)\phi(0), \quad \Phi_\pm(\phi,0) = o(1)\phi(0).
$$

Therefore we can supply equations (46) by the initial conditions:

$$
p_+(0,u) = p_-(0,u) = p_\pm(0,u) = o(1)\delta(u).
\tag{48}
$$

Then according to the standard theory of partial differential equations, the functions $p_+(\tau,u)$, $p_-(\tau,u)$, $p_\pm(\tau,u)$ can be represented in the form

$$
\begin{aligned}
p_+(\tau,u) = (\beta J)^2 \int_0^\tau d\xi \int du' K_{\tau-\xi}(u-u')(\frac{1}{2}\frac{\partial^2}{\partial u'^2}p(\xi,u') - \frac{\partial}{\partial u'}p(\xi,u'))+ \\
o(1)K_\tau(u) + \hat{d}_1(\tau,u), \\
p_-(\tau,u) = (\beta J)^2 \int_0^\tau d\xi \int du' K_{\tau-\xi}(u-u')(\frac{1}{2}\frac{\partial^2}{\partial u'^2}p(\xi,u') + \frac{\partial}{\partial u'}p(\xi,u'))+ \\
o(1)K_\tau(u) + \hat{d}_2(\tau,u), \\
p_\pm(\tau,u) = (\beta J)^2 \int_0^\tau d\xi \int du' K_{\tau-\xi}(u-u')(\frac{1}{2}\frac{\partial^2}{\partial u'^2}p(\xi,u') - 2p(\xi,u'))+ \\
o(1)K_\tau(u) + \hat{d}_3(\tau,u),
\end{aligned}
\tag{49}
$$

where the kernel $K_\xi(u)$ has the form

$$
K_\xi(u) = \frac{\exp\{-\frac{u^2}{2(\beta J)^2 \overline{q}_N \xi}\}}{\beta J\sqrt{\overline{q}_N \xi}},
\tag{50}
$$

functions $\hat{d}_{1,2,3}(\tau,u)$ are defined by the formulae

$$
\hat{d_{1,2,3}}(\tau,u) = \int_0^\tau d\xi \int du' K_{\tau-\xi}(u-u') d_{1,2,3}(u')
$$

and therefore satisfy the estimate

$$
\int \hat{d}_{1,2,3}(\tau,u)\phi(u)du \le o(1)(||\phi|| + ||\phi'||).
$$

Now, returning to formulae (26),(27) and denoting by

$$
\begin{aligned}
\psi_1(u) &= \tanh^2(u+\beta h_1)\frac{e^{4u+2\beta h_1}}{(e^{2u+\beta h_1}+e^{-\beta h_1})^2}, \\
\psi_2(u) &= \tanh^2(u+\beta h_1)\frac{e^{-2\beta h_1}}{(e^{2u+\beta h_1}+e^{-\beta h_1})^2}, \\
\psi_3(u) &= \tanh^2(u+\beta h_1)\frac{e^{2u}}{(e^{2u+\beta h_1}+e^{-\beta h_1})^2},
\end{aligned}
\tag{51}
$$

we derive from (13) by using (26), (27) and (49) that

$$
\begin{aligned}
\Delta_N = E\{\langle\sigma_1\rangle^2 \dot{q}'_{N-1}\} + o(1) = (\beta J)^2 \textstyle\int du[\psi_1(u)p_+(1,u) + \psi_2(u)p_-(1,u) + 2\psi_3(u)p_\pm(1,u)] \\
(\beta J)^2 \int_0^1 d\xi \int du\psi_1(u) \int du' K_{1-\xi}(u-u')((\frac{1}{2}\frac{\partial^2}{\partial u'^2}p(\xi,u') - \frac{\partial}{\partial u'}p(\xi,u'))+ \\
(\beta J)^2 \int_0^1 d\xi \int du\psi_2(u) \int du' K_{1-\xi}(u-u')((\frac{1}{2}\frac{\partial^2}{\partial u'^2}p(\xi,u') + \frac{\partial}{\partial u'}p(\xi,u'))+ \\
2(\beta J)^2 \int_0^1 d\xi \int du\psi_3(u) \int du' K_{1-\xi}(u-u')((\frac{1}{2}\frac{\partial^2}{\partial u'^2}p(\xi,u') - 2p(\xi,u')) + o(1) = \\
(\beta J)^2 \int_0^1 d\xi \int du \int du' \psi(u) K_{1-\xi}(u-u') p(\xi,u')) + o(1),
\end{aligned}
\tag{52}
$$

where

$$\psi(u)=\frac{1}{2}(\psi_1''(u)+\psi_2''(u)+2\psi_3''(u))+\psi_1'(u)-\psi_2'(u)-4\psi_3(u)=\cosh^{-4}(u+\beta h_1).$$

Therefore

$$\begin{gathered}\Delta_N=(\beta J)^2\int_0^1 d\xi\int du' F_\xi(u')p(\xi,u')+o(1)\le(\beta J)^2\int_0^1 F_\xi(0)d\xi\int du'|p(\xi,u')|+o(1)\le\\ (\beta J)^2\int_0^1 F_\xi(0)d\xi E\{|\dot q_+(\xi)||\dot q_-(\xi)|\}+o(1)\le\\ (\beta J)^2\int_0^1 F_\xi(0)d\xi E^{1/2}\{(\dot q_+(\xi))^2\}E^{1/2}\{(\dot q_-(\xi))^2\}+o(1)=\\ \Delta_N\cdot(\beta J)^2\int_0^1 F_\xi(0)d\xi+o(1),\end{gathered}\tag{53}$$

where

$$F_\xi(u')=\int du\int\frac{e^{-h_1^2/2h^2}}{h\sqrt{2\pi}}dh_1K_{1-\xi}(u-u')\cosh^{-4}(u+\beta h_1).$$

The first inequality in the (53) holds due the fact that $0\le F_\xi(u')\le F_\xi(0)$. The second inequality is based on the representation (45), the third is just the Schwartz inequality, and the last equality is based on Lemma 2 (note, that we have used also the fact that $|p(\xi,u')|$ does not depend on h_1). Thus (53) implies that if

$$\begin{gathered}C_N(\beta,h)\equiv\int_0^1 d\xi F_\xi(0)=\\ \frac{(\beta J)^2}{2\pi}\int_0^1 d\xi\int\int du e^{-u^2/2}\cosh^{-4}(\beta(\sqrt{J^2\overline{q}_N\xi+h^2}\,u)<1,\end{gathered}\tag{54}$$

then $\Delta_N\to 0$ and, according to result Theorem 1, the replica symmetric equations (2)-(3) hold. One can easily see that if $\beta J<1$, then $C_N(\beta,h)<1$ for any $h>0$. Thus, since the free energy is continuous with respect to h, we have replica symmetric solution for $h=0$ also. Moreover, one can see that for any β if h is large enough, then we also have replica symmetric solution. But to prove the statement of Theorem 2 we have to verify that one can replace $\overline{q}_N$ in (54) by q- the solution of equation (3).

To this end we fix β and chose h large enough to fulfil (54) (we mentioned above that it is always possible). Then, decreasing h, we reach the point $h_0(\beta)$, defined as the smallest upper bound of those ts, for which the replica symmetric solution does not hold. We will prove now that in this case $C(\beta,h_0(\beta))$ defined by (8) is not less then 1.

Indeed, since the mean free energy is the convex function with respect to h, its derivative $E\{f_N'\}=-h\beta(1-\overline{q}_N)$ is decreasing function, and the therefore there exists $\delta>0$ such that

$$\overline{q}_N(h)\ge\lim_{N\to\infty}\overline{q}_N(h_0(\beta)+0)=q$$

for any $h_0(\beta)-\delta\le h\le h_0(\beta)$. Hence, if we assume that $C(\beta,h_0(\beta))<1$, then $C_N(\beta,h)<1$ for $h_0(\beta)-\delta\le h\le h_0(\beta)$. Thus, according to (53), the replica symmetric solution holds for these h. But since this fact contradicts to the choice of $h_0(\beta)$, one can conclude that $C(\beta,h_0(\beta))\ge 1$.

Theorem 2 is proved.

Acknowledgements. The author is grateful to Prof.S.Albeverio and Prof.L.Pastur for the interest to the work.

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