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Stimulation of superconductivity by microwave radiation in wide tin films (Review Article)

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The review is devoted to an experimental study of simulation of superconductivity by microwave radiation in superconducting films. An influence of the power, frequency of microwave radiation, as well as temperature and width of superconducting films on behavior of experimental dependencies of stimulated the critical current and the current at which a vortex structure of the resistive state vanishes and the phase-slip first line appears is analyzed. The experimental studies of films with different width reveal that the effect of superconductivity stimulation by microwave field is common and occurs in both the case of uniform (narrow films) and non-uniform (wide films) distribution of superconducting current over the film width. It is shown that stimulation of superconductivity in a wide film increases not only the critical current and the critical temperature, but also the maximum current at which there is a vortex state in the film. The effect of superconductivity stimulation by microwave radiation in wide films can be described by the Eliashberg theory, which was used to explain the same phenomenon in narrow channels. For the first time it was found experimentally that when the film width increases, the range of radiation power, at which the effect of superconductivity stimulation is observed, shrinks abruptly, and hence the probability of its detection decreases. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4813655]

The review is dedicated to Academician of the NAS of Ukraine V. Eremenko, a prominent scientist in the field of magnetism and superconductivity, who generously conveys his profound knowledge and rich academic experience to young scientists, among whom once was the grateful writer of these words.

1. Introduction

For a long time there was a general opinion that effect of electromagnetic field on a superconductor should always lead to a reduction of the energy gap Δ, the critical current I_c, the critical magnetic field H_c and the critical temperature T_c. However, an increase of the critical current of a thin narrow superconducting bridge near its critical temperature under an influence of high-frequency electromagnetic field has been reported in 1966 in Ref. 1. Later this phenomenon has been observed in almost all types of superconducting weak links. This effect has been explained in the Aslamazov-Larkin theory only in 1978.[2] The phenomenon of stimulation of superconductivity has also been found in narrow superconducting channels (single-crystal filaments and thin (the thickness $d \ll \xi(T)$, $\lambda_\perp(T)$), narrow (the width $w \sim \xi(T)$, $\lambda_\perp(T)$) films). Here, $\xi(T)$ and $\lambda_\perp(T) = 2\lambda^2(T)/d$ are the coherence length and the penetration depth of a weak magnetic field perpendicular to the film, respectively, $\lambda(T)$ the London penetration depth. In 1970 G.M. Eliashberg has proposed a microscopic theory,[3] which considers the effect of electromagnetic radiation on the energy gap Δ of a superconductor. The Eliashberg theory explained the phenomenon of stimulation in a superconducting channel, and this theory did not exclude a possibility of its existence in wide films. However, decades had passed, but this phenomenon could not been found in wide films. After the discovery of high-temperature (HTS) cuprate superconductors, which caused an increased research activity, there has been a series of works devoted to the experimental study of an effect of microwave radiation on the superconducting properties of HTS films. Reference 4 was among the first which mentioned the finding of the effect of superconductivity stimulation in HTS films. That paper shows a family of current-voltage characteristics (CVC) for wide ($w \sim 10\,\mu$m) and long ($L \sim 15\,\mu$m) bridge of an epitaxial film of $YBa_2Cu_3O_{7-x}$. In the CVC it is seen that for low ($\sim 10^{-8}$ W) power the critical current I_c as well as the superconducting one I_s increase compared to the values in the absence of microwave radiation, indicating the stimulation of superconductivity by an electromagnetic field. With a further increase in power of microwave radiation the critical superconducting current decreases, and harmonic and subharmonic steps of the current appear at voltages on the bridge $V_{m,n}$, related to the frequency of an external electromagnetic field f by the Josephson relation: $V_{m,n} = (n/m)hf/2e$, where m, n are integers, h Planck's constant, e the electron charge. The authors of Ref. 4 suggest that the mechanism responsible for the increase of I_c and I_s in the investigated bridges of HTS is an energy diffusion of quasiparticles in a contact area caused by a "jitter" of the potential well due to microwave radiation.[2] It is this mechanism of stimulation of superconductivity that was also found in cuprate HTS by other experimentalists. For example, in studying the dependence of the superconducting current on the radiation power in $YBa_2Cu_3O_{7-x}$ samples[5] it was found that it is the Aslamazov-Larkin mechanism of stimulation,[2] characteristic of superconducting weak links that is responsible for increasing the superconducting current under microwave radiation. Unfortunately, preliminary studies of the effect of superconductivity stimulation in cuprate HTS caused by the Aslamazov-Larkin mechanism[2] have not been continued.

The phenomenon of superconductivity stimulation by microwave radiation in quasi-one-dimensional films (narrow channels) already belongs to classical effects in the physics of superconductivity. The experimental manifestation of this effect in a narrow channel is an increase of its critical temperature T_c and the Ginzburg-Landau critical current $I_c^{GL}(T)$. When a current flowing through a channel is greater than the current $I_c^{GL}(T)$, the narrow channel comes to the resistive current state caused solely by the appearance of phase-slip centers. In contrast, in high-quality superconducting wide films ($w \gg \xi(T)$, $\lambda_\perp(T)$) in excess of the critical current $I_c(T)$ the vortex state appears, so-called flux flow regime. A wide film is in this regime until the transport current reaches the maximum current $I_m(T)$ at which in a wide film a vortex structure of the resistance state vanishes,[6,7] and the first phase slip line (PSL) appears. In 2001, it has been experimentally observed[8] that in response to a microwave field not only the critical current $I_c(T)$ increases but the maximum current of existence of the vortex resistivity $I_m(T)$ does so. In this connection, the problem of superconductivity stimulation in wide films is particularly interesting, as it requires consideration of behavior in a microwave field of both the critical current $I_c(T)$ and the maximum current of existence of the vortex resistive state $I_m(T)$.

2. A microscopic theory of superconductivity of films, stimulated by microwave radiation

A microscopic theory of superconductivity stimulation of films, uniform in the order parameter, by a microwave field was proposed by Eliashberg[3] and developed in Refs. 9–12.

The theory applies to relatively narrow[13] and thin ($w,d \ll \xi(T)$, $\lambda_\perp(T)$) films in which the spatial distribution of microwave power and accordingly the stimulated gap are uniform over the film cross section. At the same time, the length of scattering of an electron by impurities l_i should be small compared with the coherence length.

To understand properly behavior of superconductors with an energy gap in an alternating electromagnetic field it was necessary to take into account both the processes of absorption of electromagnetic energy by quasiparticles (electrons), and the inelastic processes of scattering of the absorbed energy.

To illustrate the physical nature of the effect of superconductivity stimulation, we turn to the basic equation of Bardeen-Cooper-Schrieffer theory (BCS),[14] which relates the energy gap Δ with the equilibrium distribution function of electrons $n(\varepsilon)(e^{\varepsilon/kT}+1)^{-1}$

$$\Delta = g\int_{\Delta}^{\hbar\omega_D} d\varepsilon \frac{\Delta}{\sqrt{\varepsilon^2-\Delta^2}}[1-2n(\varepsilon)]\,. \tag{1}$$

In the theory,[3] it was shown that if a superconductor with a uniform spatial distribution of Δ is in an electromagnetic field whose frequency is lower than the frequency related with the energy gap by the ratio $\hbar\omega = 2\Delta$, and higher than the inverse relaxation time of electrons τ_ε (the relaxation time of inelastic collisions), then the equilibrium distribution function of electrons $n(\varepsilon)$ is shifted to higher energies, which leads to a steady non-equilibrium state and an increase of the superconductor's energy gap and consequently its superconducting properties. And the total number of excitations does not change. This shift in the electron distribution function, as seen in Eq. (1), results in an increase of the gap and thus enhances the superconducting properties. A change of $n(\varepsilon)$ is proportional to the field intensity E^2 (for not too large E) and the relaxation time of energy excitations τ_ε.

It should be noted that in the presence of a microwave field the Δ is variable in space and time. There is no coordinate dependence for sufficiently thin samples. And when $\omega\tau_\varepsilon \gg 1$ it turns out that temporal oscillations of Δ can also be ignored. It was also assumed that the mean free path of electrons is less than the film thickness. Otherwise it would be necessary to consider peculiarities of reflection from walls.

If we restrict our consideration to not too high intensities of electromagnetic radiation, an equation for the time-averaged Δ is as follows:

$$\left\{\frac{T_c-T}{T_c}-\frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2}-\frac{\pi l_i v_F e^2}{6T_c\hbar c^2}\left[A_0^2+\frac{A_\omega^2}{2}-\frac{3\Delta\hbar c^2}{2\pi l_i v_F e^2}G\right]\right\}\Delta = 0, \tag{2}$$

where A_0 is the potential representing a static magnetic field or a direct current, A_ω the amplitude of an electromagnetic field, v_F the Fermi velocity, l_i the mean free path of electrons under the scattering, $n_1(\varepsilon)$ the non-equilibrium part of $n(\varepsilon)$, $\zeta(3)=1.202$ the particular value of the Riemann zeta function

$$G = -\frac{2T}{\Delta}\int_{\Delta}^{\infty}\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}n_1(\varepsilon). \tag{3}$$

At low power of an external electromagnetic field

$$n_1(\varepsilon) = \frac{\alpha}{\gamma}\frac{\omega}{4T}\left(\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\varepsilon\sqrt{(\varepsilon-\omega)^2-\Delta^2}}\theta(\varepsilon-\Delta-\omega) - \frac{\varepsilon(\varepsilon+\omega)+\Delta^2}{\varepsilon\sqrt{(\varepsilon+\omega)^2-\Delta^2}}\theta(\varepsilon-\Delta) - \frac{2}{\omega}\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\sqrt{(\varepsilon-\omega)^2-\Delta^2}}\theta(\varepsilon-\Delta)\theta(\omega-\Delta-\varepsilon)\right), \tag{4}$$

where $\alpha = (1/3)v_F l_i e^2 A_\omega^2/\hbar c^2$ is proportional to the power of an external electromagnetic field, $\gamma = \hbar/\tau_\varepsilon$.

Taking into account Eq. (4), expression (3) can be written as

$$G = \frac{\omega^2}{2\Delta}\frac{\alpha}{\gamma}\int_{\Delta}^{\infty} d\varepsilon\frac{\varepsilon(\varepsilon+\omega)+\Delta^2}{\varepsilon(\varepsilon+\omega)\sqrt{(\varepsilon^2-\Delta^2)[(\varepsilon+\omega)^2-\Delta^2]}} + \frac{1}{\Delta}\frac{\alpha}{\gamma}\int_{\Delta}^{\omega-\Delta} d\varepsilon\frac{\varepsilon(\varepsilon-\omega)+\Delta^2}{\sqrt{(\varepsilon^2-\Delta^2)[(\varepsilon-\omega)^2-\Delta^2]}}\theta(\omega-2\Delta), \tag{5}$$

or

$$G = \frac{\alpha\hbar\omega}{2\gamma\Delta} f\left(\frac{\hbar\omega}{\Delta}\right). \quad (6)$$

In limiting cases, the function

$$f\left(\frac{\hbar\omega}{\Delta}\right) = \frac{\hbar\omega}{\Delta}\left[\ln\left(\frac{8\Delta}{\hbar\omega}\right) - 1\right] \text{ at } \frac{\hbar\omega}{\Delta} \ll 1,$$
$$f\left(\frac{\hbar\omega}{\Delta}\right) = \frac{\pi\Delta}{\hbar\omega} \text{ at } \frac{\hbar\omega}{\Delta} \gg 1. \quad (7)$$

Taking into account Eq. (7), Eq. (2) can be rewritten as follows:

$$\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{\pi v_F l_i e^2 A_\omega^2}{6k_B T_c \hbar c^2} \times \left[A_0^2 + A_\omega^2\left(1 - \frac{\hbar\omega}{2\pi\gamma} f\left(\frac{\hbar\omega}{\Delta}\right)\right)\right] = 0. \quad (8)$$

In Eq. (8) there is no term accounting for the interaction of an electromagnetic field with excitations, located substantially above the gap edge, which has the form[9]

$$-0.11\frac{\pi}{2}\left(\frac{\hbar\omega}{k_B T_c}\right)^2 \frac{\alpha}{\gamma}.$$

Now we can write a complete equation of the microscopic theory of superconductivity, which takes into account basic mechanisms of the interaction of a superconductor with an external electromagnetic radiation

$$\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{\pi\alpha}{2k_B T_c} \times \left\{\frac{A_0^2}{A_\omega^2} + 1 + 0.11\frac{(\hbar\omega)^2}{\gamma k_B T_c} - \frac{(\hbar\omega)^2}{2\pi\gamma\Delta}\left[\ln\left(\frac{8\Delta}{\hbar\omega}\right) - 1\right]\right\} = 0. \quad (9)$$

In this equation, the first two terms describe a temperature dependence of the equilibrium ($\alpha = 0$) superconducting gap, and the third one is a contribution of a static magnetic field or a direct current. The fourth term of the equation describes a usual pairing effect in an external microwave field, the fifth is a contribution of high-energy excitations, and the sixth term is a contribution of the interaction with an external electromagnetic field of quasiparticles located at the Fermi surface. It is this interaction that is responsible for the effect of superconductivity stimulation.[3] Effects of heating of a superconductor by an electromagnetic field are not considered in Eq. (9).

It is important to note one more circumstance. In Eq. (9) it is seen that with increasing the radiation frequency a contribution of the latter two terms increases, and an effect of second one leads to an increase of $\Delta(T,\alpha)$ for a given electromagnetic field power α. $\Delta(T,\alpha)$ is greater than $\Delta(T,\alpha = 0)$ (superconductivity stimulation), when

$$\left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k_B T_c} - \frac{(\hbar\omega)^2}{2\pi\gamma\Delta}\left(\ln\left(\frac{8\Delta}{\hbar\omega}\right) - 1\right)\right] \le 0. \quad (10)$$

At not very high frequency of external radiation the term $0.11(\hbar\omega)^2/(\gamma k_B T_c)$, which describes a contribution of high-energy excitations can be neglected, and with $\ln(8\Delta/\hbar\omega) > 1$ from Eq. (10) we obtain an expression for the lower frequency limit of the superconductivity stimulation

$$\omega_L^2 = \frac{2\pi\gamma\Delta}{\hbar^2 \ln\left(\frac{8\Delta}{\hbar\omega}\right)} = \frac{2\pi\Delta}{\hbar\tau_\varepsilon \ln\left(\frac{8\Delta}{\hbar\omega}\right)}. \quad (11)$$

3. A non-equilibrium critical current of superconducting films in a microwave field

Theoretical studies[3,9–12] considered superconductivity stimulation for narrow channels in which the equilibrium energy gap Δ and the superconducting current density j_s are distributed uniformly over the sample cross section.

According to this theory, the effect of microwave radiation on the energy gap Δ of a superconductor, through which a constant transport current with density j_s flows, is described by Eq. (9) which can be rewritten as

$$\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{2k_B T_c \hbar}{\pi e^2 D\Delta^4 N^2(0)} j_s^2 + M(\Delta) = 0, \quad (12)$$

where T_c is the critical temperature, $N(0)$ the density of states at the Fermi level, $D = v_F l_i/3$ the diffusion coefficient, v_F the Fermi velocity, and $M(\Delta)$ the non-equilibrium add-on due to the "non-equilibrium" of the distribution function of electrons[15–17]

$$M(\Delta) = -\frac{\pi\alpha}{2k_B T_c}\left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k_B T_c} - \frac{(\hbar\omega)^2}{2\pi\gamma\Delta}\left(\ln\frac{8\Delta}{\hbar\omega} - 1\right)\right], \quad \hbar\omega \ll \Delta. \quad (13)$$

Often, an experimental study of the stimulation effect assumes the measurement of the critical current, rather than the energy gap. Using Eqs. (12) and (13) one can derive an expression for the density of the superconducting current, j_s, as a function of energy gap, temperature and microwave power

$$j_s = \upsilon\Delta^2\left\{\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k_B T_c)^2} - \frac{\pi\alpha}{2k_B T_c}\left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k_B T_c} - \frac{(\hbar\omega)^2}{2\pi\gamma\Delta}\left(\ln\frac{8\Delta}{\hbar\omega} - 1\right)\right]\right\}^{1/2}, \quad (14a)$$

$$\upsilon = eN(0)\sqrt{\frac{\pi D}{2\hbar k T_c}}. \quad (14b)$$

An extremum condition of the superconducting current $\partial j_s/\partial\Delta = 0$ at given temperature and power yields a transcendental equation for the gap Δ_m, for which the maximum value j_s is reached, i.e., the critical current density

$$\frac{T_c - T}{T_c} - \frac{21\zeta(3)\Delta_m^2}{(4\pi k_B T_c)^2} - \frac{\pi\alpha}{2k_B T_c} \times \left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k_B T_c} - \frac{(\hbar\omega)^2}{4\pi\gamma\Delta_m}\left(\frac{3}{2}\ln\frac{8\Delta}{\hbar\omega} - 1\right)\right] = 0. \quad (15)$$

Thus, substituting the solution Δ_m of Eq. (15) into Eq. (14) we find an expression for the critical current in a microwave field[18]

$$I_c^P(T) = \upsilon dw\Delta_m^2 \left\{ \frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta_m^2}{(8\pi k_B T_c)^2} - \frac{\pi\alpha}{2k_B T_c} \times \left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k_B T_c} - \frac{(\hbar\omega)^2}{2\pi\gamma\Delta_m}\left(\ln\frac{8\Delta_m}{\hbar\omega} - 1 \right) \right] \right\}^{1/2}. \quad (16)$$

Without external microwave field ($\alpha = 0$), Eq. (16) is transformed into an expression for the equilibrium depairing current

$$I_c(T) = \upsilon dw\Delta_m^2 \left\{ \frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta_m^2}{8(\pi k_B T_c)^2} \right\}^{1/2}. \quad (17)$$

In this case, $\Delta_m = \sqrt{2/3}\Delta_0$, where

$$\Delta_0(T) = \pi k_B T_c \sqrt{\frac{8(T_c - T)}{7\zeta(3)T_c}} = 3.062 k_B T_c \sqrt{1 - \frac{T}{T_c}}, \quad (18)$$

the equilibrium value of the gap without transport current.

Note that the use in Eq. (17) of the Eq. (14b) with the density of states $N(0) = m^2 v_F/\pi^2\hbar^3$, calculated in a model of free electrons, results in a significant difference between the theoretical and experimental values of the equilibrium critical current, indicating a relative roughness of such an estimation for a metal (e.g., tin) used in preparation of samples. At the same time, expressing the density of states in terms of the experimentally measured quantity, the resistance of the film on a square $R^{\square} = R_{4.2}w/L$, where $R_{4.2}$ is the total film resistance at $T = 4.2$ K, we obtain an expression for $\upsilon = (edR^{\square})^{-1}\sqrt{3\pi/2k_B T_c v_F l_i \hbar}$, which when substituted into Eq. (17) leads to its good agreement with both experimental values of the equilibrium depairing current and those calculated in the Ginzburg-Landau theory, $I_c^{GL}(T)$ (see Eq. (19)).[19] This expression for the parameter υ will be used below in the formula (16) for stimulated critical current in its comparison with experimental results.

It is interesting to note that prior to Ref. 19, to our knowledge, the temperature dependencies of the stimulated critical current, arising from Eq. (16), were not compared directly with experimental data for $I_c^P(T)$. We point out, however, that attempts to compare, at least qualitatively, experimental dependencies $I_c^P(T)$ with the Eliashberg theory have been made. For example, in Ref. 14 the authors presented the depairing current of Ginzburg-Landau, using Eq. (18) for the equilibrium gap, in the form

$$I_c^{GL}(T) = \frac{c\Phi_0 w}{6\sqrt{3}\pi^2\xi(0)\lambda_\perp(0)}(1 - T/T_c)^{3/2} = K_1\Delta_0^3(T), \quad (19)$$

where $\Phi_0 = hc/2e$ is the magnetic flux quantum. Since the temperature dependence of the stimulated critical current in a narrow channel appeared to be close to equilibrium in its shape, $I_c^P(T) \propto (1 - T/T_c^P)^{3/2}$, where T_c^P is the superconducting transition temperature in a microwave field, the $I_c^P(T)$ was approximated by an expression similar to Eq. (19)

$$I_c^P(T) = K_2\Delta_P^3(T), \quad (20)$$

where the stimulated energy gap $\Delta_P(T)$ was calculated in the Eliashberg theory at zero superconducting current (Eq. (12) and $j_s = 0$). After that, assuming $K_1 = K_2$ and using the value of the microwave power as a fitting parameter, the authors of Ref. 14 fitted calculated values of $I_c^P(T)$ to experimental data with a certain degree of accuracy.

It is clear that such a comparison of experimental results with the Eliashberg theory is only a qualitative approximation, and cannot be used to obtain quantitative results.[19] First of all, Eqs. (19) and (20) contain a value of the gap in a zero-current regime ($j_s = 0$), which differs from that when there is a current. Second, the depairing curves $j_s(\Delta)$ in the equilibrium state ($P = 0$) and with a microwave field are very different.[18] Finally, as shown in Refs. 11, 14, and 18, for $T \to T_c^P - 0$ the stimulated order parameter $\Delta_P(T)$ tends to a finite (but small) value $\Delta_P(T_c^P) = (1/2)\hbar\omega$ and abruptly vanishes at $T > T_c^P$, while the critical current vanishes continuously (without a jump) at $T \to T_c^P$ and, therefore, cannot in general be satisfactorily described by the formula of the type (20). This is evidenced by a marked deviation of the dependence (20) from experimental points in the close vicinity of T_c. In this review, an analysis of experimental data is based on the exact formula (16) with using the numerical solution of Eq. (15).

4. Stimulation of superconductivity by an external microwave radiation in tin films of different widths

Upon stimulation of superconductivity by an electromagnetic field the superconducting gap Δ is, strictly speaking, variable in space and time. However, for sufficiently thin and narrow superconducting samples the dependence of Δ on coordinates can be neglected. Moreover, near T_C the relaxation time of the order parameter $\tau_\Delta \approx 1.2\tau_\varepsilon/(1 - T/T_c)^{1/2}$ is large compared to the inverse frequency of stimulating microwave radiation ($\omega\tau_\Delta \gg 1$), and the temporal oscillations of Δ can also be neglected. Therefore, the microscopic theory of Eliashberg[3] included neither the time nor spatial variations of the order parameter in a sample.

To realize experimentally the case discussed in this theory of a spatially homogeneous nonequilibrium state of a superconductor in a high-frequency field it was necessary to ensure the constancy of the energy gap over the sample volume ($w,d \sim \xi(T)$, $\lambda_\perp(T)$). This is a purely technological problem. It was also important to ensure a uniform distribution of the transport current through the sample volume. Failure to do so resulted in a non-uniform distribution of Δ due not to technological reasons, but because of the dependence of the energy gap on the transport current $\Delta(I)$. Finally, it was important to provide an efficient heat transfer from a sample. It was shown that narrow superconducting film channels deposited on relevant substrates best met these requirements. The theory proposed in Ref. 3 has been fully confirmed in experimental studies of such samples (see, e.g., Refs. 15 and 17).

For wider films, electrodynamical changes of Δ over the film width with a non-uniformly distributed current and with the presence of intrinsic vortices cannot be neglected. Therefore, the theory,[3] strictly speaking, does not apply in the case of wide films. Yet, although it is difficult to develop a theory in the case of a non-uniform distribution of Δ in a superconductor, in principle there should also be an effect of stimulation in this case.

In 2001, the stimulation of superconductivity by an external electromagnetic field has also been found in wide, $w \gg \xi(T)$, $\lambda_\perp(T)$, high-quality superconducting films of tin[8] with a non-uniform spatial distribution of $\Delta(I)$ over the sample width. It has been experimentally shown that under an external electromagnetic field not only the critical current I_c increases but also the current of formation of the first phase-slip line (see Fig. 1) does so. In Ref. 8, this current is designated as I_c^{dp}. In Ref. 20 the temperature dependencies of the current I_c^{dp} was analyzed with taking into account the nontrivial distribution of the transport current and the density of vortices over a wide film. As a result, it was shown that the current I_c^{dp} is the critical depairing current of Ginzburg-Landau, I_c^{GL}, if a film corresponds to the parameters of a vortex-free narrow channel in the temperature region near T_c. Far from T_c this current is the maximum current of existence of the vortex resistive state I_m in the Aslamazov-Lempitsky theory.[6] A physical meaning of the current I_m is that it is the maximum current, at which a steady uniform flow of intrinsic vortices of the transport current across a wide film is still possible. If it is exceeded, $I > I_m$, the vortex structure collapses, and in its place there appears a structure of phase-slip lines.[7] It is the phase that determines the resistivity of a sample for a further increase in the transport current.

In this connection, the problem of superconductivity stimulation in wide films becomes particularly interesting, because it requires consideration of the behavior in a microwave field of both the critical current and the critical temperature. An important object of the investigation is the current I_m, as well as its relation with I_c under an external electromagnetic radiation of different frequencies f and power P.

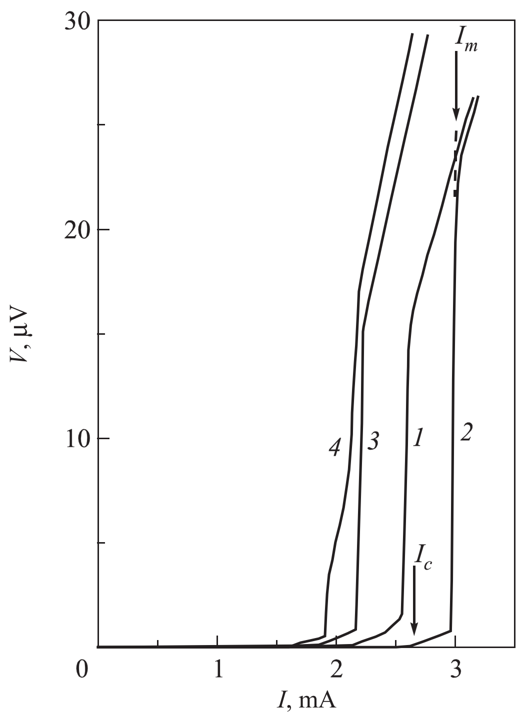

FIG. 1. A series of current-voltage characteristics of the film sample SnW5 at $T = 3.745$ K and $f = 12.89$ GHz for various levels of radiation power: the radiation power is zero (*1*), with increasing a serial number of the CVC the radiation power increases (*2–4*).

This section presents the results of studying a dependence of stimulation of the critical current I_c and the current of formation of the first PSL I_m on power and frequency of an electromagnetic field in thin (thickness $d \ll \xi(T)$, $\lambda_\perp(T)$) superconducting films as a function of their width w.[21] To understand how the effect of superconductivity stimulation manifests itself in a wide film, the authors of this work increased gradually the sample width starting from a narrow channel, and observed how the phenomenon of superconductivity stimulation was changing. As samples thin ($d \ll \xi(T)$, $\lambda_\perp(T)$) tin films which were prepared as described in Ref. 20 were used. This original technology enabled to minimize defects in both the film edge and its volume. The critical current of these samples is determined by suppression of the barrier for entering vortices when the current density at the edge of the film is of the order of j_c^{GL}, and reaches the maximum possible theoretical value,[6] indicating the absence of edge defects which create local lowering of the barrier and thus reduce the I_c. CVC were measured by a four-probe method. In measuring the CVC samples were placed in a double screen of annealed permalloy. In the area of the sample the magnetic field was: $H_\perp = 7 \times 10^{-4}$ Oe, $H_\parallel = 6.5 \times 10^{-3}$ Oe. To supply an electromagnetic radiation to the film sample it was placed in a rectangular waveguide parallel to the electric field component in the waveguide, or irradiated from the shorted end of a coaxial line, or the sample was connected to a 50-ohm coaxial line through a separating capacitance (contact method). The parameters of the samples are shown in Table 1. The temperature was measured by the vapor pressure using mercury and oil pressure gauges. In doing so, an influence of the microwave field introduced into the cryostat during the experiment on measurements of temperature in the case of using electronic thermometers was excluded. The temperature stabilization (helium vapor pressure) was provided by a membrane manostat with accuracy better than 10^{-4} K.

In Ref. 8 it was shown that long ($L \gg \xi(T)$, $\lambda_\perp(T)$) and wide ($w \gg \xi(T)$, $\lambda_\perp(T)$) superconducting films reveal the effect of increasing the critical current and the current I_m under an external microwave radiation. Fig. 1 shows a family of current-voltage characteristics of one of these films (SnW5) of the width of 42 μm for different power levels of microwave radiation with a frequency $f = 12.89$ GHz.

Here, as in Ref. 20 the notation are introduced: $I_c(T)$ is the current of voltage appearance across the sample as a

TABLE 1. Parameters of tin film samples.

Sample	L, μm	w, μm	d, nm	$R_{4.2}$, Ω	$R^{\square}$, Ω	T_c, K	l_i, nm	R_{300}, Ω
Sn1	64	1.5	90	3.05	0.071	3.834	174	59
SnW5	92	42	120	0.14	0.064	3.789	145	2.27
SnW6	81	17	209	0.185	0.039	3.712	152	3.147
SnW8	84	25	136	0.206	0.061	3.816	148	3.425
SnW10	88	7	181	0.487	0.040	3.809	169	9.156

Note: L is the length; w the width; d the sample thickness; l_i the electron mean free path, $R^{\square} = R_{4.2}w/L$ the resistance of the film on a square.

result of entering vortices of its intrinsic magnetic flux current, $I_m(T)$ is the maximum current of existence of a stable uniform flow of intrinsic vortices or the current of formation of the first phase-slip line. In Fig. 1 it is seen that the current $I_c(P)$ (see Fig. 1, CVC 2) is significantly higher than $I_c(P=0)$ (see Fig. 1, CVC 1), and $I_m(P) > I_m(P=0)$. Thus, under external radiation both I_c and I_m increase.[8]

4.1. The critical current

For a narrow channel Sn1 of the width of $w = 1.5\ \mu$m at $T/T_c = 0.994$ and $w/\lambda_\perp(T = 3.812\ \text{K}) = 0.28$ as a function of the reduced power P/P_c of microwave radiation the relative magnitude of the effect of stimulation of the critical superconducting current $I_c(P)/I_c(0)$ is shown in Fig. 2 for various frequencies of an external radiation. Here, P_c is the minimum power at which $I_c(P = P_c) = 0$. The curve *3* corresponds to a low enough frequency of radiation, 3.7 GHz, the curve *2* is plotted for the radiation frequency of 8.1 GHz; the curve *1* corresponds to the frequency of 15.4 GHz. The arrows indicate the values of powers under which the maximal effect of stimulation I_c was observed for each of the radiation frequencies. For the radiation frequency $f = 3.7$ GHz the reduced power of microwave radiation, at which a maximum of the effect is observed, equals $P/P_c = 0.25$. For the frequency $f = 8.1$ GHz, $P/P_c = 0.51$, and for $f = 15.4$ GHz, $P/P_c = 0.61$. It is seen that with increasing the radiation frequency the reduced power, at which a maximum of the stimulation effect is observed, increases.[21] Unfortunately, in the stimulation theory[3] a reduction of the effect, after the maximum, with increasing the radiation power is not considered. Therefore, a shift of maximal manifestation of superconductivity stimulation under electromagnetic radiation towards higher power with increasing the frequency the theory[3] cannot explain.

The reduced excess of the critical current as a function of the radiation frequency for films of different width is shown in Fig. 3. It is seen that with increasing the frequency the effect of exceeding the critical current $I_{c\max}(P)$ over $I_c(P=0)$ increases for both narrow (curves *1* and *2*) and wide (curve *3*) films. With further increase of the frequency this dependence passes through a maximum and then begins to decrease (not shown here). It should be noted that the frequency at which the maximum effect of stimulation of the critical current is observed, decreases with increasing the film width (for Sn1 the maximum frequency is about 30 GHz, and for SnW5 about 15 GHz).

It is interesting to note that for the film Sn1 (see Fig. 3, curve *1*) the calculation of the lower cutoff frequency of stimulation f_L from Eq. (11) gives the value of 3 GHz (indicated in Fig. 3 by a symbol Δ), which corresponds well to the experiment, as was shown previously for narrow channels.[15] It is important to emphasize that for the calculation of the lower cutoff frequency of stimulation for the sample Sn1, the value $\tau_\varepsilon = 8.3 \times 10^{-10}$ s typical of this series of samples was used.

The dependencies of the reduced critical current $I_c(P)/I_c(0)$ on the reduced power P/P_c of a microwave field for different radiation frequencies for the widest sample SnW10 of the width of $7\ \mu$m at a temperature $T = 3.777$ K

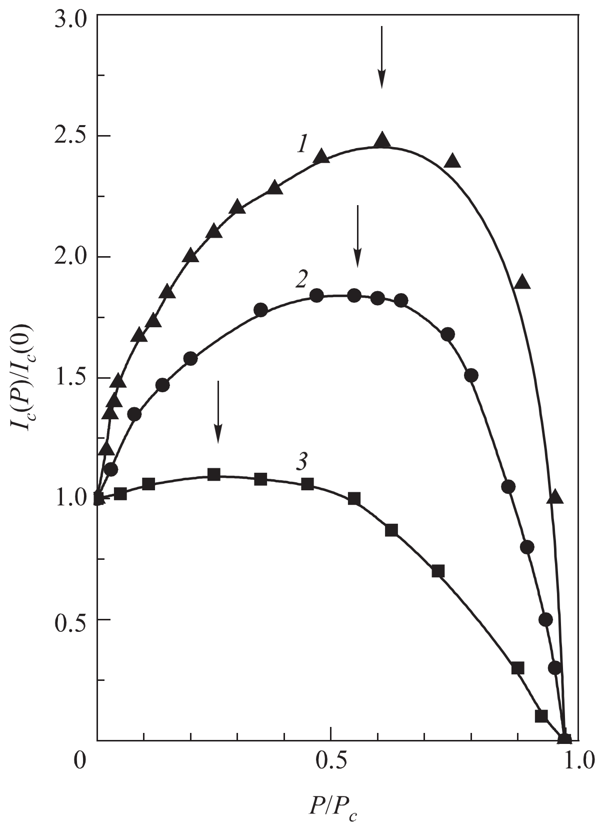

FIG. 2. The dependence of the relative critical current $I_c(P)/I_c(0)$ in the sample Sn1 on the reduced microwave radiation power P/P_C at $T = 3.812$ K for different radiation frequencies f, GHz: 15.4 (▲), 8.1 (●), 3.7 (■) ($I_c(0)$ is the critical current of the film at $P = 0$; P_C is the minimum power of electromagnetic radiation at which $I_c(P) = 0$).

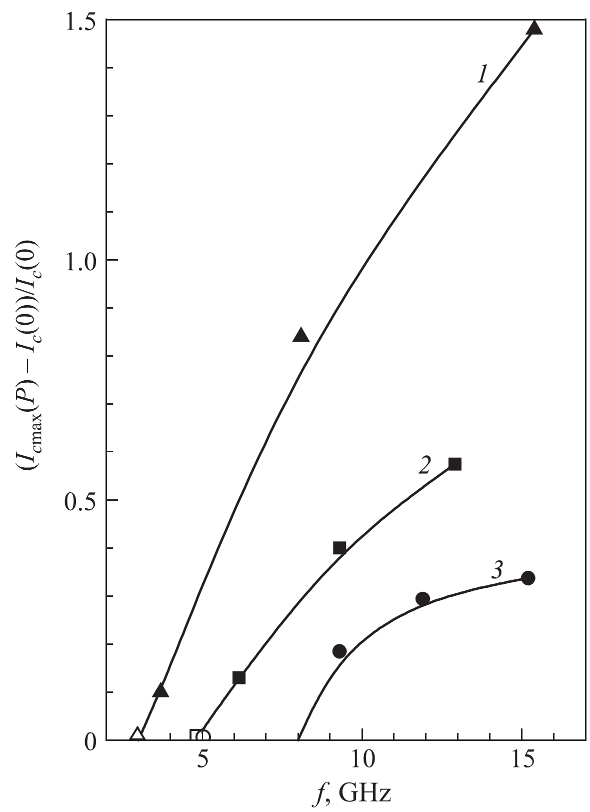

FIG. 3. The reduced value of exceeding the maximum critical current $I_{c\max}(P)$ over $I_c(0)$ as a function of the radiation frequency for the samples Sn1(▲), SnW10 (■) and SnW5 (●) at $t = T/T_c \approx 0.99$; the values of lower cutoff frequencies of superconductivity stimulation calculated by Eq. (11) for the samples Sn1 (Δ), SnW10 (□) and SnW5 (○).

($T/T_c = 0.992$) are shown in Fig. 4. At this temperature $w/\lambda_\perp = 3.56$, i.e., less than 4. As shown in Refs. 13 and 20, at this temperature, the sample SnW10 is still a narrow channel and there is no resistive part, caused by the motion of Pearl vortices, in its current-voltage characteristics. Indeed, the dependencies *1* and *2* in Fig. 4 do not differ qualitatively from those curves in Fig. 2. The arrows in Fig. 4 have the same meaning as in Fig. 2. In Fig. 4 it is seen that on increasing the radiation frequency the reduced power at which the maximum effect of superconductivity stimulation is observed, increases as it was for a narrow channel. Moreover, the calculation of the lower cutoff frequency from Eq. (11) gives the value of 4.8 GHz (denoted by a symbol □), which also agrees quite well with the experimental value f_L, as seen in Fig. 3 (curve *2*). It is important to note that to calculate the lower cutoff frequency of stimulation for the sample SnW10, the value $\tau_\varepsilon = 4.3 \times 10^{-10}$ s typical of this series of samples was used.

In Fig. 4, the experimental dependence (▼) was obtained for the relatively low radiation frequency ($f = 0.63$ GHz). This frequency is below the cutoff frequency of the effect of superconductivity stimulation, f_L, so there is only a suppression of I_c with increasing P. Since in these experimental conditions the sample SnW10 is a narrow channel, it is interesting to compare the experimental dependence (▼) and the theoretical curve *3*. In Ref. 22 it is shown that for superconducting films, the critical current of which is equal to the depairing current of Ginzburg-Landau, the following dependence of the critical current on the radiation power of electromagnetic field is valid:

$$I_c(P,\omega)/I_c(T) = [1 - (P/P_c(\omega))]^{1/2} \times [1 - (2P/((\omega\tau_\Delta)^2 P_c(\omega))]^{1/2}, \qquad (21)$$

at $\omega\tau_\Delta \gg 1$. In our case $\omega\tau_\Delta \approx 24$ and the calculated dependence (21) is shown in Fig. 4 by a dashed curve *3*. It is seen that it agrees quite well with the experimental dependence (▼) and confirms the conclusion made in Ref. 20 that at $w/\lambda_\perp < 4$ films are narrow channels. On lowering the temperature of the sample SnW10 below the crossover temperature T_{cros1},[20] the relation $w/\lambda_\perp$ increases and becomes slightly greater than 4. This is due to a gradual decrease of $\lambda_\perp(T)$ upon changing the temperature far away from T_c. As a result, the distribution of the transport current over the width of the film becomes non-uniform, but not enough to significantly affect the behavior of the film in an electromagnetic field, and consequently the form of $I_c(P)$. To observe significant differences it is necessary to lower significantly the temperature, but the effect of superconductivity stimulation decreases markedly in this case. This is due to a decrease in the number of excited quasiparticles above the gap.[3,15,17]

Therefore, to further investigate the effect of superconductivity stimulation an initially wider film (the SnW5 sample of the width of 42 μm) should be taken. In Fig. 5 for this sample at $T/T_c = 0.988$ and $w/\lambda_\perp$ ($T = 3.744$ K) = 20 there are dependencies of the reduced critical current $I_c(P)/I_c(0)$ on the reduced power P/P_c of a microwave field with different radiation frequencies. The meaning of the arrows is the same as in Figs. 2 and 4. Fig. 5 shows that the reduced power, at which the maximum effect of superconductivity stimulation is observed, increases with the radiation frequency.[21] Moreover, it is seen that descending parts of the dependencies *1*, *3–5* in Fig. 5 differ from those in Figs. 2 and 4 by a curvature sign: in Figs. 2 and 4, descending parts of the curves are convex, while in Fig. 5 they are concave. The curve *5* was obtained at the radiation frequency $f = 5.6$ GHz, and in this case the stimulation effect was not observed. The dotted curve *2* shows the calculated dependence $I_c(P)$ by Eq. (21) for the SnW5 film if the transport current in it was distributed uniformly over its width. It is seen that the curves *2* and *5* are significantly different from each other. Therefore, the concavity of the descending part of the experimental dependence *5* may well be attributed to the non-uniform current distribution across the width of the film.

In Fig. 5 the dependencies *1*, *3* and *4* were obtained for radiation frequencies 15.2, 11.9, and 9.2 GHz, respectively.

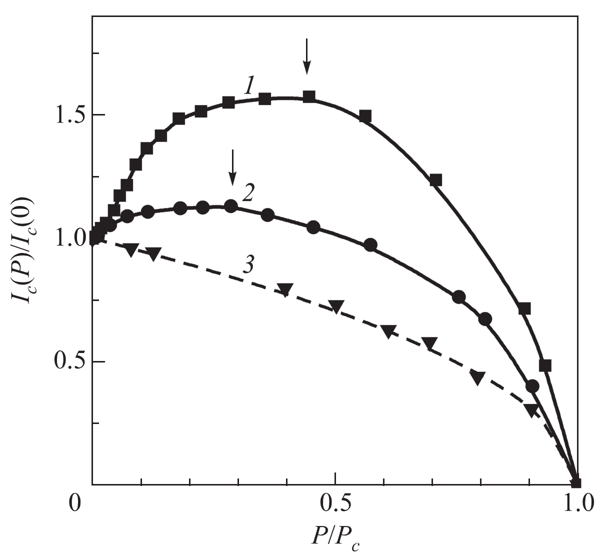

FIG. 4. The dependence of the relative critical current $I_c(P)/I_c(0)$ in the sample SnW10 on the reduced microwave radiation power P/P_C at $T = 3.777$ K for different radiation frequencies f, GHz: 12.91 (■), 6.15 (●), 0.63 (▼); dashed curve *3* is the dependence $I_c(P)/I_c(0)$ (P/P_C) calculated by Eq. (21).

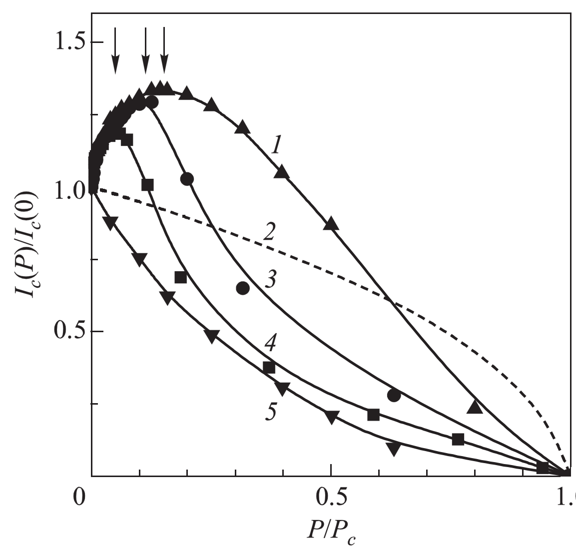

FIG. 5. The dependence of the relative critical current $I_c(P)/I_c(0)$ in the sample SnW5 on the reduced microwave radiation power P/P_C at $T = 3.744$ K for different radiation frequencies f, GHz: 15.2 (▲), 11.9 (●), 9.2 (■), 5.6 (▼). The dashed curve *2* is the dependence $I_c(P)/I_c(0)$ (P/P_C) calculated by Eq. (21).

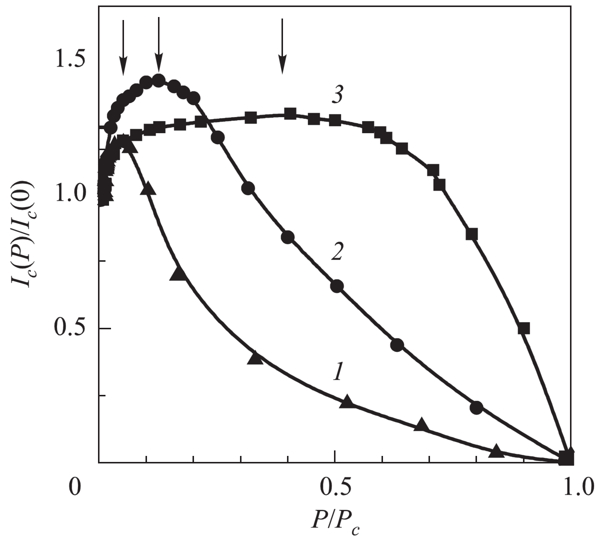

FIG. 6. The dependence of the relative critical current $I_c(P)/I_c(0)$ on the reduced microwave radiation power P/P_C with the frequency $f = 9.2$ GHz at $t = T/T_c \approx 0.99$ in different samples: SnW5 (▲), SnW6 (●) and SnW10 (■).

The concavity of descending parts can be associated, as for the curve *5*, with non-uniform current distribution over the sample width.

Interestingly, in the narrow film Sn1 the stimulation effect is already clearly seen at the radiation frequency $f = 3.7$ GHz (see Fig. 2, curve *3*), while in the film SnW5 it is not observed even at $f = 5.6$ GHz (see Fig. 5, curve *5*). The calculation of f_L for the film by the formula (11) gives the value of 5.1 GHz, which no longer corresponds to the experimental value of 8.0 GHz. It is important to emphasize that for calculation of the lower cutoff frequency of stimulation of the sample SnW5, the value $\tau_\varepsilon = 4 \times 10^{-10}$ s typical of this series of samples was used.

The dependencies $I_c(P)$ in relative units for films of different widths for the same experimental conditions are shown in Fig. 6. The arrows in Fig. 6 have the same meaning as in Figs. 2 and 4. In Fig. 6 it is seen that with growth of the film width the ratio P/P_c, at which there is a maximum stimulation effect, is reduced, and the effect of stimulation of the critical current in wider films is observed at lower radiation power, since in the microwave range the value of P_c is practically independent of frequency.[16,22] Fig. 7 shows the dependence of

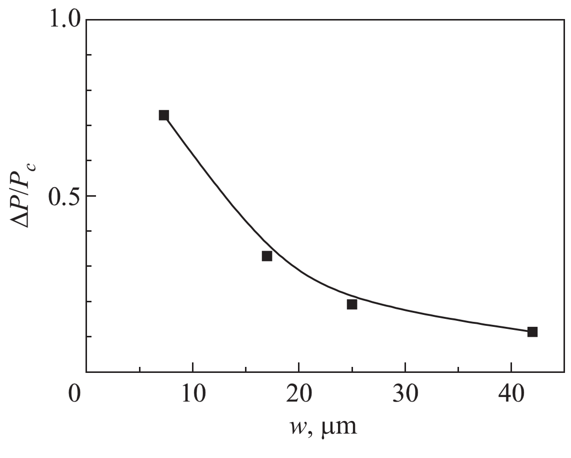

FIG. 7. A region of external radiation power $\Delta P/P_c$, where the effect of stimulation of the critical current is observed, as a function of the film width w for the frequency of 9.2 GHz at $T/T_c \approx 0.99$.

power region of external radiation, $\Delta P/P_c$, where the effect of stimulation of the critical current is observed, on the film width w at a fixed radiation frequency and temperature. From the data in the figure it follows that as the film width increases the power range ΔP, where the effect of superconductivity stimulation is observed, is reduced. Therefore, one can assume that for rather wide films ($w > 100\,\mu$m) the effect of superconductivity stimulation can be practically unrealizable in experiment as due to a very narrow power range of existence of this effect and because of its small size.

4.2. A maximum current of the existence of vortex resistivity

In Sec. 4.1 we have found out how electromagnetic radiation affects the critical current I_c of films of different widths. Another important characteristic current of a wide film is the so-called maximum current of the existence of vortex resistivity I_m. Experimentally the current I_m was studied in Ref. 20 and has the form[6]

$$I_m(T) = C I_c^{GL}(T) 1\mathrm{n}^{(-1/2)}(2w/\lambda_\perp(T)). \tag{22}$$

To date there is no theory of superconductivity stimulation, and therefore, at present the results of experimental studies of stimulation of $I_m(T)$ cannot be compared with theoretical predictions. However, form Eq. (22) obtained for the equilibrium (without external radiation) current $I_m(T)$, it can be assumed that the behavior of $I_m(P,f)$ in an electromagnetic field is determined by the effect of the radiation on $I_c^{GL}(T)$ and $\lambda_\perp(T)$.

Upon stimulation of superconductivity $I_c^{GL}(T)$ increases, and $\lambda_\perp(T)$, according to general considerations (stimulation of T_c), must decrease. The reduction rate of $\lambda_\perp(T)$ also depends on the proximity of the operating temperature T to T_c, other things being equal. Thus, it is clear qualitatively that the rate of increasing $I_m(P)$ must be lower than the growth rate of $I_c(P)$

Fig. 8 shows the experimental dependencies $I_c(P)$ and $I_m(P)$ for the film SnW5.[23] The inset shows the initial parts of the curves for a more visual representation of the growth rate of $I_c(P)$ and $I_m(P)$. In the figure it is seen that indeed with increasing the radiation power of the film its critical current $I_c(P)$ increases faster than the current $I_m(P)$. The question arises whether there is enough change in $\lambda_\perp$ under electromagnetic radiation to suppress the growth of $I_m(P)$ in comparison with an increase of $I_c(P)$. An estimation of changes in $\lambda_\perp(P)$ at the relative temperature at which the curve in Fig. 8 was measured, indicates that they are sufficiently small, and in accordance with Eq. (22) they cannot slow down significantly an increase of $I_m(P)$. Therefore, there must be another reason. An analysis of experimental data suggests that the reason may be the non-uniformity of the current distribution over the film width and the presence of resistive vortex background, which affects I_m. This background also depends on an external radiation, what Eq. (22) for the equilibrium current I_m ignores. In this context, it is necessary to draw attention to the fundamental difference between I_c and I_m. I_c always appears against a background of the pure superconducting state. So due to the transverse Meissner effect it is always first achieved at the film edges. And the larger its width with respect to $\lambda_\perp$, the more inhomogeneous distribution of the transport current is in it. In

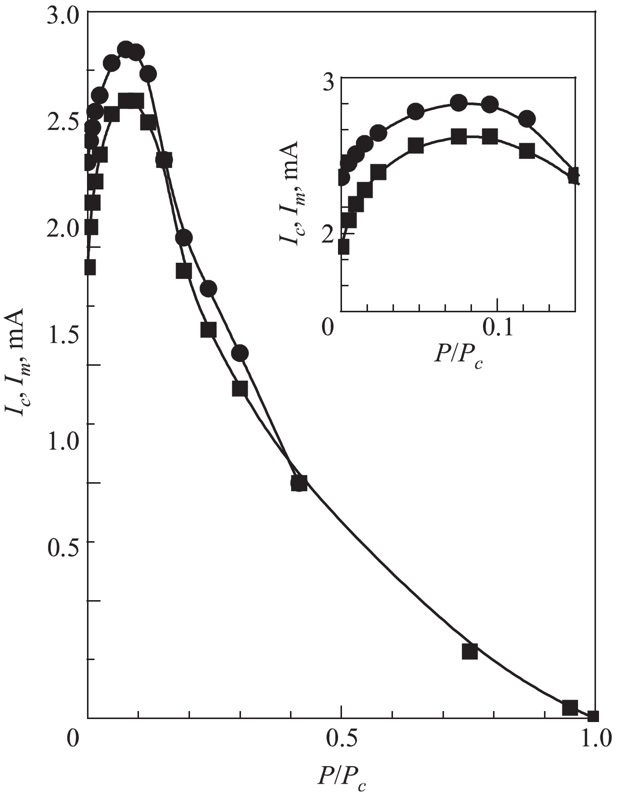

FIG. 8. The dependence of the critical current I_c (■) and the maximum current of existence of the vortex resistivity I_m (●) for the sample SnW5 on the reduced microwave power P/P_C with the frequency $f = 12.89$ GHz at $T = 3.748$ K. The inset shows an enlarged fragment of the above-mentioned dependencies.

contrast, the current I_m is the maximum current at which a uniform flow of vortices across the film is still possible. The presence of a moving vortex lattice makes the distribution of the superconducting current across the film more uniform, although specific.[6] Thus, in a wide film being in a vortex-free state at $I \leq I_c$ the current is always more non-uniformly distributed over the width than in the same film in the presence of intrinsic vortices for the currents $I_c < I \leq I_m$. Because of the above reasons, there is a need of new theory of none-quilibrium state of a wide film, which could take into account the non-uniform distribution of the transport current and the order parameter over the film width in calculating $I_c(P,f)$ and the presence of the vortex resistivity $R(P,f)$ when calculating $I_m(P,f)$.

5. Temperature dependencies of currents stimulated by microwave radiation in wide tin films

5.1. The critical current

This section presents results of systematic study of the critical current stimulation in wide superconducting films. It is established that the main properties of superconductivity stimulation in wide films with non-uniform current distribution over the cross section of a sample and those in narrow channels are very similar.[19,24] A relative moderation of the current non-uniformity in wide films near T_c allowed for using, with a little change, the theory of superconductivity stimulation in spatially homogeneous systems to interpret experimental results in wide films.

Fig. 9 shows experimental temperature dependencies of the critical current for the sample SnW10.[24] At first, a behavior of $I_c(T)$ without an external electromagnetic field (see Fig. 9, (■)) is considered. A width of the film SnW10 is relatively small ($w = 7\,\mu$m), so in the temperature range $T_{\rm cros1} < T < T_c = 3.809$ K close enough to T_c, the sample behaves like a narrow channel, and the critical current is equal to the depairing current of Ginzburg-Landau $I_c^{GL}(T) \propto (1 - T/T_c^P)^{3/2}$ which indicates the high quality of the sample. The crossover temperature $T_{\rm cros1} = 3.769$ K corresponds to the transition of the sample in the wide film regime: at $T < T_{\rm cros1}$ there is a vortex part in the CVC. The temperature dependence $I_c(T)$ at $T < T_{\rm cros1}$ initially retains the form $(1 - T/T_c)^{3/2}$, although the value of $I_c(T)$ turns out to be less than the depairing current $I_c^{GL}(T)$ due to the appearance of a non-uniform distribution of the current density and its decrease far away from the film edges. Finally, when $T < T_{\rm cros2} = 3.717$ K the temperature dependence of the critical current becomes linear $I_c(T) = I_c^{AL}(T) = 9.12 \times 10^1(1 - T/T_c)$mA, which corresponds to the Aslamazov-Lempitsky theory.[6] The latter fact confirms our earlier conclusion about the high quality of the film sample SnW10.

For measurements in a microwave field[24] the radiation power was chosen in such a way that the critical current $I_c^P(T)$ was maximal. Consider the behavior of $I_c^P(T)$ in the sample SnW10 in a microwave field with a frequency $f = 9.2$ GHz (Fig. 9, (●)). In the temperature range $(T_{\rm cros1}^P(9.2\,\text{GHz}) < T < T_c^P(9.2\,\text{GHz}))$ $(T_{\rm cros1}^P(9.2\,\text{GHz}) = 3.744\,\text{K}, T_c^P(9.2\,\text{GHz}) = 3.818\,\text{K}))$ there is no vortex part in the CVC, i.e., the sample behaves as a narrow channel. Note that $T_{\rm cros1}^P(9.2\,\text{GHz}) < T_{\rm cros1}(P=0)$, whereas $T_c < T_c^P$, i.e., at optimal stimulation the narrow channel regime retains in a

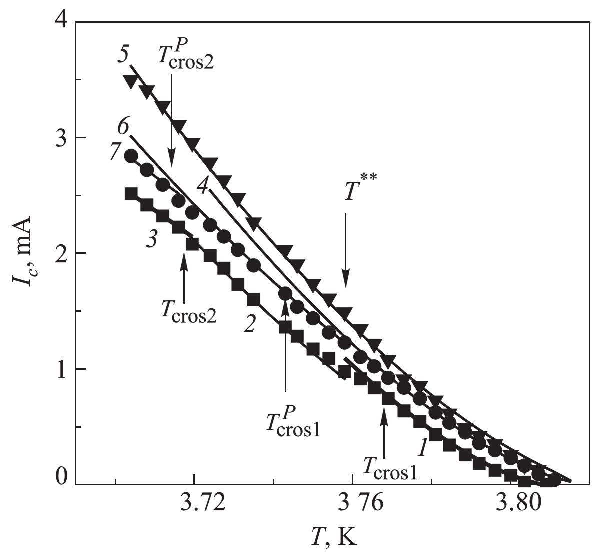

FIG. 9. The experimental temperature dependence of the critical currents $I_c(P=0)$ (■), $I_c(f = 9.2\,\text{GHz})$ (●), and $I_c(f = 12.9\,\text{GHz})$ (▼) for the sample SnW10. The theoretical dependence $I_c^{GL}(T) = 7.07 \times 10^2(1 - T/T_c)^{3/2}$ mA calculated by Eq. (19),[25,26] (curve *1*); calculated dependence $I_c(T) = 5.9 \times 10^2(1 - T/T_c)^{3/2}$ mA (curve *2*); theoretical dependence $I_c^{AL}(T) = 9.12 \times 10^1(1 - T/T_c)$ mA calculated by Eq. (27),[6] (straight line *3*); theoretical dependence $I_c(f = 9.2\,\text{GHz})$ calculated by Eq. (16) and fitting dependence $I_c(T) = 6.5 \times 10^2(1 - T/3.818)^{3/2}$ mA (curve *4*); theoretical dependence $I_c(f = 12.9\,\text{GHz})$ calculated by Eq. (16), and fitting dependence $I_c(T) = 6.7 \times 10^2(1 - T/3.822)^{3/2}$ mA (curve *5*); theoretical dependence $I_c(f = 9.2\,\text{GHz})$ calculated by Eq. (16) normalized by the curve *2*, and fitting dependence $I_c(T) = 5.9 \times 10^2(1 - T/3.818)^{3/2}$ mA (curve *6*); calculated dependence $I_c(T) = 9.4 \times 10^1(1 - T/3.818)$ mA (straight line *7*).

broader temperature range than in the equilibrium condition. In the temperature range $T^{**} = 3.760 < T < T_c^P$ the experimental values of $I_c^P(T)$ (see Fig. 9, (●)) are in good agreement with those calculated by Eq. (16) (Fig. 9, curve *4*), in which the microwave power (the value of α) is a fitting parameter.[19]

However, at $T < T^{**}$ the experimental values of $I_c^P(T)$ are lower than the theoretical curve *4*. It should be noted that such a deviation from the theory was observed in the study of narrow aluminum film channels.[14] Nevertheless, the experimental points fall well on the curve *6* (Fig. 9). This curve is calculated by the formula (16), normalized by an additional numerical factor which has provided consent to this formula at zero microwave field to the equilibrium critical current $I_c(T)$. The authors of Ref. 19 consider this as a form factor, which is understood as an estimate of non-uniformity of the current distribution over the film width. Finally, at temperatures $T < T_{\rm cros2}^P(9.2\,{\rm GHz}) = 3.717\,{\rm K}$ the temperature dependence of the stimulated critical current becomes linear (Fig. 9, straight line *7*).

Fig. 9 also shows the temperature dependence of the highest stimulated critical current of the sample SnW10 at a higher radiation frequency $f = 12.9\,{\rm GHz}$ (Fig. 9, (▼)).[24] It can be seen that, as in a narrow channel, the highest stimulated critical current increases with the radiation frequency. Note that at the given radiation frequency there is no vortex part in the CVC over the entire temperature range investigated (up to temperatures $T = 3.700\,{\rm K}$ and even a bit lower). In other words, in the temperature range $T_{\rm cros1}^P(12.9\,{\rm GHz}) < T < T_c^P(12.9\,{\rm GHz})$ ($T_{\rm cros1}^P(12.9\,{\rm GHz}) < 3.700\,{\rm K}$ and it is not shown in Fig. 9; $T_c^P(12.9\,{\rm GHz}) = 3.822\,{\rm K}$) the sample behaves as a narrow channel. Note that $T_{\rm cros1}^P(12.9\,{\rm GHz}) < T_{\rm cros1}^P(9.2\,{\rm GHz}) < T_{\rm cros1}(P=0)$, whereas $T_c < T_c^P(9.2\,{\rm GHz}) < T_c^P(12.9\,{\rm GHz})$. Thus, in conditions of optimal stimulation of superconductivity the temperature range where the sample behaves as a narrow channel increases with the radiation frequency.[24]

It is also important to note that the experimental dependence $I_c^P(T)$ (▼) at $f = 12.9\,{\rm GHz}$ is in good agreement with the theoretical one obtained in calculating the stimulated critical current by Eq. (16) for a narrow channel (Fig. 9, curve *5*) over the entire temperature range, and is well approximated by the dependence $I_c(T) = 6.7\times10^2\times(1 - T/3.822)^{3/2}\,{\rm mA}$. Hence, it follows that the temperature $T_{\rm cros1}^P$ of the transition to the wide film regime, where the vortex region appears in the CVC, as well as the deviation temperature T^{**} of the experimental dependence $I_c^P(T)$ from the dependence, calculated by Eq. (16), decrease with increasing the radiation frequency.[24]

Fig. 10 presents the temperature dependencies of the critical current I_c for the sample SnW8.[24] First, consider behavior of $I_c(T)$ without an external electromagnetic field. A width of the film is large enough ($w = 25\,\mu{\rm m}$), so this sample is a narrow channel only in the immediate vicinity of $T_c = 3.816\,{\rm K}$, and at $T < T_{\rm cros1} = 3.808\,{\rm K}$ behaves as a wide film. At $T_{\rm cros2} = 3.740\,{\rm K} < T < T_{\rm cros1}$ the temperature dependence of the critical current has the form $(1 - T/T_c)^{3/2}$, although the value of I_c is less than I_c^{GL}. At $T < T_{\rm cros2}$ the temperature dependence of the critical current becomes linear and corresponds to the Aslamazov-Lempitsky theory:[6] $I_c(T) = I_c^{AL}(T) = 1.47\times10^2(1 - T/T_c)\,{\rm mA}$.

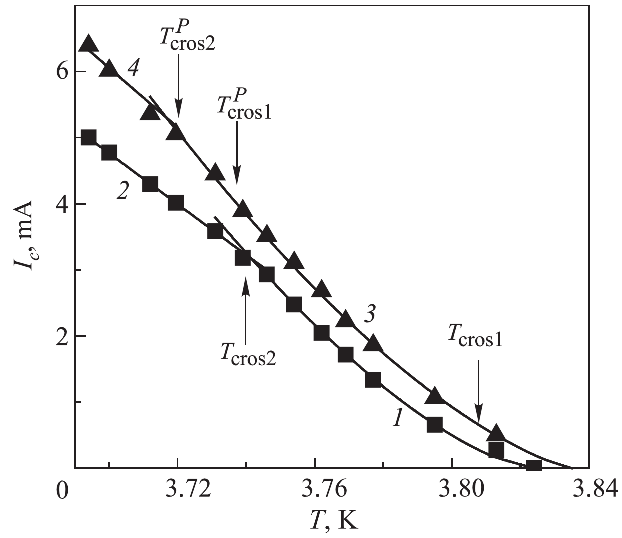

FIG. 10. The experimental temperature dependencies of the critical currents $I_c(P=0)$ (■), $I_c(f = 15.2\,{\rm GHz})$ (▲) for the sample SnW8: calculated dependence $I_c(T) = 1.0\times10^3(1 - T/T_c)^{3/2}\,{\rm mA}$ (curve *1*); theoretical dependence $I_c^{AL}(T) = 1.47\times10^2(1 - T/T_c)^{3/2}$ mA calculated by Eq. (27),[6] (straight line *2*); theoretical dependence $I_c(f = 15.2\,{\rm GHz})$ calculated by Eq. (16) normalized by the curve *1*, and fitting dependence $I_c(T) = 1.0\times10^3(1 - T/3.835)^{3/2}\,{\rm mA}$ (curve *3*); calculated dependence $I_c(T) = 1.72\times10^2(1 - T/3.835)$ mA (straight line *4*).

In a microwave field with a frequency $f = 15.2\,{\rm GHz}$, similar to the narrow sample SnW10, there is an increase of the critical temperature $T_c^P(15.2\,{\rm GHz}) = 3.835\,{\rm K}$ and a noticeable decrease of crossover temperatures: $T_{\rm cros1}^P = 3.738\,{\rm K}$ and $T_{\rm cros2}^P = 3.720\,{\rm K}$. At the same time, in order to achieve a good agreement between the experimental dependence of the highest stimulated critical current $I_c^P(T)$ and Eq. (16), it is necessary to normalize this formula by the measured equilibrium ($P = 0$) critical current $I_c(T) = 1.0\times10^3(1 - T/T_c)^{3/2}\,{\rm mA}$ over the entire temperature range measured (Fig. 10, curve *3*). In this temperature range, the critical current can be approximated by the dependence: $I_c(T) = 1.0\times10^3(1 - T/3.835)^{3/2}\,{\rm mA}$. When $T < T_{\rm cros2}^P$ the temperature dependence $I_c^P(T)$ is linear (Fig. 10, straight line *4*). From the data in Fig. 10 it follows that under stimulation of superconductivity by a microwave field even a fairly wide film behaves as a narrow channel down to low temperatures than that without radiation ($T_{\rm cros1}^P < T_{\rm cros1}$); the vortex region in the CVC in this temperature range is also absent.[24]

A qualitative similarity between obtained in Refs. 19 and 24 results of studies of wide films and experimental results of studies of narrow channels,[15] and the ability to quantitatively describe the temperature dependence of I_c^P in wide films using equations of the Eliashberg theory suggest that the mechanism of the stimulation effect is common for both wide films and narrow channels.[19,24] It lies in increasing the energy gap, caused by the redistribution of nonequilibrium quasiparticles to higher energies under a microwave field.[3] This conclusion is not entirely obvious for wide films with a non-uniform current distribution across the sample width.

A similarity of stimulation mechanisms in narrow channels and wide films can be supported by the following observations. Despite an increase of the current density, near the edges of a wide film the main current, both transport and induced by a microwave field, is distributed over the entire film width. Thus, the nonequilibrium of quasiparticles in a wide film, as in a narrow channel, is excited by a microwave field within an entire volume of a superconductor, and therefore, the effect of stimulation in wide films undergoes a certain quantitative modification due to non-uniform current distribution. In this regard, we emphasize a significant difference in the conditions of formation of nonequilibrium under a microwave field in wide films and bulk superconductors, in which up to now the effect of stimulation was not observed. In the latter case, the total current is concentrated in a thin Meissner layer of the thickness of λ near the surface of the metal, which leads to an additional relaxation mechanism: the spatial diffusion of nonequilibrium quasiparticles excited by the microwave field, from the surface into the equilibrium volume. An intensity of this mechanism is determined by time during which quasiparticles $\tau_D(T) = \lambda^2(T)/D$ leave Meissner layer, which at typical temperature is three to four orders of magnitude less than the inelastic relaxation time. This high efficiency of the diffusion mechanism of relaxation, apparently, leads to suppression of the effect of stimulation in bulk superconducting samples.[19]

The current state of a wide film, despite the semblance with the Meissner state of a bulk current-carrying superconductor is qualitatively different from the latter. While in a bulk superconductor the transport current is concentrated in a thin surface layer and decays exponentially at a distance of the London penetration depth $\lambda(T)$ away from the surface, in a wide film the current is distributed over its entire width w according to approximating power law $[x(w-x)]^{-1/2}$, where x is the transverse coordinate.[6] Thus, the characteristic length $\lambda_\perp(T) = 2\lambda^2(T)/d$ (d is the film thickness), called in the theory of the current state of wide films usually as a penetration depth of perpendicular magnetic field, in fact defines not a spatial scale of the current decay for outgoing from the edges, but a magnitude of the edge of the edge current density, acting as a "cutoff" factor in the above-mentioned law of the current density distribution at distances x, $w - x \sim \lambda_\perp$ away from the film edges.[19]

Being based on a qualitative difference between the current states in bulk and thin-film superconductors, one can argue that moderate non-uniformity of the current distribution in wide films does not cause fatal consequences for the effect of stimulation, and that the diffusion of nonequilibrium quasiparticles excited in the whole bulk of a film, making only minor quantitative deviations from the Eliashberg theory.[19]

The authors of Ref. 19 used a modeling approach to account for these deviations by introducing a numerical form factor of the current distribution in Eq. (16) for the stimulated critical current of Eliashberg. They evaluated this form factor by fitting the limiting case of Eq. (16) at zero-power microwave radiation, i.e., Eq. (17) for measured values of the equilibrium critical current. They then used the obtained values of the form factor in Eq. (16) for $P \neq 0$, to yield a good agreement with the experimental data (see Fig. 9).

Noteworthy is the question of how the Eliashberg's mechanism "works" in a wide film in narrow channels the superconductivity is destroyed due to the mechanism of homogeneous depairing of Ginzburg-Landau, while in a wide film the superconductivity is destroyed due to the emergence of vortices. We believe that in this case the stimulation of the energy gap leads to a corresponding increase of the barrier for entering vortices, and that stimulates the critical current in a wide film.[19] It is interesting to note that there are no significant features in the curves $I_c(T)$, when upon decreasing the temperature the films go from the regime of a narrow channel to the regime of a wide vortex film. One can therefore conclude that the transition between the regimes of a uniform depairing and vortex resistivity affect neither the value nor the temperature dependences of the critical current.

To complete the discussion of the effect of superconductivity stimulation, we attract an attention to the empirical fact that all the theoretical curves for $I_c^P(T)$, derived from the equations of the Eliashberg theory, are well approximated by a power law $(1 - T/T_c^P)^{3/2}$. This law is very similar to the temperature dependence of the depairing current of Ginzburg-Landau, in which the critical temperature T_c is replaced with its stimulated value T_c^P. The explicit expressions for such approximating dependencies with numerical coefficients are given in the captions to Figs. 9 and 10.

The other important result of these studies is significant expansion of temperature range near the superconducting transition temperature, where a film behaves as a narrow channel during stimulation of superconductivity: In a microwave field, the crossover temperature in the wide film regime T_{cros1}^P is significantly reduced compared to its equilibrium value T_{cros1}, while $T_c^P > T_c$. At first glance, this result is somewhat contrary to the criterion of the transition between the different regimes of a superconducting film: $w = 4\lambda_\perp(T_{cros1})$,[13] since an increase of the energy gap upon radiation implies a reduction of the $\lambda_\perp$ and, consequently, reduction of the characteristic size of vortices. This obviously makes easier the conditions for entry of vortices into a film. Consequently, the crossover temperature is expected to increase in a microwave field. However, it appears that the mechanism of an influence of microwave radiation on vortices is somewhat different. So, it was found that a wide film with a vortex region in CVC under a microwave field behaves like a narrow channel: The vortex region in CVC disappears (see, e.g., Fig. 9, (▼)). It should be noted that this kind of CVC may be in two cases. In the first case, under the influence of a microwave field there is delay in motion of vortices up to the point of their termination, i.e., vortices appear, but under a microwave radiation, they do not move. In the second case the vortices do not appear at all. Turn back to Fig. 9. In the temperature range $T_{cros1} = 3.769$ K $< T < T_c = 3.809$ K without microwave radiation the sample SnW10 is a narrow channel. Under microwave field with the frequency $f = 12.9$ GHz in this sample there is an increase of the critical current (stimulation of superconductivity) (see Fig. 9 (▼)). At the same time, it is important to emphasize that the temperature dependence of the stimulated critical current agrees well with the theoretical dependence $I_c^P(T)$ (see Fig. 9, curve *5*), plotted in accordance with the Eliashberg theory for a narrow superconducting channel with a uniform current distribution over the cross section of the sample. It is interesting to note that this theoretical dependence coincides well with experimental points (see Fig. 9 (▼)) not only in the temperature range $T_{cros1} = 3.769$ K $< T < T_c = 3.809$ K, in which the sample SnW10 is a narrow channel in the absence of microwave radiation, but at much lower temperatures (up to $T < 3.700\,\text{K}$). This behavior of $I_c^P(T)$ suggests that under the microwave field of $f = 12.9\,\text{GHz}$ in the temperature range $3.700\,\text{K} < T < T_c^P$ there are no vortices in the sample SnW10. Otherwise, in Fig. 9, in the temperature range $3.700\,\text{K} < T < 3.769\,\text{K}$ the values of $I_c^P(T)$ would be lower compared with the theoretical curve *5* calculated within the Eliashberg theory. Moreover, the crossover would be seen in the dependence $I_c^P(T)$ upon entering vortices. The suppression of vortex resistivity in a wide film by a microwave field is discussed in more detail in Ref. 25.

Thus, referring to Fig. 9, one can say the following. For the radiation frequency $f = 12.9\,\text{GHz}$ the maximum value of $I_c^P(T)$ in the sample SnW10 is realized at high ($P/P_c = 0.45$) power of an external microwave field, which prevents the formation of vortices, so the sample behaves as a narrow channel in the temperature range from T_c^P up to $T < 3.700\,\text{K}$. In this case, the curve *5* in Fig. 9, plotted using Eq. (16) of the Eliashberg theory for a narrow channel, and giving the depairing current density of Ginzburg-Landau at $P = 0$ is in a good agreement with the experimental curve $I_c^P(T)$ (Fig. 9, (▼)). On this basis, it can be argued that in this case due to a microwave field the sample becomes a narrow channel (there is no vortex region in CVC and $I_c^P(T)$ is fully consistent with the formula (16) of the Eliashberg theory, assuming a uniform distribution of the superconducting current over the cross section of the sample).

As the radiation frequency ($f = 9.2\,\text{GHz}$) decreases the power at which the maximum value of $I_c^P(T)$ is realized and consequently its influence are reduced. This leads to a smaller decrease of T_{cros1}^P with respect to T_{cros1}. It is important to note that in this case too, the experimental dependence $I_c^P(T)$ (see Fig. 9, (●)) agrees quite well with the curve *4* in Fig. 9, plotted according to Eq. (16) for a narrow channel, up to the temperature of $T^{**} = 3.760\,\text{K} < T_{\text{cros1}} = 3.769\,\text{K}$. At temperatures $T_{\text{cros1}}^P < T < T^{**}$ for the sample SnW10 there is no vortex region in CVC, but $I_c^P(T)$ deviates downward from the theoretical curve *4* in Fig. 9 plotted for a narrow channel and normalized in such a way that it gives the depairing current of Ginzburg-Landau at $P = 0$.

As can be seen from Eqs. (17) and (18) in the Eliashberg theory the expression for the critical current at $P = 0$, stimulated by a microwave field, transforms to the formula for the depairing current of Ginzburg-Landau. Like the whole theory, this is true only in the case of a narrow channel. At the same time, at temperatures $T < T_{\text{cros1}}$ the SnW10 film reveals itself as wide (there appear a vortex region in CVC), the distribution of the superconducting current over its cross section becomes non-uniform, and the critical current $I_c(T) = 5.9 \times 10^2(1 - T/T_c)^{3/2}$ mA of this film at $P = 0$ is less than the depairing current $I_c(T) = 5.9 \times 10^2(1 - T/T_c)^{3/2}$ mA, although the temperature dependence is preserved. It is interesting to note that $I_c^{GL}(T) = 7.07 \times 10^2(1 - T/T_c)^{3/2}$ for the sample SnW10. In this case, it turns out that if a normalization factor in Eq. (16) is introduced so that at $P = 0$ it will give not $I_c^{GL}(T)$ but $I_c(T)$, then using the formula one can plot a curve (see Fig. 9, curve *6*), which is in a good agreement with the experimental dependence $I_c^P(T)$. It should be noted that in this case, the normalization factor is 0.83.[19] An introduction of a universal normalization factor over the entire temperature range $T_{\text{cros2}} < T < T_{\text{cros1}}$ is possible due to the fact that the temperature dependence $I_c^P(T)$, described by Eq. (16), although is quite complex, yet is numerically very close to the law $\propto (1 - T/T_c^P)^{3/2}$, which at $P = 0$ transforms to the dependence $I_c(T) \propto (1 - T/T_c)^{3/2}$ for a wide film.

A similar situation is observed for a much wider film SnW8. This sample is a narrow channel only in the immediate vicinity of the T_c. Therefore, for temperatures $T < T_{\text{cros1}} = 3.808\,\text{K}$ Eq. (16) gives the values of the stimulated critical current that do not coincide with the experimental values of $I_c^P(T)$. However, for normalization of Eq. (16) to the equilibrium critical current $I_c(T) = 1 \times 10^3(1 - T/T_c)^{3/2}$ mA at $P = 0$ there is also a good agreement between theory and experiment (Fig. 10, curve *3*). Note that in this case too, the normalization factor of Eq. (16) of the Eliashberg theory is the same as the ratio $I_c(T)/I_c^{GL}(T)$.

Thus, we conclude that if the equilibrium critical current ($P = 0$) of a wide film has the temperature dependence $I_c(T) \propto (1 - T/T_c)^{3/2}$, typical for a narrow channel, then using the formula of the Eliashberg theory, normalized to $I_c(T)$, one can well describe the experimentally measured dependencies of the stimulated critical current $I_c^P(T)$, which are numerically very close to $(1 - T/T_c^P)^{3/2}$. In the temperature range $T < T_{\text{cros2}}^P$, where the temperature dependence of the critical current for a wide film is linear, $I_c(T) \propto 1 - T/T_c$, the temperature dependence of the stimulated critical current is also linear: $I_c^P(T) \propto 1 - T/T_c^P$. These facts, albeit indirectly, confirm the hypothesis that the mechanism of superconductivity stimulation in wide films is the same as in narrow channels.

5.2. The current of phase-slip processes

This section presents the results of experimental study of stimulation of the current at which the first PSL, $I_m(T)$, is formed, in a wide temperature range under an external microwave radiation of different frequencies.[23]

Taking into account the fact that there is no theory of superconductivity stimulation in wide films, one can try at least qualitatively to describe the effect of microwave radiation on the current $I_m(T)$ using the following considerations. In studies of the critical current stimulation in superconducting films with different width, the following experimental facts were obtained.[24] In narrow channel, the equilibrium critical current has the temperature dependence $I_c^{GL}(T) \propto (1 - T/T_c)^{3/2}$. At the same time, the stimulated critical current $I_c^P(T)$ of this channel, perfectly described by the Eliashberg theory,[3,9–12] can be well approximated by the dependence $I_c^P(T) \propto (1 - T/T_c^P)^{3/2}$.[14] Here, T_c^P is the stimulated critical temperature. In a wide (vortex) film near T_c the temperature dependence of the equilibrium critical current is $I_c(T) \propto (1 - T/T_c)^{3/2}$.[20] It turns out that the critical current stimulated by a microwave field in this case can also be well approximated by a similar dependence: $I_c^P(T) \propto (1 - T/T_c^P)^{3/2}$.[24] When $T < T_{\text{cros2}}$ in a wide film there is a linear temperature dependence of the equilibrium critical current.[20] Almost at the same temperatures the stimulated critical current can also be approximated by the linear dependence $I_c^P(T) \propto (1 - T/T_c^P)$.[24] Based on the above experimental facts, one can also try to approximate the temperature dependencies of the current $I_m^P(T)$, stimulated by a microwave field, by a dependence similar to Eq. (22) for the equilibrium case.

Fig. 11 shows the experimental temperature dependencies of the currents $I_m^P(T)$ in a microwave field and currents $I_m(T)$ in the absence of the field for the sample SnW5.[23] For clarity, in Fig. 11(b) the results are shown for a narrower temperature range near T_c than that in Fig. 11(a). A width of the film SnW5 is large enough ($w = 42\,\mu$m), so even for temperatures $T < T_{\rm cros2} = 3.740$ K there is a linear temperature dependence of the critical current,[20] what is close enough to T_c.

First, we consider the behavior of the current at which the first PSL appears, $I_m(T)$, without an external electromagnetic field (see Fig. 11, (●)). The solid curves *1* in these figures are calculations of $I_m(T)$ according to Eq. (22) with taking into account the film parameters (see Table 1)

$$I_m(T) = 2.867 \times 10^3(1 - T/T_c)^{3/2} \times 1.35\,[\ln(2 \times 42 \times (1 - T/T_c)/0.02532]^{1/2}[\text{mA}]. \tag{23}$$

As can be seen in Fig. 11, the experimental dependence $I_m(T)$ is in a good agreement with calculated one (see curve *1*).[23] The experimental dependence of the current $I_m^P(T)$ at the radiation frequency $f = 9.2$ GHz (see Fig. 11, (▼)) is well approximated by the dependence

$$I_m^P(T) = 2.869 \times 10^3(1 - T/T_{c1}^P)^{3/2} \times 1.44 \times [\ln(2 \times 42 \times (1 - T/T_{c1}^P)/0.02531]^{-1/2}[\text{mA}], \tag{24}$$

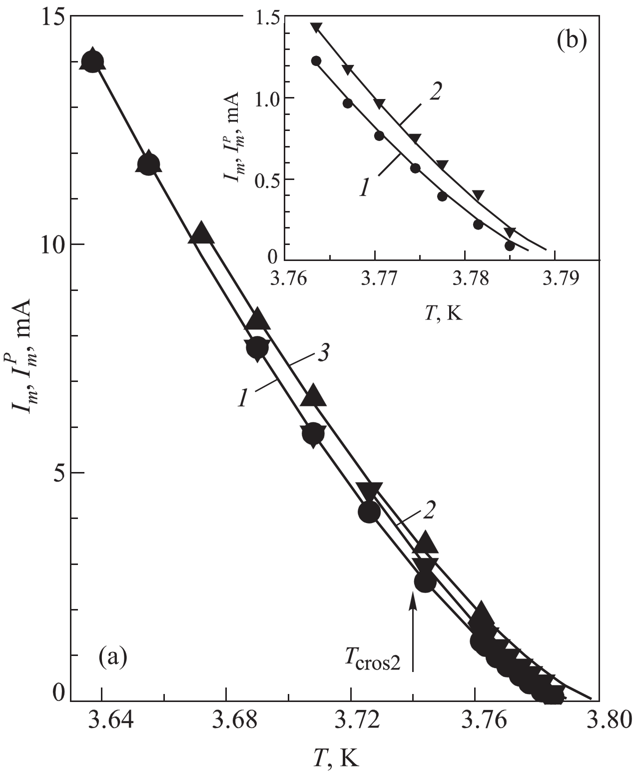

FIG. 11. The experimental temperature dependencies of the maximum current I_m of existence of stationary uniform flow of intrinsic vortices of transport current across the film SnW5: $I_m(T, P = 0)$ (●), $I_m^P(T, f = 9.2\,\text{GHz})$ (▼) and $I_m^P(T, f = 15.2\,\text{GHz})$ (▲). Curve *1* is the theoretical dependence (see Eq. (23)); curve *2* is the calculated dependence $I_m^P(T, f = 9.2\,\text{GHz})$ (see Eq. (24)); curve *3* is the calculated dependence $I_m^P(T, f = 15.2\,\text{GHz})$ (see Eq. (26)).

similar to Eq. (22) (Fig. 11, curve *2*). Here and in the calculation of the depairing current of Ginzburg-Landau, the stimulated critical temperature $T_{c1}^P = 3.791$ K was used. The experimental dependence of the current $I_m^P(T)$ at the frequency of an external electromagnetic field $f = 12.9$ GHz (not shown due to space limitations) is well approximated by

$$I_m(T) = (2.875 \times 10^3 \times (1 - T/T_{c3}^P)^{3/2}) \times 1.28 \times [\ln(2 \times 42 \times (1 - T/T_{c3}^P)/0.02529]^{(-1/2)}[\text{mA}]. \tag{25}$$

Here, the stimulated critical temperature $T_{c3}^P = 3.797$ K was also used. The experimental dependence of the current $I_m^P(T)$ at the frequency of microwave field $f = 15.2$ GHz (see Fig. 11, (▲)) is well approximated by

$$I_m^P(T) = 2.877 \times 10^3(1 - T/T_{c2}^P)^{3/2} \times 1.28 \times [\ln(2 \times 42 \times (1 - T/T_{c2}^P)/0.02528]^{-1/2}[\text{mA}], \tag{26}$$

similar to Eq. (22) (Fig. 11(a), curve *3*). Here, the stimulated critical temperature $T_{c2}^P = 3.799$ K was used.

For measurements of the current $I_m^P(T)$ of films in microwave field the radiation power was chosen in such a way that the critical current $I_c^P(T)$ was maximal. In this case the current $I_m^P(T)$ was also maximal due to some correlation of these quantities.[21]

Since the theory,[6] in which the definition $I_m(T)$ is introduced, assumes a linear temperature dependence of the critical current

$$I_c^{AL}(T) = 1.5 I_c^{GL}(0)(\pi\lambda_\perp(0)/w)^{1/2}(1 - T/T_c), \tag{27}$$

then, strictly speaking, Eq. (22) should be applicable only in the temperature range $T < T_{\rm cros2}$, where such a dependence of the critical current is observed. However, as seen in Fig. 11, Eq. (22) for the equilibrium dependence $I_m(T)$ and Eqs. (24)–(26) for the case of stimulation of $I_m^P(T)$ by an electromagnetic field sufficiently well describe the experimental dependencies in the case of $T > T_{\rm cros2}$ too. This is obviously due to the fact that at $T < T_{\rm cros2}$ and $T > T_{\rm cros2}$ the resistive current states at $I \simeq I_m$ differ little from each other: both of these states are characterized by fairly uniform current distribution over the width of the sample due to the quite dense filling of the film by a lattice of vortices.[23]

Thus, the experimental temperature dependencies of the stimulated current $I_m^P(T)$ are well approximated by Eqs. (24)–(26), similar to formula (22) for the equilibrium case of the Aslamazov-Lempitsky theory,[6] where the critical temperature T_c is replaced by the stimulated critical temperature T_c^P.[23]

Consider the behavior of $I_m^P(T)$ of the sample SnW5 in a microwave field with the frequency $f = 9.2$ GHz (Fig. 11 (▼)). It can be seen that upon irradiation of the film by microwave power there is the stimulation of $I_m^P(T, f = 9.2\,\text{GHz})$ up to $T = 3.708$ K. At temperatures $T < 3.708$ K the stimulation of $I_m^P(T)$ was not observed. For the irradiation of the sample SnW5 by microwave field with the frequency $f = 12.9$ GHz the stimulation of $I_m^P(T, f = 12.9\,\text{GHz})$ is observed up to $T = 3.690$ K. At lower temperatures the stimulation of $I_m^P(T)$ was not found. For the irradiation of the sample SnW5 by microwave field with the frequency $f = 15.2$ GHz (Fig. 11, (▲)) the stimulation of $I_m^P(T,f$

$= 15.2\,\mathrm{GHz}$) is observed up to $T = 3.655\,\mathrm{K}$. At temperatures $T < 3.655\,\mathrm{K}$ the stimulation of $I_m^P(T)$ was not observed. It should be noted that in Fig. 11 it is seen that $I_m^P(T,f = 15.2\,\mathrm{GHz}) > I_m^P(T,f = 12.9\,\mathrm{GHz}) > I_m^P(T,f = 9.2\,\mathrm{GHz})$.

Thus, an absolute value of $I_m^P(T)$ increases with the radiation frequency, and the temperature region of stimulation of $I_m^P(T)$ is extended toward lower temperatures.[23] By the way, in the same way behaves a critical current $I_c^P(T)$.[24] Let us try to find an explanation for this.

We take into account two factors. First of all, as already noted, in wide films at $I \approx I_m$ the current distribution across the width of the film is close to uniform. In this case it is wise to make use of the knowledge accumulated for narrow channels. Second, the source of stimulated critical parameters $I_m^P(T)$ and $I_c^P(T)$ of a superconductor is a non-equilibrium distribution function of quasiparticles over energy. In this case, first of all the value of the energy gap increases.[3,9–12]

With taking into account the above arguments, in Fig. 12 the curve *1* represents a temperature dependence of the equilibrium gap $\Delta_0(T)$ (in frequency units), and the curves *2* and *3* show temperature dependences of the stimulated gap $\Delta_m^P(T)$ for the sample SnW5 at the radiation frequencies of 9.2 and 15.2 GHz, assuming a uniform distribution of the transport current density over the cross section. In the figure it is seen that the upper branch of the temperature dependence of the energy gap stimulated in a superconductor, $\Delta_m^P(T)$, intersects with a similar dependence of the equilibrium gap $\Delta_0(T)$. Moreover, the higher the radiation frequency, the lower in the temperature is the point of intersection of dependencies $\Delta_m^P(T)$ and $\Delta_0(T)$ ($T_{02} = 3.762\,\mathrm{K} < T_{01} = 3.782\,\mathrm{K}$), where the stimulation of the energy gap ceases.

It should be noted that the temperature ranges where the non-equilibrium values of the energy gap and the currents exceed the equilibrium values do not coincide. The reason for this lies in a significant difference between the curves of depairing $I_s(\Delta)$ in equilibrium and non-equilibrium cases.[15]

It is important to note that the maximum stimulated critical temperatures $T_{c1}^P = 3.791\,\mathrm{K}$ at the radiation frequency of 9.2 GHz and $T_{c2}^P = 3.791\,\mathrm{K}$ at the frequency of microwave field 15.2 GHz, derived from theoretical curves *2* and *3* in Fig. 12, are in a good agreement with the values of T_c^P, obtained by fitting the experimental curves $I_m^P(T)$ (see Fig. 11).

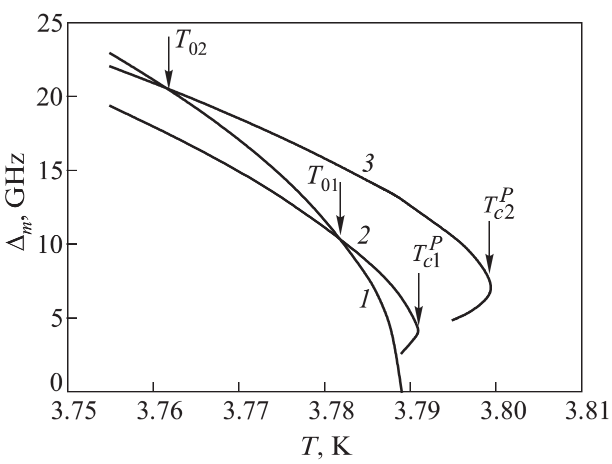

FIG. 12. The calculated dependencies of the equilibrium (curve *1*) and stimulated by microwave field the gap for the sample SnW5 (curve *2*, $f = 9.2\,\mathrm{GHz}$; curve *3*, $f = 15.2\,\mathrm{GHz}$).

6. Conclusion

In the present review, the behavior of the critical current I_c and the maximum current I_m at which in a wide film a vortex structure of the resistive state disappears and the first phase-slip line arises is analyzed in thin superconducting films of different width, located in a microwave field. A stimulation of superconductivity by an external electromagnetic field was found in wide $w \gg \xi(T)$, $\lambda_\perp(T)$ superconducting films[8] with a non-uniform spatial distribution of the current over the sample width. The superconductivity stimulation in a wide film increases not only the critical current I_c, but also the maximum current at which there is a vortex resistive state, I_m.[8] In the framework of the Eliashberg theory an equation for the stimulated critical current was derived and expressed in terms of experimentally measured quantities.[19,24] A comparison of experimental temperature dependencies of the stimulated critical current with those calculated in the framework of the Eliashberg theory revealed a good agreement between them.[19,24] It was shown[24] that near the superconducting transition the temperature dependence of the stimulated critical current in not very wide films ($w < 10\,\lambda_\perp(T)$) appears to be numerically very close to the law $(1 - T/T_c^P)^{3/2}$ for the equilibrium critical current of depairing when T_c is replaced with the stimulated critical temperature T_c^P. It was found[24] that for sufficiently wide films ($w > 10\,\lambda_\perp(T)$) the stimulated critical current has a linear temperature dependence $I_c^P(T) \propto 1 - T/T_c^P$, similar to that in the equilibrium theory of Aslamazov-Lempitsky with replacement of T_c by T_c^P. Experimental dependencies of the stimulated critical current I_c^P and the stimulated current of formation of the first PSL, I_m^P, on power and frequency of microwave radiation were obtained in thin (thickness $d \ll \xi(T)$, $\lambda_\perp(T)$) superconducting films of different width w.[21,23,24] For the first time it was found experimentally that when the film width increases, the range of radiation power, at which the effect of superconductivity stimulation is observed, shrinks abruptly, and hence the probability of its detection decreases.[21] This statement is an answer to the question on much delayed discovery of the stimulation effect in wide films. It is established that with an increase of the film width the ratio P/P_c, at which there is a maximum stimulation effect, is reduced, and the effect of stimulation of the critical current in wider films is observed at lower radiation power, since in the microwave range the value of P_c is practically independent of frequency.[16,22] The power, at which the maximum effect of stimulation is observed, increases with the frequency of microwave radiation. It was found that when the film width increases, the curvature sign of a descending section in the dependence $I_c^P(P)$ is changed.[21]

Studies of the critical current stimulated by a microwave radiation confirmed an earlier conclusion, based on studies of the equilibrium critical current (in the absence of external radiation),[20] that narrow channels are films which satisfy to the relation $w/\lambda_\perp \leq 4$. For them, according to a theory[3] the calculated values of lower boundary frequencies of superconductivity stimulation correspond to experimental values. In much wider films there appears a dependence of characteristic parameters of stimulation effect on the film width. In this connection, to describe a nonequilibrium state of wide films ($w/\lambda_\perp > 4$) in electromagnetic fields it is necessary to develop a theory, which in contrast to the Eliashberg theory[3] initially takes into account a non-uniform current distribution and the presence of vortices of its own magnetic flux.

An unexpected effect of electromagnetic field on the current I_m, which cannot be considered as a trivial influence of the radiation on the $I_c^{GL}(T)$ and $\lambda_\perp(T)$, was found.[23] The experimental temperature dependencies of the stimulated current $I_m^P(T)$ are well approximated by formulas which are similar to the formula (22) for an equilibrium case of the Aslamazov-Lempitsky theory,[6] in which the critical temperature T_c is replaced by the stimulated critical temperature T_c^P. It was found that an absolute value of $I_m^P(T)$ increases with the radiation frequency, and the temperature region of stimulation of $I_m^P(T)$ is extended toward lower temperatures.[23]

One more important fact is worth mentioning. References 8, 19, 21, 23–25 present the main results of a study of superconductivity stimulated by microwave radiation in wide films. In those works, the equilibrium critical current in wide films reached a maximum possible value—a value of the depairing current, and corresponded to the critical current of the Aslamazov-Lempitsky theory. A significant excess of this depairing critical current obtained in the Aslamazov-Lempitsky theory in the absence of external fields was observed under microwave radiation. This indicates the existence of superconductivity stimulation in wide films and the negligible effect of overheating, if it occurs, including an overheating of an electronic system in a superconductor.

Thus, experimental studies of films with different width showed that the effect of superconductivity stimulation by microwave radiation is common, and occurs in both the case of uniform (narrow films) and non-uniform (wide films) distribution of the superconducting current over the film width.

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