

Electromagnetic and acoustic waves in layered organic conductors (a review)

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The review is devoted to theoretical investigations of propagation of electromagnetic and acoustic waves in layered conductors of organic origin. Attention is focussed on spectroscopic possibilities for studying the electron structure of organic quasi-two-dimensional conductors, which is of great importance for understanding physical processes in these materials. High-frequency and magnetoacoustic effects considered in this review are typical of quasi-two-dimensional conductors and quite informative. The analysis of these effects makes it possible to study in detail the electron energy spectrum and relaxation properties of charge carriers in layered conductors. © 1999 American Institute of Physics. [S1063-777X(99)00111-5]

1. INTRODUCTION

The search for new materials in the sixties attracted the attention of researchers to conductors of organic origin with a layered or filamentary structure. Intense experimental investigations of physical properties of organic conductors were stimulated in the hope of obtaining superconductors with high critical parameters just among quasi-one-dimensional filamentary conductors in which a superconducting transition can theoretically occur at high temperatures. Many years of efforts made by physicists and chemists to obtain a large number of new organic conductors culminated in the synthesis of organic quasi-one-dimensional conductors with a superconducting transition temperature T_c of the order of several kelvins as well as layered organic superconductors with a record-high superconducting transition temperature $T_c \cong 13$ K. Although these values of T_c are lower than for some intermetallic compounds, the interest towards the electronic properties of organic conductors remains unabated.

Layered conductors of organic origin are attractive for experimenters to a considerable extent due to their peculiar behavior in strong magnetic fields and a number of phase transitions under comparatively low pressures. Their electrical conductivity along layers is several orders of magnitude higher than electrical conductivity along the normal \mathbf{n} to the layers, and the critical magnetic field at which superconductivity is violated depends considerably on its orientation relative to the layers. Under the action of applied pressure, the superconducting transition temperature of the β -modification of tetrathiafulvalene salt (BEDT-TTF)₂JBr₂ increases approximately by a factor of three.¹ Such a sensitive reaction of the system of charge carriers to crystal deformation indicates that acoustoelectronic phenomena in layered conductors with a quasi-two-dimensional electron energy spectrum apparently possess peculiar properties.

The interest in investigations of organic conductors with a layered structure is also due to the variety of various phase

states of these compounds and the possibility of changing the ground state with external agencies.

Shubnikov–de Haas magnetoresistance oscillations observed in tetraselenotetracene halides and a large family of tetrathiafulvalene-based ion-radical salts with a charge transport in magnetic fields of the order of several tens tesla indicate that these compounds possess the metal-type conductivity. This allows us to describe the electron processes in such conductors on the basis of the concept of quasiparticles carrying an electric charge e , which are similar to conduction electrons in metals. Strong anisotropy of the electrical conductivity of a layered conductor is apparently associated with strong anisotropy of the velocity of charge carriers $\mathbf{v} = \partial \varepsilon(\mathbf{p}) / \partial \mathbf{p}$ on the Fermi surface $\varepsilon(\mathbf{p}) = \varepsilon_F$, i.e., their energy $\varepsilon(\mathbf{p})$ weakly depends on the momentum component $p_z = \mathbf{p} \cdot \mathbf{n}$ along the normal \mathbf{n} to the layers.

The Fermi surface of quasi-two-dimensional conductors is open and weakly corrugated along the p_z -axis. The corrugated planes can be rolled into a cylinder whose base lies in a unit cell of the momentum space so that the Fermi surface of layered conductor can be presented as a system of weakly corrugated cylinders or a system of planes corrugated weakly along the p_z -axis. Small closed cavities belonging to anomalously small groups of charge carriers can also be present.

The mean free path l of charge carriers in experimentally investigated layered conductors attains values of several micrometers, and the radius of curvature r of conduction electrons in strong magnetic fields that may be induced in actual practice can be much smaller than l . Under these conditions, it is appropriate to formulate the inverse problem of reconstruction of the electron energy spectrum with the help of experimental investigation of kinetic phenomena in a magnetic field.

Galvanomagnetic phenomena and quantum oscillation effects in low-dimensional conductors of organic origin have been investigated experimentally by many authors. In recent years, several publications appeared,^{2–7} in which the results of experimental studies of high-frequency phenomena were reported (including the discovery of cyclotron resonance in

the layered conductor α -(BEDT-TTF)₂KHg(SCN).

High-frequency parameters of layered and filamentary conductors are undoubtedly quite informative, and their analysis will make it possible to determine in fine details the electron energy spectrum and relaxation properties of charge carriers. Here we shall consider the propagation of electromagnetic and acoustic oscillations in organic quasi-two-dimensional conductors, choosing these oscillations from the variety of waves that can propagate in current-carrying media.

2. ENERGY SPECTRUM OF LAYERED CONDUCTORS

A unit cell of a crystal in layered organic conductors contains a large number of atoms, and the separation a between layers is much larger than atomic spacing in a layer. As a result, the overlapping of wave functions for electrons belonging to different layers is quite small, and we can use the strong-coupling approximation for dispersion relations for charge carriers:

$$\varepsilon(\mathbf{p}) = \sum_{n=0}^{\infty} \varepsilon_n(p_x, p_y) \cos\left(\frac{anp_z}{h}\right). \tag{2.1}$$

Here h is Planck's constant and $\varepsilon_n(p_x, p_y)$ are assumed to be arbitrary functions of their arguments. However, the maximum values ε_n^{\max} at the Fermi surface decrease significantly with increasing n so that $\varepsilon_1^{\max} = \eta\varepsilon_F \ll \varepsilon_F$, and $\varepsilon_{n+1}^{\max} \ll \varepsilon_n^{\max}$, where η is the quasi-two-dimensionality parameter of the spectrum.

Shubnikov-de Haas quantum oscillations are observed virtually for all organic conductors of the family of tetrathiafulvalene salts.⁸⁻¹⁹ This points to the presence of closed sections of the Fermi surface by the plane $p_H = \mathbf{p} \cdot \mathbf{H}/H$ for such conductors, and the large value of the oscillation amplitude suggests the presence of a group of charge carriers for which the states with the Fermi energy are located on weakly corrugated cylinder in the momentum space, such a group of conduction electrons dominating over the remaining charge carriers with the Fermi energy.

The model of a Fermi surface of a quasi-two-dimensional conductor in the form of a weakly corrugated cylinder (Figs. 1 and 2) is in good agreement with the experimental investigations of galvanomagnetic phenomena and Shubnikov-de Haas oscillations in many layered complexes of organic origin with charge transport. Among other things, the results of theoretical calculations based on this model are in complete accord with the experimentally observed quantum oscillations of magnetoresistance of tetrathiafulvalene salts (BEDT-TTF)₂JBr₂ and (BEDT-TTF)₂J₃. However, the substitution of the complex MHg(SCN)₄ for halogens in these salts, where M is a metal of the group (K, Rb, Tl), leads to a more complex dependence of resistance on magnetic field. According to band analysis of the electron energy spectrum,²⁰ the Fermi surface of (BEDT-TTF)₂MHg(SCN)₄ salts contains, apart from a weakly corrugated cylinder, two quasi-one-dimensional sheets. Although the presence of a magnetic field affect the dynamic properties of charge carriers with a quasi-one-dimensional spectrum only slightly, the existence of such a

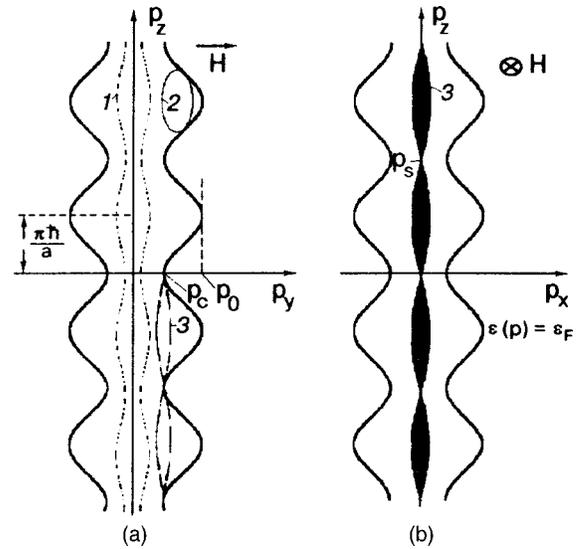


FIG. 1. Various types of electron trajectories in momentum space in a magnetic field parallel to the layers: open trajectories (curves 1), closed electron orbits (curve 2), and a self-intersecting orbit containing a saddle point p_c (curve 3). The cross section of the Fermi surface by the plane $p_y = p_c$ separates the regions of open and closed electron trajectories; (a) and (b) show different projections of the Fermi surface.

charge carrier group can change significantly the dependence of electromagnetic and acoustic impedances on the magnitude of a strong magnetic field.

Yamagiji²¹ used a rather simplified model of the Fermi surface in theoretical calculations of the magnetoresistance anisotropy of layered conductors, while Zimbovskaya²² analyzed the rf properties by using the energy spectrum of

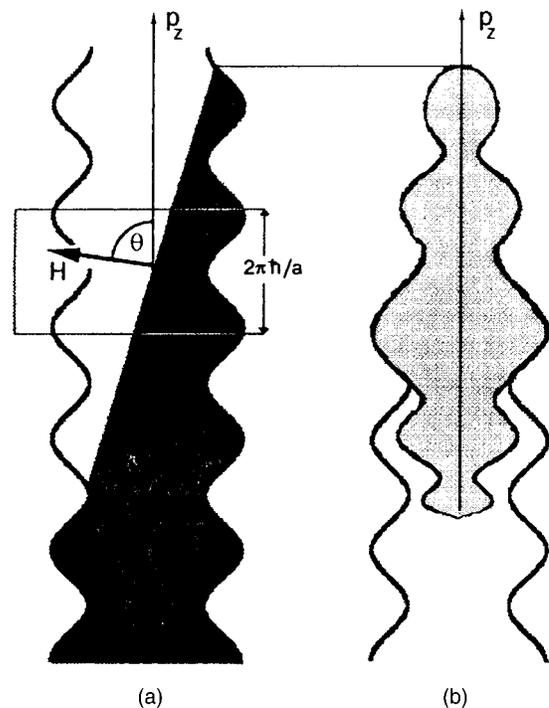


FIG. 2. Electron trajectories in momentum space in a magnetic field (θ is the angle formed by the magnetic field vector with the normal to the layers); (a) and (b) show different projections of the Fermi surface.

charge carriers of an exotic form with kinks on the Fermi surface. Under such assumptions, spectroscopic potentialities of studying electron processes in organic conductors in a magnetic field were underestimated or even disregarded altogether. We shall consider here the high-frequency and magnetoacoustic effects in organic conductors under the most general assumptions concerning the form of quasi-two-dimensional electron energy spectrum (2.1).

The quasi-one-dimensional energy spectrum of charge carriers will not be specified either. We shall only assume that the coefficients A_{000} and A_{100} in the expression for the dependence of energy on quasimomentum

$$\varepsilon_l(\mathbf{p}) = \sum_{nml}^{\infty} A_{nml} \cos\left(\frac{a_1 n p_x}{h}\right) \cos\left(\frac{a_2 m p_y}{h}\right) \cos\left(\frac{a l p_z}{h}\right) \quad (2.2)$$

are much larger than all the remaining coefficients A_{nml} . The dimensions a_1 and a_2 of a unit cell of the crystal lattice in the xy plane of the layers can also differ considerably. In the case when these planes are not the symmetry planes of the crystal, we must take into account additional phase in the arguments of the cosines in formulas (2.1) and (2.2), which changes sign upon the substitution of $-\mathbf{p}$ for \mathbf{p} . This will not alter the wave spectrum in layered conductors considerably, and so there is no need to complicate the solution of the given problem. Thus, we shall use below the dispersion relation for charge carriers in the form (2.1) and (2.2), assuming that the coefficients A_{nml} and the functions $\varepsilon_n(p_x, p_y)$ are arbitrary.

3. COMPLETE SET OF EQUATIONS

An acoustic wave in a conductor always generates a varying electromagnetic field accompanying it. However, the perturbation of the electron subsystem of a conductor by an electromagnetic wave incident on its surface can also excite elastic oscillations in it. Consequently, the system of equations describing the propagation of waves in a conductor contains the equation of the theory of elasticity for ionic displacement \mathbf{u} , i.e.,

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \lambda_{ijlm} \frac{\partial u_{lm}}{\partial x_j} + F_i, \quad (3.1)$$

as well as Maxwell's equations

$$\text{curl } \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}; \quad \text{curl } \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}; \quad \text{div } \mathbf{B} = 0. \quad (3.2)$$

Here ρ and λ_{ijlm} are the density and elastic tensor of the crystal, $u_{lm} = (1/2)(\partial u_l / \partial x_m + \partial u_m / \partial x_l)$ is the strain tensor, and c the velocity of light.

In view of a quite high number density of charge carriers, Poisson's equation can be reduced to the electroneutrality condition of the conductor, and hence the continuity condition for charge flux in the asymptotic approximation in reciprocal density of conduction electrons assumes the form

$$\text{div } \mathbf{j} = 0. \quad (3.3)$$

The magnetization \mathbf{M} induced by an external magnetic field in conductors without a spontaneous magnetic moment is usually small, and there is no need to distinguish between the magnetic induction \mathbf{B} and the magnetic field $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}(\mathbf{B})$ except at ultra-low temperatures. At quite low temperatures, when the inclusion of charge carrier energy quantization in a magnetic field is significant, the amplitude of quantum oscillations of magnetization as a function of $1/B$ can become comparable with \mathbf{B} , and the difference $\mathbf{B} - 4\pi\mathbf{M}(\mathbf{B})$ can become an infinitely small quantity. In this case, the wave process is essentially nonlinear even for small wave amplitude.^{23,24}

If $M(\mathbf{B}) \ll B$, Eqs. (3.2) can be reduced to a high degree of accuracy to the equation

$$\text{curl } \text{curl } \mathbf{E} - \frac{\omega^2}{c^2} \mathbf{E} = \frac{4\pi i \omega}{c^2} \mathbf{j}. \quad (3.4)$$

In the case of a small wave amplitude, it is sufficient to confine the analysis to the linear approximation in weak perturbation of the electron system, and the wave process can be regarded as monochromatic with frequency ω so that the differentiation with respect to time is equivalent to multiplication by $(-i\omega)$, which is taken into account in Eq. (3.4). This assumption does not violate in any way the generality of the problem since in view of the linearity of equations relative to the displacement of ions, the electric field $\mathbf{E}(\mathbf{r}, t)$, and the magnetic field of the wave, the generalization to the case of an arbitrary time dependence of the fields is trivial and can be reduced to the summation of various harmonics of the solution of the system of equations (3.1)–(3.3).

The perturbation of the electron system by crystal deformation leads to a renormalization of the conduction electron energy,²⁵ i.e.,

$$\delta\varepsilon = \lambda_{ij}(\mathbf{p}) u_{ij} \quad (3.5)$$

and to the emergence of the force

$$F_i = \frac{1}{c} [\mathbf{j} \times \mathbf{H}]_i + \frac{m}{e} i \omega j_i + f_i^d, \quad (3.6)$$

exerted by electrons on the crystal lattice.

The electric current density

$$j_i = -\frac{2}{(2\pi\hbar)^3} \int e v_i \psi \frac{\partial f_0}{\partial \varepsilon} d^3 p \equiv \langle e v_i \psi \rangle \quad (3.7)$$

and the deforming force density^{26,27}

$$f_i^d = \frac{\partial}{\partial x_k} \langle \Lambda_{ik} \psi \rangle, \quad (3.8)$$

characterizing the response of the electron system to perturbation are functionals of the charge carrier distribution function $f = f_0\{\varepsilon(\mathbf{p}) + i\omega\mathbf{p} \cdot \mathbf{u}\} - \psi \partial f_0 / \partial \varepsilon$, where $f_0\{\varepsilon(\mathbf{p}) + i\omega\mathbf{p} \cdot \mathbf{u}\}$ is the equilibrium Fermi function in a reference frame moving with the vibrating lattice at a velocity $-i\omega\mathbf{u}$. The nonequilibrium correction to this velocity should be determined by solving the kinetic equation closing the complete system of equations of the problem and having the form

$$\mathbf{v} \frac{\partial \psi}{\partial \mathbf{r}} + \frac{\partial \psi}{\partial t} + \left(\frac{1}{\tau} - i\omega \right) \psi = g. \quad (3.9)$$

Here the function $g = -\omega \Lambda_{ij}(\mathbf{p}) u_{ij} + e \tilde{\mathbf{E}} \cdot \mathbf{v}$ takes into account the perturbation of the system of charge carriers by the electric field

$$\tilde{\mathbf{E}} = \mathbf{E} - \frac{i\omega}{c} [\mathbf{u} \times \mathbf{H}] + \frac{m\mathbf{u}\omega^2}{e} \quad (3.10)$$

and by crystal deformation.

The components $\lambda_{ij}(\mathbf{p})$ of the deformation potential tensor in the kinetic equation (3.9) and in expression (3.8) for the deforming force density are given in the form taking into account the conservation of the number of charge carriers, i.e.,

$$\Lambda_{ik}(\mathbf{p}) = \lambda_{ik}(\mathbf{p}) - \langle \lambda_{ik}(\mathbf{p}) \rangle / \langle 1 \rangle. \quad (3.11)$$

The collision operator in the equation for ψ is taken in the approximation of the relaxation time τ for charge carriers, and the time t is a coordinate in momentum space, which indicates the position of a charge on its trajectory in a magnetic field in accordance with the equation of motion

$$\frac{\partial \mathbf{p}}{\partial t} = \frac{e}{c} [\mathbf{v} \times \mathbf{H}]. \quad (3.12)$$

The kinetic equation must be supplemented with the boundary condition taking into account the scattering of charge carriers at the conductor surface coinciding, say, with the plane $x=0$:

$$\begin{aligned} \psi(\mathbf{p}_+, 0) = & q(\mathbf{p}_-) \psi(\mathbf{p}_-, 0) + \int d^3 p W(\mathbf{p}, \mathbf{p}_+) \\ & \times \{1 - \Theta[v_x(\mathbf{p})]\} \psi(\mathbf{p}, 0). \end{aligned} \quad (3.13)$$

Here the specular reflection parameter $q(\mathbf{p})$ is the probability that a conduction electron incident on the sample surface with a momentum \mathbf{p}_- has after reflection a momentum \mathbf{p}_+ connected with \mathbf{p}_- through the specular reflection condition presuming the conservation of the energy of the charge and of the component of its momentum along the scattering boundary. The specular reflection parameter is connected with the scattering indicatrix $W(\mathbf{p}, \mathbf{p}_+)$ through the relation^{28,29}

$$q(\mathbf{p}_-) = 1 - \int d^3 p W(\mathbf{p}, \mathbf{p}_+) \{1 - \Theta[v_x(\mathbf{p})]\}, \quad (3.14)$$

where $\Theta(\zeta)$ is the Heaviside function.

In a bulk conductor whose size is much larger than the mean free path l of charge carriers, most of them do not collide with the sample surface during their mean free time. If we are interested in "bulk" effects that are not associated with interaction of a small group of charge carriers with the sample surface, there is no need to use the boundary condition, and the function ψ can be presented in the form

$$\psi = \int_{-\infty}^t dt' g[x+x(t')-x(t)] \exp[\nu(t'-t)], \quad (3.15)$$

where $\nu = 1/\tau - i\omega$, and $x(t) = \int^t v_x(t) dt$.

Let us suppose that a wave propagates along the normal to the surface of a conductor occupying the half-space $x \geq 0$. Using the Fourier method, we continue evenly $\mathbf{u}(x)$ and $\tilde{\mathbf{E}}(x)$ to the region of negative values of x and obtain for the fourier component

$$u_i(k) = 2 \int_0^\infty dx u_i(x) \cos kx \quad (3.16)$$

of ion displacement and for the electric field

$$\tilde{E}_i(k) = 2 \int_0^\infty dx \tilde{E}_i(x) \cos kx \quad (3.17)$$

the following system of algebraic equations:

$$\frac{4\pi i\omega}{c^2} j_\alpha(k) = 2E'(0) + k^2 E_\alpha(k) - \left(\frac{\omega}{c}\right)^2 E_\alpha(k), \quad (3.18)$$

$$\alpha = y, z,$$

$$j_x(k) = 0 \quad (3.19)$$

$$\begin{aligned} -\omega^2 \rho u_i(k) = & -\lambda_{ixix} [2u'_\alpha(0) + k^2 u_i] + (im\omega/e) j_i(k) \\ & + c^{-1} [\mathbf{j}(k) \times \mathbf{H}]_i + ik \langle \Lambda_{ix} \psi \rangle. \end{aligned} \quad (3.20)$$

The fluxes characterizing the response of the electron system to a perturbation can be presented with the help of the solution of the kinetic equation in the following form:

$$j_i(k) = \sigma_{ij}(k) \tilde{E}_j(k) + a_{ij}(k) k \omega u_j(k), \quad (3.21)$$

$$\langle \Lambda_{ix} \psi(k) \rangle = b_{ij}(k) \tilde{E}_j(k) + c_{ij}(k) k \omega u_j(k), \quad (3.22)$$

where the Fourier transforms of electrical conductivity $\sigma_{ij}(k)$ and of acoustoelectronic tensors $a_{ij}(k), b_{ij}(k)$ and $c_{ij}(k)$ are defined as

$$\sigma_{ij}(k) = \langle e^2 v_i \hat{R} v_j \rangle; \quad a_{ij}(k) = \langle e v_i \hat{R} \Lambda_{jx} \rangle, \quad (3.23)$$

$$b_{ij}(k) = \langle e \Lambda_{ix} \hat{R} v_j \rangle; \quad c_{ij}(k) = \langle \Lambda_{ix} \hat{R} \Lambda_{jx} \rangle. \quad (3.24)$$

Here

$$\begin{aligned} \hat{R} g \equiv & \int_{-\infty}^t dt' g(t') \exp\{ik[x(t')-x(t)] + \nu(t'-t)\}, \\ g(t) = & \omega \Lambda_{ji}(t) k_i u_j(k) + e \mathbf{v}(t) \cdot \tilde{\mathbf{E}}(k). \end{aligned} \quad (3.25)$$

Substituting expressions (3.21) and (3.22) into Eqs. (3.18)–(3.20), we obtain a system of linear algebraic equations in $u_i(k)$ and $\tilde{E}_i(k)$. The problem of distribution of electric field and the field of displacement of ions in a conductor will be solved completely if we apply the inverse Fourier transformation to its solutions.

The condition for the existence of a nontrivial solution of the obtained system of equations (i.e., the equality to zero of its determinant) is a dispersion equation. The imaginary components of the roots of the dispersion equation determine the damping factors of the acoustic and electromagnetic waves, while the real components of these roots describe renormalizations of the velocities of the waves.

4. PROPAGATION OF ELECTROMAGNETIC WAVES IN LAYERED CONDUCTORS

The equations in the theory of elasticity and Maxwell's equations turn out to be coupled weakly when the mutual transformation of electromagnetic and acoustic waves is hampered. In this case, the propagation of acoustic waves in conductors can be investigated without using Maxwell's equations, and the problem of propagation of electromagnetic waves can be solved to a sufficiently high degree of accuracy without using equations in the theory of elasticity.

We consider the propagation of electromagnetic waves in a layered conductor. Their attenuation length depends considerably on the polarization of the incident wave. A linearly polarized wave with the electric field directed along the normal to the layers penetrates into the conductor to a considerably larger depth than a wave with the electric field directed along the layers.

The surface impedance and the penetration depth of the varying electric field of the wave can easily be determined by solving the system of equations (3.18), (3.7), and (3.9) with the boundary condition (3.13). The solution of the kinetic equation (3.9) allows us to find the relation between the Fourier transforms of current density and electric field:

$$j_i(k) = \sigma_{ij}(k)E_j(k) + \int dk' Q_{ij}(k, k')E_j(k'), \quad (4.1)$$

where

$$\begin{aligned} \sigma_{ij}(k) &\equiv 2e^3 H/c(2\pi h)^3 \\ &\times \int dp_H \int_0^T dt v_i(t, p_H) \int_0^t dt' v_j(t', p_H) \\ &\times \exp\{\nu(t' - t)\} \cos k\{x(t', p_H) - x(t, p_H)\} \\ &\equiv \langle e^2 v_i \hat{R} v_j \rangle. \end{aligned} \quad (4.2)$$

The kernel of the integral operator $Q_{ij}(k, k')$ depends considerably on the state of the sample surface, i.e., on the probability of specular reflection of charge carriers at the surface.

In the cases when the relation between the Fourier transforms of current density and electric field is local, i.e., the contribution of electrons colliding with the sample boundary to the alternating current is considerably smaller than the contribution from "bulk" electrons, the electric field attenuation length is determined by the imaginary component of the roots of the dispersion equation

$$\det \left\{ \left(k^2 - \frac{\omega^2}{c^2} \right) \delta_{\alpha\beta} - \frac{4\pi i \omega}{c^2} \bar{\sigma}_{\alpha\beta}(k) \right\} = 0, \quad (4.3)$$

where

$$\bar{\sigma}_{\alpha\beta}(k) = \sigma_{\alpha\beta}(k) - \frac{\sigma_{\alpha x}(k)\sigma_{x\beta}(k)}{\sigma_{xx}(k)}; \quad \alpha, \beta = (y, z). \quad (4.4)$$

Under the conditions of normal skin effect, when the mean free path of charge carriers is smaller than the skin depth, the relation between current density and electric field is local to a high degree of accuracy, i.e.,

$$j_i(x) = \sigma_{ij} E_j(x), \quad (4.5)$$

and the component of the electrical conductivity matrix $\sigma_{ij} = \sigma_{ij}(0)$ have the same form as in a uniform electric field. The electrical conductivity $\sigma_{zz} = \eta^2 \sigma_0$ across the layers is proportional to the square of the quasi-two-dimensionality parameter of the electron energy spectrum, and σ_0 has the same order of magnitude as the electrical conductivity along the layers in a uniform electric field. In this case, the dispersion equation (4.3) implies that the attenuation depth δ_{\parallel} of the electric field $E_z(\mathbf{r})$ is larger than the attenuation depth δ_{\perp} of the electric field along the layers by a factor of $1/\eta$, i.e.,

$$\delta_{\perp} = \delta_{\parallel} \eta. \quad (4.6)$$

Under the conditions of anomalous skin effect, when the skin depth δ_{\parallel} is much smaller than the mean free path l of charge carriers, the relation between δ_{\perp} and δ_{\parallel} has the form

$$\delta_{\perp} = \delta_{\parallel} \eta^{2/3}, \quad (4.7)$$

since the tensor components $\sigma_{ij}(k)$ are inversely proportional to the wave number k for $kl \gg 1$.

In a magnetic field, the relations between δ_{\perp} and δ_{\parallel} are more diversified.

Let us consider the propagation of electromagnetic waves in a layered conductor in a magnetic field $\mathbf{H} = (H \sin \varphi, H \cos \varphi \sin \theta, H \cos \varphi \cos \theta)$, tilted by the angle φ to the conductor surface $x_s = 0$.

The integral term in the boundary condition (3.13) ensures the absence of current through the sample surface, but in the range of high frequencies ω , the solution of the kinetic equation weakly depends on this functional.³⁰ Disregarding this functional for $\varphi = 0$ and assuming the absence of charge carrier drift along the x -axis along open electron orbits, we can write the solution of the kinetic equation in the form

$$\begin{aligned} \psi(t, p_H, x) &= \int_{\lambda}^t dt' e\nu(t', p_H) \cdot \mathbf{E}[x(t', p_H) - x(\lambda, p_H)] \\ &\times \exp\{\nu(t' - t)\} + q(\lambda, p_H)[1 - q(\lambda, p_H)] \\ &\times \exp\{\nu(2\lambda - T)\}^{-1} \int_{\lambda}^{T-\lambda} dt' \\ &\times e\nu(t', p_H) \cdot \mathbf{E}[x(t', p_H) - x(\lambda, p_H)] \\ &\times \exp\{\nu(t' - t + 2\lambda - T)\}, \end{aligned} \quad (4.8)$$

where $T = 2\pi/\Omega = 2\pi m^*c/eH$ is the period of motion of a charge in the magnetic field, m^* the effective cyclotron mass of conduction electrons, and λ is the root of the equation

$$x(t, p_H) - x(\lambda, p_H) = \int_{\lambda}^t v_x(t', p_H) dt' = x. \quad (4.9)$$

which is nearest to t .

Conduction electrons for which $\{x(t, p_H) - x_{\min}\} < x$ do not collide with the sample surface, and we must put $\lambda = -\infty$ for such electrons.

In a magnetic field tilted to the sample surface, conduction electrons either penetrate to the bulk of the sample after several collisions with the boundary, or tend to approach this surface. The relative fraction of the latter electrons is not

large, and they make a small contribution to the alternating current. The contribution of the remaining electrons to the current for $\varphi \cong 1$ is naturally determined by the type of their interaction with the sample surface, but the state of the surface affects only insignificant factor of the order of unity in the expression for surface impedance.

4.1. Normal skin effect

We shall apply the term normal skin effect to penetration of an electromagnetic field to the bulk of a sample under the condition when the current density $\mathbf{j}(\mathbf{r})$ is determined to a high degree of accuracy by the value of the electric field $\mathbf{E}(\mathbf{r})$ at the same point \mathbf{r} . In a strong magnetic field parallel to the conductor surface, charge carriers with closed orbits drift in the momentum space along the sample surface. If the diameter $2r$ of their orbits is much smaller than the skin depth, the main contribution to rf current comes from carriers separated from the surface $x_s=0$ by a distance greater than $2r$. These conduction electrons do not collide with the sample surface, and it is expedient to use the approximation of local coupling between the current density and the electric field of the wave to calculate the surface impedance in the asymptotic approximation in the small parameter r/δ in the absence of open cross sections of the Fermi surface.

The asymptotic expression for the tensor component $\sigma_{ij}(k)$ for $kr \ll 1$ has the same form as in a uniform electric field so that the electric current E_y for $kr \ll 1$ attenuates at distances

$$\delta_{\perp} \cong \delta_0 = c(2\pi\omega\sigma_0)^{-1/2} \quad (4.10)$$

for any relation between the mean free path of charge carriers and the skin depth.

For $\eta \ll 1$, each of the components σ_{zx} and σ_{xz} is at least proportional to η^2 so that $\bar{\sigma}_{zz} \cong \sigma_{zz}$. The asymptotic form of $\sigma_{zz}(k)$ for small angles θ is equal to $\sigma_0\eta^2$ in order of magnitude, and the attenuation length δ_{\parallel} of the electric field E_z is larger than δ_{\perp} by a factor of $1/\eta$ as in zero magnetic field if the corrugation of the Fermi surface is not very small and $\eta \gg \delta_0\omega/c$. For $\omega \gg \sigma_0\eta^2$, the skin depth

$$\delta_{\parallel} = \frac{\delta_0^2\omega}{c\eta^2} \left(1 + \frac{r^2\omega^2}{c^2} \right)^{-1/2} \quad (4.11)$$

increases with the magnetic field, attaining its limiting value $\omega\delta_0^2/c\eta^2$.^{31,32}

For significant values of θ , there exists a sequence of values of $\theta = \theta_c$ for which the asymptotic behavior of σ_{zz} changes considerably, as well as the behavior of the quantity³³⁻³⁵ $\bar{\sigma}_{zz}$ which satisfies the expression

$$\begin{aligned} \bar{\sigma}_{zz}(k, \eta, \theta) = & \frac{ae^3\tau TH \cos \theta}{4\pi^2 h^4 c} \sum_n n^2 I_n^2 + \sigma_0 \eta^2 \{ \eta^2 f_1(\theta) \\ & + \gamma^2 f_2(\theta) + (kr)^2 f_3(\theta) \}, \end{aligned} \quad (4.12)$$

where the f_i stand for functions of θ of the order of unity, and

$$I_n(\theta) = \int_0^T dt \varepsilon_n(t) \cos\{anp_y(t)\tan \theta/h\}. \quad (4.13)$$

For $\theta = \theta_c$, when $I_1(\theta_c)$ vanishes, the value of $\bar{\sigma}_{zz}$ decreases abruptly for small η , $\gamma = (\Omega\tau)^{-1}$, ω/Ω , and kr .

As a result, the penetration depth for the electric field E_z increases considerably for $\theta = \theta_c$, and the angular dependence of impedance acquires a series of narrow peaks. For $\tan \theta \gg 1$, these peaks are repeated periodically, with a period determined by the separation between the stationary phase points on the electron orbit, where $\mathbf{k} \cdot \mathbf{v} = \omega$, which are close to turning points ($v_x = 0$). Since the phase velocity of the wave $v_{\varphi} = \omega/k = (\omega\tau)^{-1/2} \omega c / \omega_0 \eta$ is much smaller than the Fermi velocity v_F of conduction electrons, the separation between stationary phase points on the electron orbit can be regarded to be equal to the diameter of the orbit to a high degree of accuracy.

The height of sharp peaks for $\theta = \theta_c$ in pure conductors at low temperatures, when $l\eta^2 > \delta_0$, decreases with increasing magnetic field, and conversely, for $l\eta^2 < \delta_0$, it increases in proportion to $l\delta_0/r\eta$ if $l\eta < r < \delta_0/\eta$. At not very high frequencies, when the displacement current is smaller than the conduction current, the solution of the dispersion equation (4.3) for $\theta = \theta_c$ can be represented in the form of the interpolation formula

$$\delta_{\parallel} = l \left(\frac{r^2 + \delta_0^2 \eta^{-2}}{r^2 + l^2 \eta^2} \right)^{1/2}. \quad (4.14)$$

In the case of extremely low electrical conductivity along the normal to the layers, when $\omega > \sigma_0 \eta^2 (\eta^2 + r^2/l^2)$, the skin depth δ_{\parallel} has the form

$$\begin{aligned} \delta_{\parallel} = & (\delta_0/\eta^2) \{ 1 + (r/l\eta)^2 + (r\omega/c\eta)^2 \}^{-1/2} \\ & \times \{ 1 + (r\eta/\delta_0)^2 \}, \end{aligned} \quad (4.15)$$

and the electric field attenuation depth along the normal to the layers is again equal to δ_0/η^2 in a strong magnetic field when $r < (l^2\eta^2 + \delta_0^2/\eta^2)^{1/2}$. In the range of moderate magnetic fields in which the relation $\delta_0/\eta \ll r \ll \delta_{\parallel}$ holds for $\theta = \theta_c$, the impedance as a function of magnetic field has a minimum since for $r \gg l\eta$ the skin depth

$$\delta_{\parallel} = lr\eta/\delta_0 \quad (4.16)$$

is inversely proportional to the magnetic field, i.e., decreases with increasing magnetic field.³²⁻³⁵

For $\delta_{\perp} \ll r \ll \delta_{\parallel}$, the attenuation length of the electric field $E_z(x)$ depends weakly on the type of reflection of charge carriers at the sample surface as before, but the penetration depth for the electric field $E_y(x)$ is quite sensitive to the state of the conductor surface if the value of δ_{\perp} is smaller than or comparable to the mean free path of charge carriers. In this range of magnetic fields, normal skin effect can take place only for $\delta_{\perp} \gg l$, when the local relation between the current density and electric field is observed for any polarization of the wave. The asymptotic expression $\bar{\sigma}_{yy}(k)$ for $kl \ll 1$ coincides with σ_0 to within a numerical factor of the order of unity, and hence δ_{\perp} coincides in order of magnitude with δ_0 . However, the penetration depth of the electric field $E_z(x)$ in the sample depends considerably on the magnetic field orientation.

A peculiar dependence of the attenuation length of the electric field $E_z(x)$ is observed for $\theta = \pi/2$, when, apart from

the drift of charge carriers along the magnetic field, a fan of various drift directions is possible in the xy plane for conduction electrons belonging to open cross sections of the Fermi surface. In this case, the dependence of σ_{zz} on the magnitude of a strong magnetic field ($\gamma_0 = 1/(\Omega_0\tau) \ll 1$, where Ω_0 is the frequency of electron rotation in a magnetic field orthogonal to the layers) can be presented by the following interpolation formula:

$$\sigma_{zz} = \sigma_0 \gamma_0^2 \eta^2 (\gamma_0^2 + \eta)^{-1/2}, \quad (4.17)$$

which is valid for any orientation of the magnetic field in the xy plane, i.e., for any angle of its inclination to the sample surface $x_s = 0$.

Using formulas (4.3) and (4.17), we can easily verify that the value of δ_{\parallel} increases with the magnetic field in proportion to $H^{1/2}$ for $\eta^{1/2} \ll \gamma_0 \ll 1$, while the attenuation length $\delta_{\parallel} \cong \delta_0 / \gamma_0 \eta^{3/4}$ of the electric field along the normal to the layers increases linearly with the magnetic field for $\eta^2 \ll \gamma_0 \ll \eta^{1/2}$.

The solution of the dispersion equation (4.3) for φ differing from zero has the form

$$k = \frac{(2\pi\omega)^{1/2}(1+i)}{2c} \{ \sigma_0^{-1} + \sigma_{zz}^{-1} \pm [(\sigma_{zz}^{-1} - \sigma_0^{-1})^2 - (2H \cos \theta \sin \varphi / Nec)^2]^{1/2} \}^{-1/2}, \quad (4.18)$$

where N is the charge carrier density.

This formula shows that in the extremely strong magnetic field, when $\gamma_0 \ll \eta^2$, helicoidal waves can propagate. For $\varphi \cong 1$, one of the roots of the dispersion equation describes attenuation of electric field along the layers at distances of the order of

$$\delta_{\perp} = \delta_0 \left(1 + \frac{\sigma_{zz}}{\sigma_0 \gamma_0^2} \right)^{1/2}. \quad (4.19)$$

It can easily be seen that the penetration depth for the electric field E_y increases as the magnetic field increases in proportion to H when $\gamma_0 \ll \eta$. The electric field directed along the normal to the layers for $\gamma_0 \gg \eta^2$ attenuates at distances³⁵

$$\delta_{\parallel} = \delta_0 (\sigma_0 / \sigma_{zz})^{1/2}, \quad (4.20)$$

i.e., at distances of the order of δ_0 / η as in zero magnetic field.

In the presence of an additional group of charge carriers with a quasi-one-dimensional energy spectrum, high-frequency properties of layered conductors are quite sensitive not only to the polarization of the incident wave, but also to the direction of propagation of electromagnetic field in the plane of the layers.³⁶⁻³⁸ If the reflection of charge carriers at the conductor surface is close to specular, the relation between the Fourier transforms of current density and electric field can be regarded as local to a fairly high degree of accuracy even for an indefinitely large mean free path of charge carriers:

$$j_i(k) = \{ \sigma_{ij}(k) + \sigma_{ij}^{(1)}(k) \} E_j(k). \quad (4.21)$$

Here $\sigma_{ij}^{(1)}(k)$ is the contribution to the rf electrical conductivity from charge carriers with the energy spectrum (2.2), in which we retain only a few terms by putting

$$A_{100} = U, \quad A_{010} = \eta_1 U \ll U, \quad A_{001} = \eta_2 U \ll U.$$

The contribution to $\tilde{\sigma}_{\alpha\beta}(k)$ from charge carriers with a quasi-one-dimensional energy spectrum is mainly determined by the component $\sigma_{xx}^{(1)}(k)$ which has the following form accurate to small corrections proportional to η_1^2 and η_2^2 :

$$\sigma_{xx}^{(1)}(k) = \sigma_1(k) = \frac{\sigma_1}{1 + (kl_1)^2}, \quad (4.22)$$

where $l_1 = v_{01}\tau_1 / (1 - i\tau_1)$; σ_1 is the contribution of this group of charge carriers to electrical conductivity along the x -axis in a uniform electric field, τ_1 the mean free time of charge carriers with the energy spectrum (2.2), and $v_0 = (Ua_1/h) \sin[(\epsilon_F - A_{000})/U]$.

The magnetic field dependence of $\sigma_{ij}^{(1)}(k)$ is manifested only in the next terms of expansion into a power series in the small parameters η_1 and η_2 :

$$\sigma_{yy}^{(1)}(k) = \sum_{\pm} \frac{\eta_1^2 \sigma_1 a_2^2 U^2 / 4h^2 v_0^2}{1 + (k \pm eHa_2 \cos \theta / ch)^2 l_1^2}, \quad (4.23)$$

$$\sigma_{zz}^{(1)}(k) = \sum_{\pm} \frac{\eta_2^2 \sigma_1 a^2 U^2 / 4h^2 v_0^2}{1 + (k \pm eHa \sin \theta / ch)^2 l_1^2}, \quad (4.24)$$

whose inclusion does not affect significantly the skin depth of electromagnetic field attenuation.

The asymptotic behavior of the components of $\tilde{\sigma}_{\alpha\beta}(k)$ in strong magnetic fields ($\gamma = 1/\Omega\tau \ll 1$), i.e.,

$$\tilde{\sigma}_{yy}(k) = \frac{\sigma_1(k) \{ \gamma^2 \sigma_0 + \sigma_{zz} \tan^2 \theta \} + \gamma^2 \sigma_0^2}{\sigma_1(k) + \gamma^2 \sigma_0}, \quad (4.25)$$

$$\tilde{\sigma}_{yz}(k) = \tilde{\sigma}_{zy}(k) = \frac{\sigma_1(k)}{\sigma_1(k) + \gamma^2 \sigma_0} \sigma_{zz} \tan \theta, \quad (4.26)$$

$$\tilde{\sigma}_{zz}(k) = \sigma_{zz} + \sigma_{zz}^{(1)}(k). \quad (4.27)$$

is very sensitive to the emergence of a group of charge carriers with a quasi-one-dimensional energy spectrum.

We have omitted here insignificant numerical factors of the order of unity and small corrections of the order of $(kr)^2$ in the expression for σ_{zz} , i.e., the contribution of charge carriers with a quasi-two-dimensional spectrum to the current is taken into account, as before, in the approximation valid for normal skin effect.

If σ_1 and σ_0 are of the same order of magnitude, the value of $\tilde{\sigma}_{yy}(k)$ does not attain saturation in strong magnetic fields as in the case of $\sigma_1 = 0$ and turns out to be much smaller than σ_0 in a fairly wide range of magnetic fields. This leads to a considerable increase in the conductor transparency.

The dispersion equation (4.3) taking into account relations (4.25)–(4.27) makes it possible to determine the length of attenuation of electromagnetic fields in a strong magnetic field:

$$\delta_1 \cong \delta_0 / \eta, \quad \delta_2 \cong \delta_0 / \gamma, \quad (4.28)$$

where $\delta_0 = \{c/2\pi\omega(\sigma_0 + \sigma_1)\}^{1/2}$.

If σ_1 is much smaller than σ_0 , but $\sigma_1 \geq \gamma^2 \sigma_0$, the expression for δ_2 should be supplemented with the small factor $(\sigma_1/\sigma_0)^{1/2}$. For $\sigma_1 \ll \gamma^2 \sigma_0$, the attenuation lengths for the electric fields $E_z(x)$ and $E_y(x)$ differ significantly ($\delta_{\parallel} = \delta_1$ and $\delta_{\perp} \equiv \delta_0$, respectively), by the electric fields along and across the layers for $\sigma_1 \geq \gamma^2 \sigma_0$ contain both components with considerably different attenuation lengths δ_1 and δ_2 . Consequently, in pure conductors for which $l\eta \gg \delta_0$, not only the field $E_z(x)$, but also the field $E_y(x)$ attenuate over distances considerably longer than the mean free path of charge carriers in magnetic fields for which $r \ll \delta_0$.

When an electromagnetic wave propagates along the y -axis, the presence of a group of charge carriers with a quasi-one-dimensional energy spectrum does not affect significantly the attenuation length of electromagnetic waves. As in the case of a single group of charge carriers with the dispersion relation (2.1), the electric field along the layers attenuates over distances of the order of δ_0 , and the electric field along the normal to the layers penetrates a quasi-two-dimensional conductor to the depth δ_{\parallel} for which the above formulas (4.11), (4.14)–(4.16) are valid. The effect of charge carriers with spectrum (2.2) on the propagation of electromagnetic waves becomes significant when $\cos \alpha \gg \gamma^2 \sigma_0/\sigma_1$, where α is the angle between the wave vector and the predominant direction of the velocity of charge carriers with a quasi-one-dimensional energy spectrum.

Thus, analyzing the dependence of surface impedance on the magnetic field during the propagation of an electromagnetic wave in two different directions in the plane of the layers, we can determine unambiguously the presence of a quasi-one-dimensional cavity on the Fermi surface and its contribution of the electrical conductivity of an organic conductor.

4.2. Anomalous skin effect

With increasing frequency of an electromagnetic wave, the skin depth δ decreases, and the relation between current density and electric field becomes essentially nonlocal for $\delta \leq 2r$. In this case, Maxwell’s equations are of the integral type even in the Fourier representation.³⁹ Hartmann and Luttinger⁴⁰ proposed a correct solution of these equations in a magnetic field for some special cases. If we disregard numerical factors of the order of unity, we can obtain a reasonable solution of the physical problem, i.e., determine the dependence of surface impedance and other characteristics of waves in a conductor on physical parameters, with the help of a correct estimation of the contribution of the integral term in formula (4.1) to the Fourier transform of the high-frequency current. In a magnetic field parallel to the sample surface, for $\delta_{\perp} \leq r$, the contribution of charge carriers colliding with the sample surface to the current is significant. In the case of a nearly specular reflection of charge carriers by the sample boundary (the width of scattering indicatrix for charge carriers $w \ll r^{3/2}/l\delta_{\perp}^{1/2}$), the contribution of conduction electrons “sliding” along the sample surface and remaining in the skin layer to the rf current is quite large. In this case, the asymptotic expression for $\tilde{\sigma}_{yy}(k)$ for large k has the form

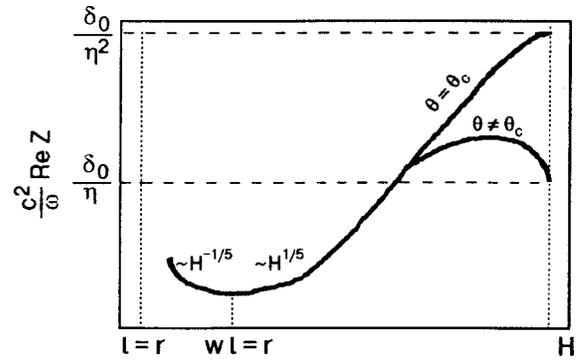


FIG. 3. Dependence of surface impedance on the magnitude of a strong magnetic field ($r \ll l$) parallel to the surface of the conductor $x_s = 0$. The width w of the indicatrix of charge carrier scattering at the sample surface can be determined from the position of the minimum.

$$\tilde{\sigma}_{yy}(k) = \frac{\omega_0^2}{\Omega(kr)^{1/2}(w+r/l)}. \tag{4.29}$$

Using the dispersion equation (4.3), we can easily determine the attenuation length of electric fields, i.e.,

$$\delta_{\perp} = \delta_0^{6/5} r^{-1/5} (w+r/l)^{2/5}; \quad \delta_{\parallel} = \delta_0 / \eta. \tag{4.30}$$

In the range of not very strong magnetic fields, where $\delta_{\perp} \ll r \ll l$, the impedance has a minimum for $r = wl$, and its position determines uniquely the width of indicatrix of charge carrier scattering at the sample boundary (Fig. 3).

Under the conditions of extremely anomalous skin effect, when the depth of electromagnetic wave penetration in the conductor is the smallest parameter of the problem having the dimensions of length (i.e., not only δ_{\perp} , but also δ_{\parallel} is much smaller than r and l), the values of δ_{\perp} and δ_{\parallel} are connected through a universal relation in a magnetic field parallel to the sample surface for $w \ll r^{3/2}/l\delta_{\parallel}^{1/2}$.³³

$$\delta_{\perp} = \delta_{\parallel} \eta^{4/5}. \tag{4.31}$$

If $w \gg r^{3/2}/l\delta_{\perp}^{1/2}$ and $\delta_{\perp} \ll r \ll l$, the contribution to the rf current mainly comes from charge carriers that do not interact with the sample surface, and the relation between δ_{\perp} and δ_{\parallel} has the form (4.7).

In the intermediate case when $r^{3/2}/l\delta_{\parallel}^{1/2} \ll w \ll r^{3/2}/l\delta_{\perp}^{1/2}$, only δ_{\perp} depends considerably on w for $w \geq r/l$:

$$\delta_{\parallel} = r^{1/3} (\delta_0 / \eta)^{2/3}, \quad \delta_{\perp} = w^{2/5} \delta_0^{6/5} r^{-1/5}. \tag{4.32}$$

In the absence of open electron orbits, conduction electrons carry information on the field in the skin layer to the bulk of the conductor in the form of narrow spikes predicted by Azbel.⁴¹ The transport of electromagnetic field to the bulk of the conductor and the screening of the incident wave at the surface $x_s = 0$ are mainly accomplished by charge carriers moving in phase with the wave almost parallel to the sample surface. For $\eta \leq \delta/r$, almost all of charge carriers participate in the formation of electromagnetic field spikes.⁴² The intensity of the spikes at distances from the sample surface multiple to the diameter of the electron orbit in the direction of the x -axis has the same order of magnitude in the collisionless limit. The inclusion of scattering of conduction electrons in the bulk of the conductor leads to field attenua-

tion in a spike at distances of the order of the mean free path of charge carriers. Thus, there are two scales of electromagnetic field attenuation length under the conditions of anomalous skin effect. Apart from the skin depth, the electromagnetic field penetrates into the bulk of the sample to a depth of the order of the mean free path of charge carriers.

For $\eta \gg \delta/r$, only an insignificant fraction of charge carriers of the order of $(\delta/r\eta)^{1/2}$ participates in the formation of spikes. The spread in the diameters of orbits of such carriers in the vicinity of the extremal diameter is comparable with the skin depth. As a result, with increasing distance from the surface $x_s=0$, the intensity of each next spike acquires an additional small factor $(\delta/r\eta)^{1/2}$ apart from the exponential factor $\exp\{-x/l\}$ taking into account attenuation of waves in the spike over the mean free path l .

As the angle θ approaches $\pi/2$, closed electron orbits become strongly elongated along the x -axis, and the spike mechanism of penetration of electromagnetic field in the bulk of the sample is replaced by the electron transport of the varying field in the form of Reuter–Sondheimer weakly attenuating quasi-waves^{39,43–46} when the diameter of the orbits in this direction exceeds the mean free path l .

4.3. Weakly attenuating Reuter–Sondheimer waves

The drift of charge carriers along the normal to the sample surface facilitates the transport of electromagnetic field from the skin layer to the bulk of the conductor over a distance smaller than or of the order of the mean free path l of charge carriers. For $\theta = \pi/2$, the drift of charge carriers along open trajectories leads to penetration of electromagnetic field over a distance $x \leq l$ even in a magnetic field parallel to the surface $x_s=0$.

In order to determine the electric field in the bulk of the sample with the help of inverse Fourier transformation

$$E_j(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk E_j(k) \exp\{-ikx\} \quad (4.33)$$

we continue $E_j(k)$ analytically to the entire complex k -plane and close the integration contour in formula (4.33) with an arc of infinitely large radius in the half-plane where $\text{Im} k \geq 0$. The skin depth is determined by the poles of the integrand in formula (4.33), while weakly attenuating waves are associated with integration along the cuts drawn from the branching point of the function $E_j(k)$. It can easily be verified that the tensor component $\sigma_{ij}(k)$ for indefinitely small η display a root singularity of the form

$$\sigma_{zz}(k) = (\omega_0^2 \eta^2 / \nu) \{ (\alpha_+^2 - 1)^{-1/2} + (\alpha_-^2 - 1)^{-1/2} \}; \quad (4.34)$$

$$\Delta \sigma_{yy}(k) = \nu (\omega_0 / k \nu)^2 \{ (k \nu / \nu)^2 + 1 \}^{1/2}, \quad (4.35)$$

where ω_0 is the frequency of plasma oscillations of charge carriers, $\nu = v_x^{\text{max}} \cong v_F$, and $\alpha_{\pm} = i(k \nu \pm \Omega) / \nu$. For $\eta \ll 1$, the time variation of the electron velocity v_x in the magnetic field $\mathbf{H} = (0, H, 0)$ does not exceed $\nu \eta^{1/2}$ so that away from the saddle points on the Fermi surface, charge carriers move in the momentum space along the p_z -axis virtually without acceleration over a distance equal to the period of a unit cell

during the time $T = 2\pi \hbar c / aeH v_x$. In this case, Ω appearing in the expression for α_{\pm} is equal to $aeH \nu / \hbar c$.

The kernel of the integral operator $Q_{ij}(k, k')$ as a function of k also possesses a similar singularity.

The electromagnetic field decreases in proportion to $x^{-3/2} \exp(-x/l)$ over distances from the sample surface which exceed considerably either $r = \nu / \Omega$, or the displacement of an electron during the wave period $2\pi \nu / \omega$. For $\Omega \gg \omega$, the slowly decreasing varying electric field

$$E_z(x) = E_z(0) \eta^{-4/3} (c / \omega_0)^{4/3} (\nu / \omega)^{2/3} r^{-1/2} x^{-3/2} \times \exp\{ix/r - x/l\} \quad (4.36)$$

oscillates upon variation of H at large distances $x \gg r$.

The attenuation of the electric field $E_y(x)$ over the mean free path of charge carriers for $\eta \leq 1$ has the form

$$E_y(x) = E_y(0) (c / \omega_0)^{4/3} (\nu / \omega)^{2/3} (\nu / \nu)^{1/2} x^{-3/2} \times \exp\{-x/l + ix\omega / \nu\}, \quad (4.37)$$

$$\nu / \omega \ll x \ll \nu / \omega \eta$$

and is independent of the magnetic field.

The oscillatory dependence of $E_y(x)$ on the magnetic field is manifested only in small corrections proportional to η^2 .

For values of η that are not small in zero magnetic field, the functions $\sigma_{yy}(k)$ and $\sigma_{zz}(k)$ have a logarithmic singularity for $k_1 = i\nu / v_1$ and $k_2 = i\nu / v_2$, where v_1 is the electron velocity at the reference point on the Fermi surface in the x -direction and v_2 the projection of the velocity v_x at the saddle point of the Fermi surface, at which connectedness of the line $v_x = \text{const}$ changes.⁴⁴ For indefinitely small η , these branching points of the rf conductivity tensor component become closer, and the logarithmic singularity changes into a root singularity for $\eta = 0$.⁴⁷ For small η , we choose the integration contour in the k -plane along the cut lines drawn from the branching points k_1 and k_2 parallel to the imaginary axis so that we can bypass both branching points simultaneously. In this case, the electric field $E_z(x)$ away from the skin layer assumes the form

$$E_z(x) = -2E_z'(0) \left\{ \int_{k_1}^{k_1+i\infty} dk \left[k^2 - \frac{\omega^2}{c^2} - \frac{4\pi i \omega}{c^2} \sigma_{zz1}(k) \right]^{-1} \times \exp(ikx) + \int_{k_2+i\infty}^{k_2} dk \left[k^2 - \frac{\omega^2}{c^2} - \frac{4\pi i \omega}{c^2} \sigma_{zz2}(k) \right]^{-1} \exp(ikx) \right\}. \quad (4.38)$$

We can neglect the integral along lines connecting the branching points k_1 and k_2 and assume that $\sigma_{zz1}(k)$ is the value of the function $\sigma_{zz}(k)$ at the left bank of the cut drawn from the point k_1 , while $\sigma_{zz2}(k)$ is its value at the right bank of the cut drawn from the point k_2 . For definiteness, we assume that v_1 is greater than v_2 . If we disregard anisotropy of the dispersion relation (2.1) for charge carriers in the plane of the layers, the diagonal components of the rf electrical conductivity tensor for $k_1 \leq k \leq k_2$ assume the form

$$\sigma_{yy}(k) = \frac{\omega_0^2 \eta}{\pi^3} \int_0^\pi d\alpha \int_0^{\pi/2} d\varphi \frac{\sin^2 \varphi}{\nu + ik\nu \cos \varphi (1 + \eta \cos \alpha)^{1/2}}. \quad (4.39)$$

$$\sigma_{zz}(k) = \frac{\omega_0^2 \eta^2}{\pi^3} \int_0^\pi \sin^2 \alpha d\alpha \times \int_0^{\pi/2} d\varphi \frac{1}{\nu + ik\nu \cos \varphi (1 + \eta \cos \alpha)^{1/2}}. \quad (4.40)$$

It can easily be seen that the rf electrical conductivity component $\sigma_{zz}(k)$ is proportional to $(\nu + ik\nu)^{-1/2}$, for $\eta \ll 1$, while $\sigma_{yy}(k)$ is proportional to $(\nu + ik\nu)^{1/2}$, i.e., both component have a root singularity for $k = i\nu/\nu$. In the case of a considerable corrugation of the Fermi surface, when $\eta \cong 1$, the root singularity is replaced by a logarithmic singularity for $k = i\nu/\nu(1 + \eta)^{1/2}$ and $k = i\nu/\nu(1 - \eta)^{1/2}$. After the integration with respect to φ , the integrands in (4.39) and (4.40) have a root singularity for $k = i\nu/\nu(1 + \eta \cos \alpha)^{1/2}$. As a result of simple calculations, we arrive at the following expression for the electric field component weakly attenuating at large distances from the skin layer:

$$E_y(x) = E_y(0)(c/\omega_0)^{4/3}(\nu/\omega)^{2/3}x^{-3/2}(\nu/\nu)^{1/2} \times \int_0^\pi d\alpha \exp\left\{-\frac{\nu x}{\nu(1 + \eta \cos \alpha)^{1/2}}\right\}. \quad (4.41)$$

At large distances from the sample surface, the electric field along the normal to the layers can be described by the same formula if we supplement the integrand in the integral with respect to α with the factor $\eta^{-4/3} \sin^2 \alpha$. For $x \gg \nu/\omega\eta$, the integrand in formula (4.41) is a rapidly oscillating alternating function, and the main contribution to this integral comes from small neighborhoods of the stationary phase point $\alpha = (0, \pi)$. As a result of simple calculations, we obtain

$$E_y(x) = E_y(0)(c/\omega_0)^{4/3}(\nu/\omega)^{2/3}x^{-2}\eta^{-1/2} \times \left[\exp\left\{-\frac{\nu x}{\nu(1 + \eta)^{1/2}}\right\} + \exp\left\{-\frac{\nu x}{\nu(1 - \eta)^{1/2}}\right\} \right], \quad (4.42)$$

$x \gg \nu/\omega\eta.$

In the above formulas, we have omitted insignificant numerical factors of the order of unity. The pre-exponential factor in formula (4.42) is inversely proportional to x^2 as in normal metals. Such an asymptotic behavior in quasi-two-dimensional conductors is observed only in the range of high frequencies, where $\omega\tau \gg 1/\eta$. Essentially different asymptotic forms of electric fields at such frequencies can be explained by tracing the phase of the wave carried by conduction electrons with different velocity components \mathbf{v}_x from the skin layer. At the instant t , electrons carry over a distance x the information on electromagnetic wave with a phase lag $\omega\Delta t = \omega x/\mathbf{v}_x$. Averaging over different values of \mathbf{v}_x by the formula

$$E(x) \sim \int d\mathbf{v}_x \exp\{-i\omega t + i\omega x/\mathbf{v}_x\} \quad (4.43)$$

we can easily see that a weakly attenuating wave which propagates with the electron velocity \mathbf{v}_1 at the reference point of the Fermi surface is formed by charge carriers whose velocity \mathbf{v}_x differs from \mathbf{v}_1 by the quantity $\Delta\mathbf{v}_x \ll \mathbf{v}_1^2/\omega x$. If $\mathbf{v}_1 - \mathbf{v}_2 \cong \nu\eta$ is smaller than $\Delta\mathbf{v}_x$, i.e., $x \ll \nu/\omega\eta$, formula (4.37) is valid for $E_y(x)$, while in the opposite limiting case, when $\Delta\mathbf{v}_x \ll \nu\eta$, weakly attenuating waves described by formula (4.42) are formed by electrons from small neighborhoods on the Fermi surface near the saddle and reference points.

In a magnetic field, charge carriers belonging to one of the "banks" of the central open cross section of the Fermi surface, on which the velocity \mathbf{v}_x varies with time periodically in the interval between \mathbf{v}_2 and \mathbf{v}_1 , move most rapidly to the bulk of the sample. Weakly attenuating waves propagate at a velocity equal to the extremal value $\bar{\mathbf{v}}_x$ and are described by formulas (4.36) and (4.37).

Weakly attenuating waves in a magnetic field tilted from the plane of the layers have a similar form. If the magnetic field lies in the xy plane, i.e., $\theta = \pi/2$, a weakly attenuating wave with φ differing noticeably from zero propagates at a velocity $\bar{\mathbf{v}}_x$ equal to the drift velocity of charge carriers belonging to the open cross section of the Fermi surface containing the reference point along the p_x -axis. The asymptotic form of the electric field $E_y(x)$ is described by (4.37), and its oscillatory dependence on the magnetic field orthogonal to the axis of the corrugated cylinder is manifested, as before, only in small corrections proportional to η^2 .

When electromagnetic waves propagate along the normal to the layers (along the z -axis), charge carriers can carry information on the field in the skin layer to the bulk of the sample only over a distance of the order of $l\eta$, which exceeds the skin depth only for very small values of η .

The weakly attenuating electric field component can easily be determined with the help of relation (4.33) in which x should be replaced by z . Without a loss in generality of the given problem, we shall confine our analysis only to the first two terms in expression (2.1) for $\varepsilon(\mathbf{p})$, assuming that $\varepsilon_1(p_x, p_y)$ is a constant quantity equal to $\eta\nu_0 h/a$, where ν_0 coincides in order of magnitude with the characteristic Fermi velocity \mathbf{v}_F of charge carriers along the layers.

If the magnetic field is orthogonal to the layers, the Fourier components $\sigma_{ij}(k)$ of the electrical conductivity tensor assume the form

$$\sigma_{ij}(k) = \frac{2e^2}{(2\pi\hbar)^3} \times \sum_n \int dp_z 2\pi m^* \frac{\mathbf{v}_i^{(-n)} \mathbf{v}_j^{(n)}}{\nu + ik\nu_F \eta \sin(ap_z/h) + in\Omega}. \quad (4.44)$$

After simple calculations, we obtain

$$\sigma_{ij}(k) = \omega_0^2 \sum_n C_{ij}^{(n)} \{(k\nu_0\eta)^2 + (\gamma_n\Omega)^2\}^{-1/2}, \quad (4.45)$$

where

$$\gamma_n = \gamma + in,$$

$$v^{(n)} = (1/T) \int_0^T dt v_i(t, p_z) \exp(-in\Omega t),$$

and $C_{ij}^{(n)}$ are numerical factors of the order of unity. For $i = j$, all these factors are real-valued and positive, while in Hall's nondissipative components they are imaginary as a rule and change sign upon inversion of i and j so that a helicoidal wave attenuating over a distance $l_{\text{hel}} = \delta_0(\Omega\tau)^{3/2}$ is formed in a strong magnetic field for $\Omega \gg kv_0\eta$.

For moderate magnetic fields in which $kr\eta \gg 1$, Hall's nondissipative Fourier components $\sigma_{ij}(k)$ are of the same order of magnitude as the dissipative diagonal components, and all of them possess a root singularity for $k = k_{\pm} = (\omega \pm \Omega + i/\tau)/(v_F\eta)$. In this region of magnetic fields, electromagnetic field penetrates in the bulk of the sample only in the form of a Reuter–Sondheimer quasiwave

$$\mathbf{E}(z) = \mathbf{E}(0) \left(\frac{c}{\omega_0}\right)^{4/3} \left(\frac{v\eta}{\omega}\right)^{1/6} z^{-3/2} \exp\{ik_{\pm}z\},$$

$$z \gg v\eta/\omega. \tag{4.46}$$

4.4. Cyclotron resonance

In all organic conductors synthesized at present, the mean free path l of charge carriers is not large ($l \leq 10 \mu\text{m}$) so that the frequency of electromagnetic waves in the rf and microwave regions is much lower than the electron collision frequency $1/\tau$, and the time dispersion can be disregarded while calculating the skin depth. However, the frequency of electromagnetic wave in the millimeter and submillimeter regions at low temperatures can be comparable to the collision frequency for charge carriers, and the interaction of conduction electrons with electromagnetic field is of resonant type, when the wave frequency ω is equal or multiple to the frequency Ω of their rotation in a magnetic field.

In a magnetic field orthogonal to the sample surface $z_s = 0$, cyclotron resonance can take place at multiple frequencies $\omega = n\Omega$ in the case of essentially anisotropic spectrum of charge carriers in the plane of the layers. The shape of the resonance curve can be determined easily by using formula (4.45) for $\sigma_{ij}(k)$. Resonance takes place for $r\eta \ll \delta_0$, but it is manifested most clearly when $l\eta \leq \delta_0$. If $l\eta \ll r$ in this case, all charge carriers with a quasi-two-dimensional energy spectrum participate in the formation of resonance effect. In the case of an isotropic spectrum of charge carriers in the plane of the layers, i.e., for $\varepsilon_0(p_x, p_y) = \varepsilon_0(p_{\perp})$, where $p_{\perp} = (p_x^2 + p_y^2)^{1/2}$, we have only one resonance value of the magnetic field satisfying the condition $\omega = \Omega$.

Diagonalizing the tensor $\sigma_{ij}(k)$, we obtain the following expression for the diagonal components of surface impedance:

$$Z_{\mu} = -\frac{8i\omega}{c^2} \int_0^{\infty} \frac{dk}{k^2 - 4\pi i\omega c^{-2}\sigma_{\mu}(k)}. \tag{4.47}$$

Under favorable conditions for cyclotron resonance, i.e., for $l\eta \ll \{r, \delta_0\}$, the resonance value of the impedance is $Z_{\mu}^{\text{res}} = 8\pi\omega\delta_0/c^2$, and the resonance line width is $(H - H^{\text{res}})/H^{\text{res}} \cong \gamma$. Away from the resonance we have Z_{μ}

$\cong \gamma^{-1}Z_{\mu}^{\text{res}}$. If $l\eta \gg \delta_0$, both terms in the braces of formula (4.45) have the same order of magnitude, and the resonance line is “blurred.”

The detection of cyclotron resonance at multiple frequencies would make it possible to analyze in detail the energy spectrum of charge carriers, but the observation of this effect requires long mean free paths of charge carriers. The cyclotron resonance observed by Polisski *et al.*⁶ in (BEDT–TTF)₂ReO₄(H₂O) for only one resonance value of magnetic field cannot be regarded as an evidence of isotropic spectrum of charge carriers in the plane of the layers. The information on the dispersion relation of charge carriers in this compound can be refined by analyzing the Azbel–Kaner resonance⁴⁸ in a magnetic field parallel to the sample surface, at which the cyclotron resonance at multiple frequencies takes place for any shape of the electron energy spectrum.

5. PROPAGATION OF ACOUSTIC WAVES

In an analysis of sound absorption in ordinary metals, the inclusion of electromagnetic waves accompanying an acoustic wave is essential in the range of strong magnetic fields, when the radius of curvature r of charge carrier trajectories is much smaller than not only the mean free path of the carriers, but also the acoustic wave length k^{-1} . If, however, the inequality

$$1 \ll kr \ll kl. \tag{5.1}$$

is satisfied, the attenuation of sound in a metal is mainly determined by the deformation mechanism associated with the renormalization of electron energy in the field of the wave. In low-dimensional conductors, the role of electromagnetic fields excited by sound is significant in a wider range of magnetic fields, including fields satisfying the inequality (5.1). In this region of magnetic fields, the sound absorption coefficient Γ oscillates upon a change in reciprocal magnetic field. If the magnetic field is orthogonal to the wave vector \mathbf{k} , and the trajectories of charge carriers in the momentum space are closed, the amplitude of oscillations in a normal metal is small in comparison with the smoothly varying component of Γ since oscillations are formed by a small group of charge carriers with a diameter of orbits close to the extremal diameter. This effect predicted by Pippard⁴⁹ is associated with periodic repetition of the conditions of effective interaction of a charge with an acoustic wave, when the number of wave lengths corresponding to the diameter of the electron orbit changes by unity. If the vectors \mathbf{k} and \mathbf{H} are not orthogonal, the average velocity of a charge in the direction of propagation of the sound differs from zero for any shape of the Fermi surface, i.e., charge carriers drift in the direction of wave propagation. The existence of points at which the interaction with the wave is most effective on such a trajectory leads to a resonant dependence of the sound absorption coefficient on reciprocal magnetic field. In ordinary metals, periodic variations of Γ with $1/H$, which are not associated with quantization of the motion of charge carriers with an amplitude much larger than the minimum value of Γ , are possible only in the presence of drift along \mathbf{k} .⁵⁰

In contrast to conventional metals, the formation of Pipard oscillations in low-dimensional conductors involves virtually all charge carriers on the Fermi surface since the diameters of their orbits are close in value. As a result, the amplitude of periodic variations of electrical conductivity and other acoustoelectronic coefficients with $1/H$ increases abruptly, and absorption is of the resonant type.^{51–54} In this case, we cannot obtain even an order-of-magnitude estimate of the sound absorption coefficient without taking into account electromagnetic fields correctly.

5.1. Longitudinal wave propagating along the layers

Let us consider a longitudinal acoustic wave ($\mathbf{u} = (u, 0, 0)$) propagating along the layers in a quasi-two-dimensional conductor in a magnetic field \mathbf{H} . Using formulas (3.19)–(3.21), we can write the system of equations (3.18) after elimination of the field \tilde{E}_x in the form

$$\begin{aligned} (\tilde{a}_{yx}k\xi + iH_z/c)\omega u + (\xi\sigma_{yy} - 1)\tilde{E}_y + \xi\tilde{\sigma}_{yz}\tilde{E}_z &= 0, \\ (\tilde{a}_{zx}k\xi - iH_z/c)\omega u + \xi\tilde{\sigma}_{zy}\tilde{E}_y + (\xi\sigma_{zz} - 1)\tilde{E}_z &= 0, \\ (\omega^2 - s^2k^2)\rho u + [ik\tilde{c}_{xx} + c^{-1}(\tilde{a}_{yx}H_z - \tilde{a}_{zx}H_y)]k\omega u \\ + [ik\tilde{b}_{xy} + c^{-1}(\tilde{\sigma}_{yy}H_z - \tilde{\sigma}_{zy}H_y)]\tilde{E}_y + [ik\tilde{b}_{xz} \\ + c^{-1}(\tilde{\sigma}_{yz}H_z - \tilde{\sigma}_{zz}H_y)]\tilde{E}_z &= 0, \end{aligned} \quad (5.2)$$

where

$$\begin{aligned} s &= (\lambda_{xxxx}/\rho)^{1/2}, \quad \xi = 4\pi i\omega/(k^2c^2 - \omega^2), \\ \tilde{\sigma}_{\alpha\beta} &= \sigma_{\alpha\beta} - \frac{\sigma_{\alpha x}\sigma_{x\beta}}{\sigma_{xx}}, \quad \tilde{a}_{\alpha j} = a_{\alpha j} - \frac{a_{xj}\sigma_{\alpha x}}{\sigma_{xx}}, \\ \tilde{b}_{i\beta} &= b_{i\beta} - \frac{b_{ix}\sigma_{x\beta}}{\sigma_{xx}}, \quad \tilde{c} = c_{ij} - \frac{b_{ix}a_{xj}}{\sigma_{xx}}, \\ \alpha, \beta &= y, z. \end{aligned}$$

For $\omega\tau \ll 1$, the root of the dispersion equation describing an acoustic wave is close to ω/s , and we can write it in the form

$$k = \omega/s + k_1. \quad (5.3)$$

In the case of weak corrugation of the Fermi surface ($\eta \ll 1$), the expression for k_1 has the form

$$\begin{aligned} k_1 &= \frac{ik^2}{2\rho s} \frac{1}{1 - \xi\tilde{\sigma}_{yy}} \left\{ \xi(\tilde{a}_{yx}\tilde{b}_{xy} - \tilde{c}_{xx}\tilde{\sigma}_{yy}) + [\tilde{c}_{xx} - i(\tilde{a}_{yx} \right. \\ &\quad \left. - \tilde{b}_{xy})] \frac{H_z}{kc} + \tilde{\sigma}_{yy} \frac{H_z^2}{k^2c^2} \right\} \Bigg|_{k=\omega/s}. \end{aligned} \quad (5.4)$$

Vectors \mathbf{H} and \mathbf{k} are orthogonal. In a magnetic field $\mathbf{H} = (0, H \sin \theta, H \cos \theta)$ orthogonal to the direction of wave propagation, the solution of the kinetic equation in the Fourier representation can be written in the form

$$\begin{aligned} \psi &= \frac{\int_{t-T}^t dt' g(t') \exp\{i\mathbf{k}[x(t') - x(t)] + \nu(t' - t)\}}{1 - \exp(-\nu T)} \\ &\equiv \hat{R}g, \end{aligned} \quad (5.5)$$

where T is the period of rotation of charges in the magnetic field. In the range of magnetic fields for which the inequality (5.1) is satisfied, the interaction with the acoustic wave is most effective for charge carriers moving in phase with the wave. Such carriers make the main contribution to the components of acoustoelectronic tensors which can easily be calculated with the help of the stationary phase method. The amplitude of their oscillations with $1/H$ is large if the quasi-two-dimensionality parameter η satisfies the condition $kr\eta \ll 1$ for which the spread in the diameter of electron orbits $\Delta D \cong 2r\eta$ becomes much smaller than the acoustic wave length. Let a charge pass through two stationary phase points at which $k v_x = \omega$ during the period of motion T . Then the following expressions hold⁵⁵ for σ_{yy} and a_{yx} for $\eta \rightarrow 0$:

$$\begin{aligned} \sigma_{yy}(k) &= (G/kD)(1 - \sin kD); \\ a_{yx}(k) &= -i(G\Lambda_{xx}/e v kD) \cos kD, \end{aligned} \quad (5.6)$$

where $D = cD_p/(eH \cos \theta)$, D_p being the averaged diameter of the Fermi surface along the p_y axis, $G = 4vD_p e^2 \tau / [ac(2\pi h)^2]$, and Λ_{xx} the value of the quantity $\Lambda_{xx}(\mathbf{p})$ at the stationary phase points.

It can easily be verified that the value of $\tilde{\sigma}_{yy}$ is mainly determined by the σ_{yy} component, and hence the denominator in formula (5.4) for k_1 decreases significantly for $kD = 2\pi(n + 1/4)$. This leads to the emergence of sharp peaks of the sound absorption coefficient Γ , which are repeated periodically with the period

$$\Delta\left(\frac{1}{H}\right) = \frac{2\pi e \cos \theta}{kcD_p}. \quad (5.7)$$

The height

$$\Gamma_{\text{res}} = \frac{\omega\tau}{D} = \frac{\omega}{v} \Omega\tau \quad (5.8)$$

of these resonance peaks is proportional to H for $l \ll kr^2$.

Regions of high acoustic transparency in which the absorption coefficient has the form

$$\Gamma = \frac{\omega\tau}{D} \left[\left(\frac{D}{l}\right)^2 + (kD\eta)^2 \right]. \quad (5.9)$$

are situated away from the resonance (in regions where $\sin kD$ differs considerably from unity).

We can easily obtain explicit expressions for Γ for arbitrary $kr\eta$. Let us consider by way of an example a layered quasi-two-dimensional conductor for which the dispersion relation for charge carriers has the form

$$\varepsilon(\mathbf{p}) = \frac{p_x^2 + p_y^2}{m} + \eta \frac{h}{a} v_0 \cos\left(\frac{ap_z}{h}\right), \quad v_0 = 2\varepsilon_F/m, \quad (5.10)$$

and the deformation potential tensor components $\Lambda_{ik}(\mathbf{p})$ can be represented in the form

$$\Lambda_{ik}(\mathbf{p}) = \Lambda_{ik}^{(0)}(\mathbf{p}) + \eta L_{ik} \cos\left(\frac{ap_z}{h}\right), \quad (5.11)$$

where

$$\Lambda_{ik}^{(0)}(\mathbf{p}) = -\frac{1}{m} \begin{bmatrix} p_x^2 - m\varepsilon_F & p_x p_y & 0 \\ p_x p_y & p_y^2 - m\varepsilon_F & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

the matrix components L_{ik} coinciding the Fermi energy in the order of magnitude.

Let us write the expressions for some components of acoustoelectronic tensors obtained in the main approximation in the small parameters $\gamma = (\Omega\tau)^{-1}$ and $(kD)^{-1}$ for a magnetic field orthogonal to the layers.^{52,53}

$$\begin{aligned} \sigma_{yy} &= \frac{4Ne^2}{m\nu\pi kD} [1 - J_0(\zeta)\sin kD], \\ \sigma_{yx} &= -\sigma_{xy}^{(2)} = \frac{4Ne^2}{m\nu\pi kD} J_0(\zeta)\cos kD, \\ c_{xx} &= \frac{Nm\nu_0}{\nu\pi kD} [1 + J_0(\zeta)\sin kD], \end{aligned} \quad (5.12)$$

where N is the number density of charge carriers with a quasi-two-dimensional dispersion relation, J_0 Bessel's function of $\zeta = kR\eta$, and $R = 2hc/(eHa)$. The diameter D of the electron orbit in the case under investigation has the form $D = 2c\nu_0 m/(eH)$.

For $\zeta \gg 1$, the corrugation of the Fermi surface is quite strong, and absorption coefficient behaves as in an ordinary isotropic metal:

$$\Gamma = \Gamma_0 \Omega_0 \tau \left[1 + \left(\frac{2}{\pi\zeta} \right)^{1/2} \cos \left(\zeta - \frac{\pi}{4} \right) \sin(kD) \right] \Big|_{k=\omega/s}, \quad (5.13)$$

where $\Omega_0 = eH/(mc)$; $\Gamma_0 = Nm\omega\nu_0/(4\pi\rho s^2)$ is the energy absorption coefficient for acoustic waves in zero magnetic field.

For $\zeta \ll 1$, specific features of the quasi-two-dimensional conductor are manifested, and Γ is given by

$$\begin{aligned} \Gamma &= \Gamma_0 \Omega_0 \tau \\ &\times \operatorname{Re} \left[\frac{(\pi\gamma)^2 + \zeta^2/2 + i\mu[1 + \sin kD]}{1 - \sin kD + (\pi\gamma)^2/2 + \zeta^2/2 + 9/8(kD)^{-2} + i\mu} \right] \Big|_{k=\omega/s}, \end{aligned} \quad (5.14)$$

where $\mu = \pi\nu_0 c^2 \omega / 2s^3 \omega_0^2 \Omega\tau$, ω_0 being the frequency of plasma oscillations. If the latter is comparable with the value typical of ordinary metal ($10^{15} - 10^{16} \text{ s}^{-1}$), the parameter μ in the ultrasonic frequency range is quite small, and periodic variations of $\Gamma(1/H)$ have the form of giant resonance oscillations (Fig. 4). Such a behavior of Γ is typical of any conductor with a quasi-two-dimensional dispersion relation for charge carriers.

Vectors \mathbf{H} and \mathbf{k} are not orthogonal. Let us now consider the case when the magnetic field $\mathbf{H} = (H \sin \varphi, 0, H \cos \varphi)$ is not orthogonal to the vector \mathbf{k} . In this case, the value of the velocity component \bar{v}_x along the direction of the wave vector averaged over the period differs from zero, and the solution of the kinetic equation has the form

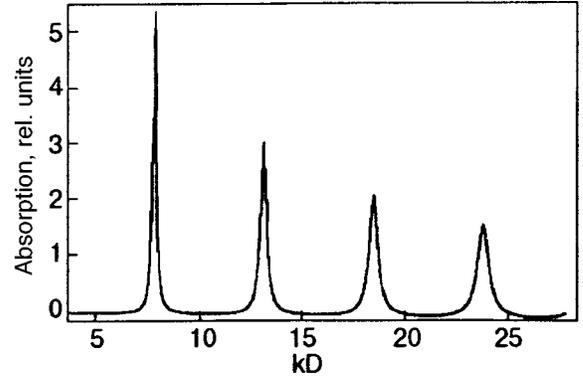


FIG. 4. Dependence of the absorption coefficient of a longitudinal acoustic wave on the reciprocal magnetic field $D \propto 1/H$ in relative units.

$$\begin{aligned} \psi &= \frac{\int_{t-T}^t dt' g(t') \exp\{i\mathbf{k}[x(t') - x(t)] + \nu(t' - t)\}}{1 - \exp[-\nu T - i\mathbf{k}\bar{\mathbf{v}}_x T]} \\ &\equiv \hat{R}g. \end{aligned} \quad (5.15)$$

It follows from the equation of motion (3.9) for a charge with the dispersion relation (2.1) that its velocity components averaged over the period T satisfy the relation

$$\bar{\mathbf{v}}_x = \tan \varphi \bar{\mathbf{v}}_z; \quad \bar{\mathbf{v}}_\alpha = \frac{1}{T} \int_0^T v_\alpha(t_H) dt_H. \quad (5.16)$$

The displacement of an electron over the period of motion along the wave vector is given by

$$\begin{aligned} \bar{\mathbf{v}}_x T &= -\tan \varphi \sum_{n=1}^{\infty} \frac{an}{h} \int_0^T dt \varepsilon_n(t, p_H) \sin \frac{anp_z}{h} \\ &= -\tan \varphi \sum_{n=1}^{\infty} \frac{an}{h} \int_0^T dt \varepsilon_n(t, p_H) \sin \left[\frac{anp_H}{h \cos \theta} \right. \\ &\quad \left. - \frac{1}{h} anp_x(t, p_H) \tan \varphi \right]. \end{aligned} \quad (5.17)$$

If we take into account the fact that p_x and p_y , and hence ε_n depends weakly on the integral of motion $p_H = p_x \sin \varphi + p_z \cos \varphi$ in a magnetic field, the drift velocity of electrons along \mathbf{k} in the main approximation in the small parameter η of quasi-two-dimensionality of the electron energy spectrum assumes the form

$$\bar{\mathbf{v}}_x = -\tan \varphi \operatorname{Im} \sum_{n=1}^{\infty} \frac{an}{h} \exp \left\{ \frac{ianp_H}{h \cos \varphi} \right\} I_n(\tan \varphi), \quad (5.18)$$

where

$$I_n(\tan \varphi) = \frac{1}{T} \int_0^T dt \varepsilon_n(t) \exp \left\{ -\frac{i}{h} anp_x(t) \tan \varphi \right\}. \quad (5.19)$$

These relations are valid for $\Omega\tau \equiv (eH\tau \cos \varphi / mc) \gg 1$, i.e., when $\cos \varphi$ differs from zero considerably.

It can be easily seen that the main term in formula (5.18) proportional to $I_1(\tan \varphi)$ vanishes for certain values of $\tan \varphi$, and there exists a large number of values of the angle $\varphi = \varphi_c$ in the vicinity of zeros of the function $I_1(\tan \varphi)$, for

which the drift velocity \bar{v}_x of charge carriers along the acoustic wave vector coincides with the velocity s of propagation of the acoustic wave, and their interaction with the wave is most effective. As a result, we can expect the presence of narrow peaks in the dependence of the damping decrement of acoustic waves on the angle φ .

Using the stationary phase method, we can easily calculate the acoustoelectronic tensor components in the presence of electron drift along \mathbf{k} also. For example, for the dispersion relation (5.10) for charge carriers, we obtain the following expression for σ_{yy} for small η :

$$\sigma_{yy} = \frac{4Ne^2}{\pi m \nu k D} \left\{ \frac{1 - \sin kD}{(1 + \alpha^2)^{1/2}} + \frac{(\pi \gamma)^2}{3} \left(1 + \frac{1}{2} \sin kD \right) + \pi \gamma \sin kD \left(1 - \frac{1}{(1 + \alpha^2)^{1/2}} \right) \right\}. \quad (5.20)$$

Here $D = 2v_0/\Omega$, $\alpha = kl\eta \tan \varphi J_0(ah^{-1}mv \tan \varphi)$. The component σ_{yy} oscillates with reciprocal magnetic field, and its complex periodic dependence on the angle φ can be described in terms of the quantity α . The remaining acoustoelectronic coefficients behave similarly.

For $\alpha \ll 1$, we can easily obtain the following expression for k_1 ^{56,57}:

$$k_1 = \frac{i\omega N m \nu}{4\pi \rho s^2} \times \frac{2\pi \sin^2 kD [1 - (1 + \alpha^2)^{-1/2}] + \pi^2 \gamma}{1 - \sin kD + [(\pi \gamma)^2/2](1 + \alpha^2)^{1/2} + \pi \gamma [(1 + \alpha^2)^{1/2} - 1]}. \quad (5.21)$$

If $\alpha \ll 1$, we obtain

$$k_1 = \frac{i\omega N m \nu}{4\pi \rho s^2} \frac{\pi \alpha^2 \sin^2 kD + \pi^2 \gamma}{1 - \sin kD + (\pi \gamma)^2/2 + \pi \gamma \alpha^2/2}. \quad (5.22)$$

For $\gamma^{1/2} \ll \alpha \ll 1$, the oscillating terms exceed the smoothly varying terms not only in the denominator, but also in the numerator of formula (5.21). This leads to giant oscillations of the sound absorption coefficient $\Gamma = \text{Im } k_1$ upon a variation of the reciprocal magnetic field as well as the angle φ between \mathbf{H} and \mathbf{n} . In the case, when the displacement of charge carriers along \mathbf{k} during their mean free time is much larger than the acoustic wave length, these oscillations also take place. Then we can write the following expression for k_1 :

$$k_1 = \frac{i\omega N m \nu}{4\pi \rho s^2} \frac{2\pi \sin^2 kD + \pi^2 \gamma}{1 - \sin kD + \pi \gamma \alpha}, \quad 1 \ll \alpha \ll 1/\gamma. \quad (5.23)$$

Thus, the existence of even a small displacement of charge carriers along \mathbf{k} affects significantly the sound absorption Γ . For $\sin kD = 1$, the function $\Gamma(H)$ attains its maximum value

$$\Gamma_{\max} = \frac{\Gamma_0 \Omega \tau}{(1 + \alpha^2)^{1/2}}. \quad (5.24)$$

A slight deviation of $\sin kD$ from unity leads to a strong decrease in Γ which has the minimum value $\Gamma_{\min} = \Gamma_0/\Omega\tau$ for $\sin kD = -1$ if $\alpha^2 \ll \gamma \ll 1$. For $\gamma \leq 3\alpha^2/2 \ll 1$, the minimum of $\Gamma(H)$ is shifted towards the values of H for which $\sin kD$ is

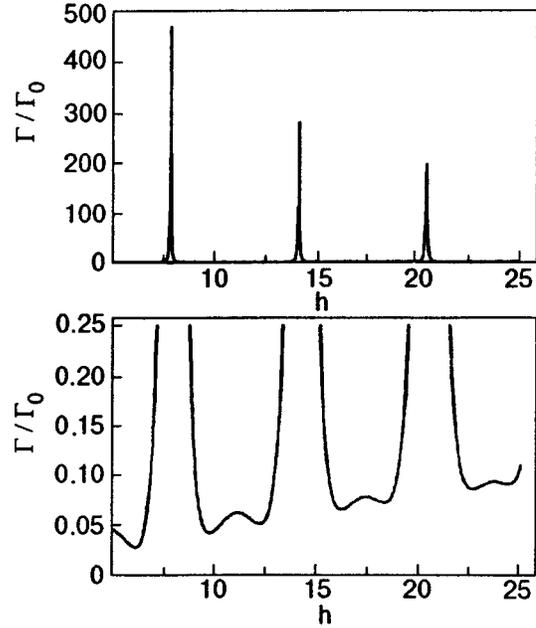


FIG. 5. Dependence of the absorption coefficient Γ/Γ_0 on $h = H_0/H$ ($H_0 = 2\omega cm v_0/es$) for $kl = 10^3$, $\eta = 10^{-2}$, $x = \tan \varphi = 1.5 \times 10^{-2}$. The upper and lower figures differ in scale.

close to zero, and the function $\Gamma(H)$ has a local peak $\Gamma = \Gamma_0 \alpha^2$ for $\sin kD = -1$. This peak increases with α and attains the value Γ_0 of the sound absorption coefficient in zero magnetic field for $\alpha \geq 1$. At the same time, the main peak decreases with increasing α and approaches the local maximum. For $\sin kD = -1$, the absorption coefficient oscillates with a large amplitude exceeding the minimum value of Γ by a factor of $\Omega\tau$.

Figures 5, 6, and 7 show the dependence of absorption coefficient on the quantity $h = H_0/H$ ($H_0 = 2\omega v_0 mc/se$)

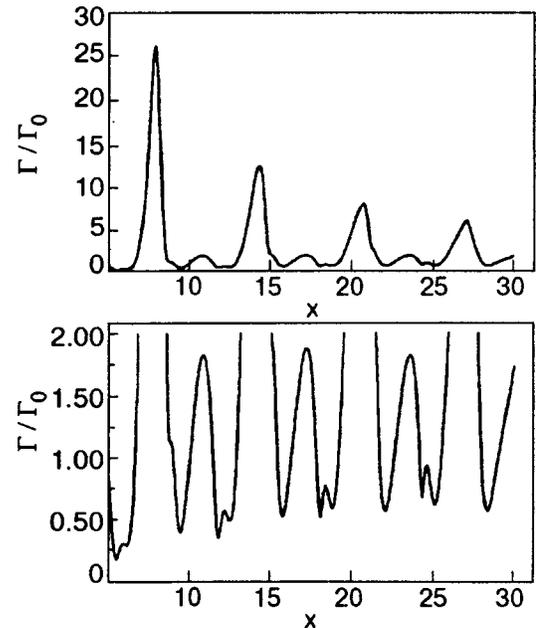


FIG. 6. Dependence of the absorption coefficient Γ/Γ_0 on $x = \tan \varphi$ for $kl = 10^2$, $\eta = 10^{-2}$. The upper and lower figures differ in scale.

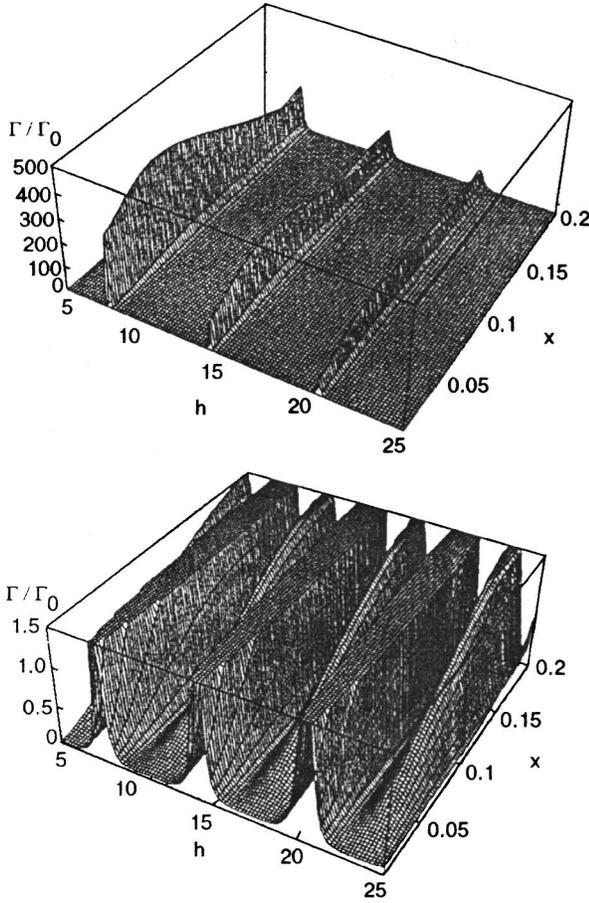


FIG. 7. Dependence of the absorption coefficient Γ/Γ_0 on h and $x = \tan \varphi$ for $kl = 10^3$, $\eta = 10^{-2}$. The upper and lower figures differ in scale.

and on $\tan \varphi$.

It can easily be seen that the dependence of Γ on $1/H$ and $\tan \varphi$ described above remains valid for an arbitrary form of the quasi-two-dimensional electron energy spectrum. If the electron orbit contains only two stationary phase points, the value of $D = cD_p/eH$ is determined by the diameter D_p of the Fermi surface in a direction orthogonal to the vectors \mathbf{k} and \mathbf{H} .

Noticeable manifestation of the effect of drift of charge carriers on the oscillatory dependence of Γ on $1/H$ at ultrasonic frequencies ($\omega \cong 10^8 \text{ s}^{-1}$) is determined by certain requirements. For example, we must use perfect samples with a large mean free path of charge carriers and strong magnetic fields of the order of 10 T. In this range of magnetic fields, the Shubnikov–de Haas effect is manifested clearly in compounds of tetrathiafulvalene, which indicates that the condition $\Omega \tau \gg 1$ is satisfied, and at the same time the separation between quantized electron energy levels is much smaller than not only the Fermi energy, but also the quantity $\eta \varepsilon_F$. Under these conditions, a semiclassical description of non-equilibrium processes is valid. In stronger magnetic fields, the quantization of electron energy levels is significant, but the effects described above must also be observed.

Presence of a quasi-one-dimensional group of charge carriers. In order to clarify the role of a quasi-one-dimensional group of charge carriers in attenuation of acous-

tic waves, we consider a simple model of the energy spectrum for a two-band conductor. We assume that the dispersion relation (5.10) is valid for one group of charge carriers, while the other group has a quasi-one-dimensional dispersion relation of the form

$$\varepsilon_1(\mathbf{p}) = \pm \mathbf{p} \cdot \mathbf{N} v_1 + \eta_1 \frac{h}{a} v_1 \cos\left(\frac{a n p_z}{h}\right). \quad (5.25)$$

Here $\eta_1 \ll 1$ and v_1 is the velocity of an electron with the Fermi energy on a quasi-one-dimensional sheet of the Fermi surface. The vector $\mathbf{N} = (\cos \beta, \sin \beta, 0)$ is oriented in the plane of the layers and forms an angle β with the direction of wave propagation.

In this case, for calculating acoustoelectronic tensors, we must carry out integration in formulas (3.21) over all sheets of the Fermi surface, and each component is the sum of the contributions from quasi-two-dimensional and quasi-one-dimensional ($\sigma_{ij}^{(1)}, a_{ij}^{(1)}, b_{ij}^{(1)}, c_{ij}^{(1)}$) groups of charge carriers.

The existence of preferred direction of the velocities of charge carriers in the quasi-one-dimensional group is manifested in the dependence of their deformation potential $\Lambda_{ij}^{(1)}$ on the angle β . If crystal deformation does not lead to a redistribution of charges between electron groups, we can naturally assume [bearing in mind relation (3.12)] that $\Lambda_{ij}^{(1)}$ vanishes in the main approximation in the small parameter η_1 . If we put $\Lambda_{xx}^{(1)} = \eta_1 \varepsilon_F \cos \beta$, the expressions for the contributions to acoustoelectronic coefficients from the electrons of the quasi-one-dimensional group assume the form

$$\sigma_{ij}^{(1)} = h \beta \frac{N_1^2 e^2 v_1^2}{\nu \varepsilon_F} N_i N_j, \quad i, j = x, y;$$

$$c_{xx}^{(1)} = \eta_1^2 h \beta \frac{N_1 \varepsilon_F \cos^2 \beta}{\nu},$$

$$a_{xx}^{(1)} = b_{xx}^{(1)} = i \eta_1 h \beta \frac{N_1 e v_1}{\nu} kl \cos^3 \beta,$$

$$a_{yx}^{(1)} = b_{xy}^{(1)} = i \eta_1 h \beta \frac{N_1 e v_1}{\nu} kl \cos^2 \beta \sin \beta,$$

$$h_\beta = [1 + (kl)^2 \cos^2 \beta]^{-1}. \quad (5.26)$$

Here $l = v_1 \tau$ and N_1 is the number density of charge carriers with the quasi-one-dimensional dispersion relation. The contribution to the acoustoelectronic coefficients from the quasi-two-dimensional group of charge carriers have the form of (5.12) and similar relations.

In the main approximation in the small parameters $(\Omega \tau)^{-1}$, $(kD)^{-1}$, the absorption coefficient for a longitudinal acoustic wave has the form⁵⁸

$$\Gamma = \Gamma_0 \Omega_0 \tau \frac{1 - J_0^2(\zeta) + kD g_\beta [1 + J_0(\zeta) \sin kD] + \eta_1^2 kD f_\beta^2 \cos^2 \beta [1 - J_0(\zeta) \sin kD]}{1 - J_0(\zeta) \sin kD + kD g_\beta} \Big|_{k=\omega/s} \quad (5.27)$$

The functions

$$f_\beta = \frac{N_1}{N} \frac{(kl)^2 \cos^2 \beta}{1 + (kl)^2 \cos^2 \beta} \quad \text{and} \quad g_\beta = \frac{N_1}{N} \frac{\sin^2 \beta}{1 + (kl)^2 \cos^2 \beta}$$

do not exceed unity when the number densities of charge carriers of both electron groups are equal. In expression (5.4), we have neglected unity in comparison with the quantity $|\xi \bar{\sigma}_{yy}|$. This corresponds to the inequality $c^2 \omega^2 D / s^3 \omega_0^2 \tau \ll 1$ which is satisfied in the ultrasonic frequency range if the frequency of plasma oscillations ω_0 in a quasi-two-dimensional conductor is of the same order of magnitude as in an ordinary metal. Insignificant numerical factors in formula (5.27) have been omitted.

The presence of a group of charge carriers with a quasi-one-dimensional dispersion relation leads to considerable anisotropy in attenuation of an acoustic wave in the plane of the layers. If the wave propagates along the preferred direction of velocities of electrons belonging to this group ($\beta = 0$), the sound absorption coefficient can be represented in the form

$$\Gamma = \Gamma_0 \left(\Omega_0 \tau \frac{1 - J_0^2(\zeta)}{1 - J_0(\zeta) \sin kD} + \eta_1^2 \frac{N_1}{N_2} \frac{\omega \tau}{s} v_0 \right) \Big|_{k=\omega/s} \quad (5.28)$$

For $\zeta \ll 1$, the corrugation of the quasi-two-dimensional cavity on the Fermi surface is quite small, and the first term in formula (5.28) assumes the form of sharp resonance peaks. The resonant dependence of Γ on H^{-1} can be observed by measuring the derivative of Γ with respect of reciprocal magnetic field. In this case, charge carriers belonging to the quasi-one-dimensional group make a contribution to the ‘‘background’’ component of Γ .

When the angle β deviates from zero, the resonant nature of the dependence $\Gamma(H^{-1})$ is preserved as long as the inequality $\pi/2 - \beta > (kD)^{1/2}/kl$ is satisfied. When the value

of the angle β approaches $\pi/2$, the resonant behavior of the sound absorption coefficient changes for giant oscillations which assume the following form for $\beta = \pi/2$:

$$\Gamma = \Gamma_0 \Omega_0 \tau \{1 + J_0(\zeta) \sin kD\} \cong \Gamma_0 \Omega_0 \tau \left\{ 1 + \sin kD - \frac{\zeta^2}{4} \sin kD \right\} \Big|_{k=\omega/s} \quad (5.29)$$

For $\sin kD = -1$, the absorption coefficient Γ assumes its minimum value which is the smaller, the weaker the corrugation of the Fermi surface.

Figures 8 and 9 show the dependence of absorption coefficient on h and $\cos \beta$.

The peaks on the experimentally observed dependence of Γ on the magnitude and orientation of magnetic field are considerably less sharp than those in Figs. 5–9 since the value of kl in the layered conductors studied at present considerably exceeds unity only in the region of hypersonic frequencies.

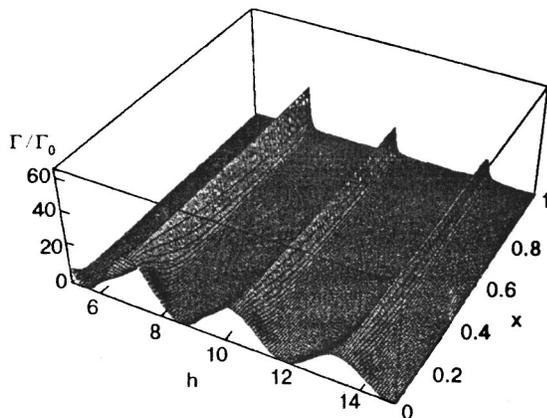


FIG. 8. Dependence of the absorption coefficient Γ/Γ_0 on $h=H_0/H$ ($H_0=2\omega cm v_0/es$) and $x=\cos \beta$ for $\eta=\eta_1=10^{-2}$, $N_1/N_2=1$, and $kl=10^2$.

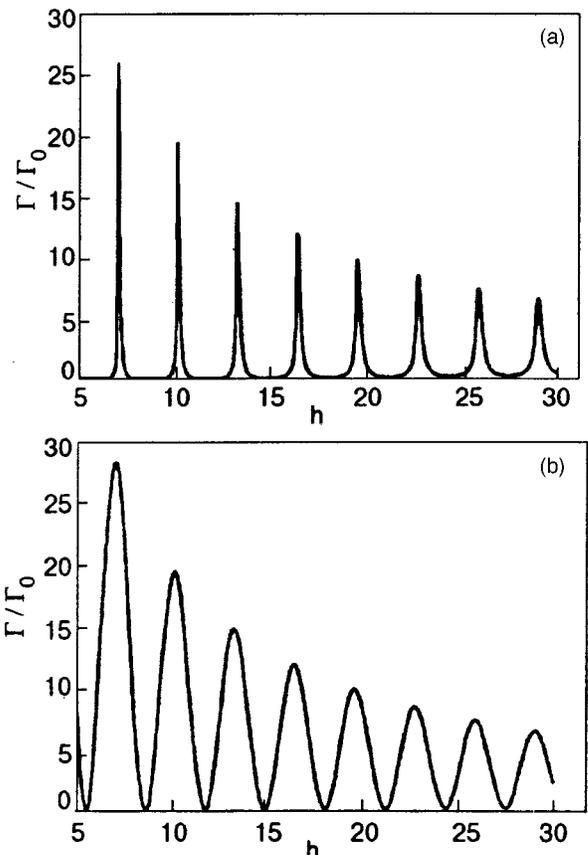


FIG. 9. Cross sections of the curve in Fig. 8 by the planes $x=1$ (a) and $x=0$ (b).

5.2. Transverse wave propagating along the layers

In the case of transverse polarization of an acoustic wave $\mathbf{u}=(0, u_y, u_z)$, the magnetic field $\mathbf{H}=(0, H \sin \theta, H \cos \theta)$ oriented perpendicularly to the wave vector appears in Maxwell's equations

$$\tilde{E}_\alpha = \frac{m\omega^2}{e} u_\alpha + \xi j_\alpha; \quad \alpha=y, z \tag{5.30}$$

only in expressions for acoustoelectric coefficients. Using formulas (3.21), we can write these equations in the form

$$j_y(1 - \xi \tilde{\sigma}_{yy}) - j_z \xi \tilde{\sigma}_{yz} = \left(k\omega \tilde{a}_{yy} + \frac{m\omega^2}{e} \tilde{\sigma}_{yy} \right) u_y + \left(k\omega \tilde{a}_{yz} + \frac{m\omega^2}{e} \tilde{\sigma}_{yz} \right) u_z,$$

$$-j_y \xi \tilde{\sigma}_{zy} + j_z(1 - \xi \tilde{\sigma}_{zz}) = \left(k\omega \tilde{a}_{zy} + \frac{m\omega^2}{e} \tilde{\sigma}_{zy} \right) u_y + \left(k\omega \tilde{a}_{zz} + \frac{m\omega^2}{e} \tilde{\sigma}_{zz} \right) u_z. \tag{5.31}$$

Let us consider the propagation of a transverse acoustic wave in a conductor with one group of charge carriers possessing a quasi-two-dimensional energy spectrum. Supplementing Eqs. (5.31) with equations (3.2) from the theory of elasticity, we obtain a system of equation whose compatibility condition

$$\begin{vmatrix} 1 - \xi \tilde{\sigma}_{yy} & -\xi \tilde{\sigma}_{yz} & \chi_{yy} & \chi_{yz} \\ -\xi \tilde{\sigma}_{zy} & 1 - \xi \tilde{\sigma}_{zz} & \chi_{zy} & \chi_{zz} \\ (i\omega m/e) + ik\xi \tilde{b}_{yy} & ik\xi \tilde{b}_{yz} & (\omega^2 - s_y^2 k^2)\rho + \varphi_{yy} & \varphi_{yz} \\ ik\xi \tilde{b}_{zy} & (i\omega m/e) + ik\xi \tilde{b}_{zz} & \varphi_{zy} & (\omega^2 - s_z^2 k^2)\rho + \varphi_{zz} \end{vmatrix} = 0 \tag{5.32}$$

is the dispersion equation of the problem. Here $s_y = (\lambda_{yzyx}/\rho)^{1/2}$ and $s_z = (\lambda_{zxzx}/\rho)^{1/2}$ are the velocities of acoustic waves polarized along the y- and z-axes, respectively, and

$$\chi_{\alpha\beta} = -k\omega \tilde{a}_{\alpha\beta} - \frac{m\omega^2}{e} \tilde{\sigma}_{\alpha\beta},$$

$$\varphi_{\alpha\beta} = ik \left[k\omega \tilde{c}_{\alpha\beta} + \frac{m\omega^2}{e} \tilde{b}_{\alpha\beta} \right]. \tag{5.33}$$

The elastic moduli tensor components λ_{yxzx} and λ_{zyyx} vanish if the xy plane is the symmetry plane of the crystal.⁵⁹ Otherwise, these components must be taken into account, but this does not change the final results significantly.

In view of strong anisotropy of the energy spectrum for charge carriers, the absorption of acoustic waves polarized along and across the layers has essentially different forms. It can easily be verified that the series expansion in small η of acoustoelectronic tensor components with at least one index z starts with quadratic or higher-order terms in η . Retaining only quadratic terms in η in Eq. (5.31), we obtain

$$\left\{ [(\omega^2 - s_y^2 k^2)\rho + \varphi_{yy}](1 - \xi \tilde{\sigma}_{zz})\chi_{yy} \left(i \frac{\omega m}{e} + ik\xi \tilde{b}_{yy} \right) \right\} \times [(\omega^2 - s_z^2 k^2)\rho + \varphi_{zz}] \left(1 - \xi \tilde{\sigma}_{zz} - \chi_{zz} i \frac{\omega m}{e} \right) = 0. \tag{5.34}$$

The multiplicity of this equation implies that in the approximation quadratic in η , acoustic waves polarized along the y- and z-axes do not interact with each other. Equating the ex-

pression in the braces in (5.34) to zero, we obtain the dispersion equation for the wave polarized along the y-axis. Its solution can be presented in the form $k = \omega/s_y + k_2$, where

$$k_2 = \frac{i}{2\rho s_y^2 (1 - \xi \tilde{\sigma}_{yy})} \left[\xi k\omega (\tilde{a}_{yy} \tilde{b}_{yy} - \tilde{c}_{yy} \tilde{\sigma}_{yy}) + \frac{m\omega^2}{e} (\tilde{a}_{yy} + \tilde{b}_{yy})k\omega \tilde{c}_{yy} + \frac{m^2 \omega^3}{k e^2} \tilde{\sigma}_{yy} \right]_{k=\omega/s_y}. \tag{5.35}$$

The denominator in this expression has the same form as in formula (5.4) for k_1 . It follows hence that the absorption of a transverse acoustic wave polarized along the y-axis in a conductor with a single quasi-two-dimensional group of charge carriers is of resonance type like the absorption of a longitudinal wave.

The deviation of the second root of Eq. (5.34) from σ/s is described by the formula

$$k_3 = \frac{i}{2\rho s_z^2} \left[\frac{m\omega^2}{e} \left(\frac{\tilde{a}_{zz}}{1 - \xi \tilde{\sigma}_{zz}} + \tilde{b}_{zz} \right) + \left(\frac{m\omega}{e} \right)^2 \frac{s_z \tilde{\sigma}_{zz}}{1 - \xi \tilde{\sigma}_{zz}} + \frac{\omega^2}{s_z} \tilde{c}_{zz} \right]_{k=\omega/s_z}. \tag{5.36}$$

It can easily be verified that the last term in the brackets in formula (5.36) has the highest order of magnitude. Its contribution to the absorption coefficient is decisive and has the form

$$\Gamma \cong \Gamma_0 \eta^2 \frac{l}{D} (1 + \sin kD). \tag{5.37}$$

The peculiarity of quasi-two-dimensional energy spectrum of charge carriers for waves with the above polarization is manifested in stronger magnetic fields also, when $kD \ll 1$. In this case, the orientation magnetoacoustic effect is manifested in a strong oscillatory dependence of absorption coefficient on the angle formed by the magnetic field with the normal to the layers.^{53,60}

Electron orbits in the momentum space are cross sections of the Fermi surface by the plane $p_H = \text{const}$, where p_H is the momentum component along the magnetic field. Consequently, integrating over the Fermi surface for calculating acoustoelectronic tensors by formulas (3.21), we can conveniently use the variables ε, t , and p_H . If we substitute $p_z = p_H/\cos \theta - p_y \tan \theta$ into the integrands containing the expressions

$$\Lambda_{zz}(\mathbf{p}) = \sum_{n=1}^{\infty} \Lambda_n(p_x, p_y) \cos \frac{anp_z}{h},$$

$$v_z(\mathbf{p}) = - \sum_{n=1}^{\infty} n \varepsilon_n(p_x, p_y) \frac{a}{h} \sin \frac{anp_z}{h}, \quad (5.38)$$

it can easily be verified that the corresponding acoustoelectronic coefficients are complex periodic functions of the angle θ formed by the directions of magnetic field and the normal to the layers. All the orbits in a quasi-two-dimensional conductor are almost indistinguishable, and hence the momentum components p_x and p_y depend on p_H weakly. This allows us to obtain explicit dependence of acoustoelectronic coefficients on θ and to make sure that they vanish for certain values of the angle $\theta = \theta_c$ in the approximation quadratic in the parameter η . When $\tan \theta \gg 1$, but $\cos \theta \gg 1/\Omega\tau$, the values of θ_c are repeated with a period $\Delta(\tan \theta) = 2\pi h/D_p$. These oscillations are associated with the motion of charge carriers in strongly elongated orbits in the momentum space, which intersect a large number of unit cells in the reciprocal lattice, and the period of oscillations is connected with a change in this number by unity.

In the case when the dispersion relation for charge carriers has the form (5.10), and the deformation potential is described by formula (5.11), the absorption coefficient has the form

$$\Gamma = \eta^2 \Gamma_0 \frac{\omega \tau v_0}{s} J_0^2(\xi). \quad (5.39)$$

where $\xi = (a v_0 m/h) \tan \theta$. At points where Bessel's function $J_0(\xi)$ vanishes, we must take into account the next terms in the expansion in small parameters kD and s/v .

5.3. Acoustic wave propagating across the layers

In order to solve the system of equations (3.1)–(3.3) in the case when a wave propagates across the layers, we must carry out Fourier transformations in the coordinate z considering that the solution of the kinetic equation has the form

$$\psi = \int_{-\infty}^t dt' g[(z + z(t') - z(t)) \exp[\nu(t' - t)]],$$

$$z(t) = \int^t v_z(t') dt'. \quad (5.40)$$

We consider the propagation of a longitudinal acoustic wave $\mathbf{u} = (0, 0, u)$ in a magnetic field $\mathbf{H} = (0, H \sin \theta, H \cos \theta)$. The system of equations for the Fourier components of ion displacement and electric field in this case has the form

$$(\tilde{a}_{xz} k \xi + i H_y / c) \omega u + (\xi \sigma_{xx} - 1) \tilde{E}_x + \xi \tilde{\sigma}_{xy} \tilde{E}_y = 0,$$

$$\tilde{a}_{yz} k \xi \omega u + \xi \tilde{\sigma}_{yx} \tilde{E}_x + (\xi \sigma_{yy} - 1) \tilde{E}_y = 0,$$

$$(\omega^2 - s^2 k^2) \rho u + [ik \tilde{c}_{zz} + c^{-1} \tilde{a}_{xz} H_y] k \omega u + [ik \tilde{b}_{zx} + c^{-1} \tilde{\sigma}_{xx} H_y] \tilde{E}_x + [ik \tilde{b}_{zy} + c^{-1} \tilde{\sigma}_{xy} H_y] \tilde{E}_y = 0, \quad (5.41)$$

where

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \frac{\sigma_{\alpha z} \sigma_{z\beta}}{\sigma_{zz}}, \quad \tilde{a}_{\alpha z} = a_{\alpha z} - \frac{a_{zz} \sigma_{\alpha z}}{\sigma_{zz}},$$

$$\tilde{b}_{z\beta} = b_{z\beta} - \frac{b_{zz} \sigma_{z\beta}}{\sigma_{zz}}, \quad \tilde{c}_{zz} = c_{zz} - \frac{b_{zz} a_{zz}}{\sigma_{zz}},$$

$$s = (\lambda_{zzzz} / \rho)^{1/2}.$$

Acoustoelectronic coefficients are defined by formulas (3.21) in which

$$\hat{R}g = \int_{-\infty}^{\tau} dt' g(t') \exp\{ik[z(t') - z(t)] + \nu(t' - t)\}.$$

We shall describe the results of analysis of the dispersion equation of the system (5.41), which is carried out for $\omega\tau \ll 1$ for an acoustic wave propagating, as before, along the layers.

If the magnetic field is directed along the normal to the layers ($\theta = 0$), the absorption is mainly determined by renormalization of the charge carrier energy under the action of deformation. In the case when the deformation potential is described by formula (5.11), the absorption coefficient satisfies the following expression:

$$\Gamma = \Gamma_0 \frac{1}{kl} \{ [1 + (\eta kl)^2]^{1/2} - 1 \}, \quad (5.42)$$

which has the form

$$\Gamma = \Gamma_0 \eta^2 kl. \quad (5.43)$$

for $kl\eta \ll 1$. Here $l = \tau v_0$.

If, however, the angle θ differs from zero, but is not very close to $(\pi/2)(\cos \theta \gg 1/\Omega_0\tau)$, the absorption coefficient for $kl\eta \ll 1$ is described by the formula

$$\Gamma = \frac{\Gamma_0}{2} \left\{ \eta^2 kl J_0^2(\xi) + \frac{\sin^2 \theta}{kl} \left(\frac{\Omega_0 \omega c^2}{\omega_0^2 s^2} \right) \right\}, \quad (5.44)$$

which coincides with formula (5.43) for $\theta = 0$.

The first term in formula (5.44) is determined by deformation interaction of electrons with the acoustic wave and describes angular oscillations of absorption coefficient. The second term is associated with the electromagnetic field excited by the acoustic wave and differs from zero even for

$\eta \rightarrow 0$. For values of angles for which Bessel's function $J_0(\xi)$ does not vanish, the dependence of Γ on kl has a minimum for

$$kl\eta = \frac{\sin \theta}{J_0(\xi)} \frac{\Omega_0 \omega c^2}{\omega_0^2 s^2},$$

associated with the competition between the two mechanisms of absorption.

If the magnetic field is oriented along the plane of the layers ($\theta = \pi/2$), almost all charge carriers move in open orbits, and the attenuation of the acoustic wave for $kR\eta \ll 1$ [$R = hc/(aeH)$] is described by the formula

$$\Gamma = \Gamma_0 \eta^2 kR. \quad (5.45)$$

Thus, the dependences of Γ on the magnitude and direction of the applied magnetic field are quite diverse and can give rich information for studying the properties of charge carriers in low-dimensional conducting structures.

6. CONCLUSION

Wave processes in layered organic conductors in a strong magnetic field are quite sensitive to the form of the electron energy spectrum, and their experimental study will provide detailed and reliable information on the dispersion relation and relaxation properties of charge carriers.

Organic conductors are also interesting for applications owing to the diversity of high-frequency and magnetoacoustic phenomena typical of conductors with low-dimensional electron energy spectra. The acoustic transparency stimulated by a magnetic field undoubtedly facilitates the perfection of acoustoelectronic devices. Such a strong dependence of the intensity of the wave penetrating in the bulk of the sample on its polarization makes it possible to use even thin plate of layered conductors, whose thickness is considerably larger than the skin depth, but smaller than or of the order of the mean free path of charge carriers, as filters through which waves with a definite polarization can pass. We shall consider our task to be fulfilled and the publication of this review as expedient if the variety of weakly attenuating waves typical of quasi-two-dimensional conductors considered by us here draws the attention of experimenters.

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