

## ELECTRONIC PROPERTIES OF METALS AND ALLOYS

## High-temperature quantum size oscillations of the conductivity in thin metal films

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The longitudinal electrical conductivity of a thin conducting film in the presence of electron and hole groups of charge carriers is calculated by the Kubo method under conditions such that the carrier motion is spatially quantized. It is shown that in the case of elastic scattering on impurities there are quantum high-temperature oscillations of the conductivity which are relatively insensitive to the temperature smearing of the Fermi level. © 2003 American Institute of Physics. [DOI: 10.1063/1.1630719]

## 1. INTRODUCTION

The quantum size effect (QSE) in thin films has been well studied both experimentally and theoretically (see Ref. 1 and the literature cited therein). In spite of this, several experimentally observed phenomena have not been given an exhaustive theoretical explanation. For example, experimental studies of the QSE in bismuth<sup>2–5</sup> find evidence of the presence of quantum size oscillations of the conductivity at  $T \gg \Delta E$  ( $\Delta E$  is the distance between adjacent subbands formed by the size quantization, and  $T$  is the temperature in energy units), while at the same time the theoretical calculation (see Refs. 6 and 7) predicts that they should be strongly attenuated. Interestingly, a similar situation also arises in the case of magnetic quantization, as is attested to by a series of experimental studies of the Shubnikov–de Haas (SdH) effect in bismuth (see Refs. 8–10 and references cited therein). These high-temperature oscillations cannot be explained in the framework of the usual theory of the SdH effect (see, e.g., Ref. 11).

In this paper we consider a mechanism for the appearance of quantum high-temperature oscillations (QHTOs) in the presence of an elastic scattering potential. This mechanism, which was proposed in Ref. 12 for semiconductors placed in a quantizing magnetic field, consists in the following. In the case of two valleys, the electron density of states has a feature at values of the energy corresponding to the bottom of a subband (i.e., the point of the subband with the lowest value of the energy for the electron valley and the highest energy for the hole valley). The intensity of intervalley scattering increases substantially when the energies corresponding to the bottom of any two subbands in the different valleys coincide. This condition of a maximum in the scattering intensity will be sensitive to changes in the position of the subbands. As the quantization conditions are varied (i.e., the magnetic field strength in the case of quantization by a magnetic field or the thickness of the sample in the case of the QSE) the subbands from the different valleys will shift along the energy scale with different rates, periodically “overtaking” one another. In the case of valleys with differ-

ent signs of the charge carriers the subbands will be shifted in opposite directions. Of course, this condition must hold in the region of temperature smearing of the Fermi level.

This mechanism is also used to explain the high-temperature oscillations in bismuth in a quantizing magnetic field (the case of two valleys with different signs of the charge carrier was studied in Refs. 13–15) and in the case of a quasi-two-dimensional dispersion relation.<sup>16,17</sup>

## 2. STATEMENT OF THE PROBLEM

We consider a thin metal film with mirrorlike boundaries,  $z=0$  and  $z=L_z$ , and two groups of charge carriers: electron and hole. For simplicity we shall assume that the Fermi surface consists of two pockets with a quadratic dispersion relation in each, i.e., the energy  $\varepsilon$  of the charge carriers in the electron  $\varepsilon_e$  and hole  $\varepsilon_h$  valleys is equal:

$$\begin{aligned}\varepsilon_e &= \frac{p_x^2 + p_y^2}{2m_e} + \alpha_e n^2, \\ \varepsilon_h &= \varepsilon_{OL} - \frac{p_x^2 + p_y^2}{2m_h} - \alpha_h n^2,\end{aligned}\quad (1)$$

where  $m_{e,h}$  are the masses of the electrons and holes, respectively,  $\varepsilon_{OL}$  is the value of the band overlap, and

$$\alpha_{e,h} = \frac{\pi^2 \hbar^2}{2m_{e,h} L_z^2}.$$

We shall take into account only the scattering on a short-range impurity, the effective radius of which is much smaller than the de Broglie wavelength of an electron on the Fermi surface, and assume that the impurity centers are uniformly distributed over the volume of the film with a density  $n_{\text{imp}}$ . For simplicity we shall assume that the matrix elements of the impurity potential operator responsible for transitions within the electron and hole groups and for intervalley transitions are of the same order of magnitude, i.e.,

$$\hat{V}_{ee} \approx \hat{V}_{hh} \approx \hat{V}_{eh}. \quad (2)$$

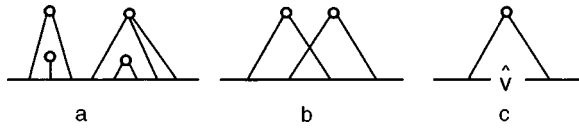


FIG. 1. Examples of the diagrams taken into account in the averaging over the distribution of impurity centers (a); a “crossing” diagram (b); a diagram in which the velocity operator leads to vanishing of the corresponding contribution after averaging over the position of the impurity center (c).

A severalfold difference in the values of these quantities will not qualitatively influence the result but will only affect the amplitude ratio of the various modes of oscillation while preserving the qualitative picture of the QSE.

### 3. CONDUCTIVITY AT ZERO TEMPERATURE

We write the expression for the conductivity in the form<sup>18</sup>

$$\sigma_{xx} = \frac{2\pi\hbar}{T} e^2 \int_{-\infty}^{\infty} dE f^0(E) (1 - f^0(E)) \times \langle \text{Tr} \hat{v}_x \delta(\hat{H} - E) \hat{v}_x \delta(\hat{H} - E) \rangle_{\text{imp}}, \quad (3)$$

where  $f^0(E)$  is the Fermi–Dirac distribution function,  $\hat{H} = \hat{\varepsilon} + \sum_i V(\hat{\mathbf{r}} - \mathbf{R}_i)$  is the energy operator with the impurity taken into account (the impurity contribution to the energy operator is equal to the sum of the potentials of the individual impurity centers, which are randomly distributed at the points  $\mathbf{R}_i$ ), the summation in the trace  $\text{Tr}$  is done over the one-electron complete quantum set  $(n, P_x, P_y)$ , and the angle brackets denote averaging over the distribution of impurity centers.

In averaging over the distribution of impurity centers we use the standard diagram technique, given, e.g., in Ref. 19 or in more detail for the case of an elastic impurity in Ref. 20. Simple estimates show that in expression (3) the contributions containing “crossing” diagrams (see Fig. 1b) can be neglected.<sup>20</sup> The contribution of expressions of the form in Fig. 1c, which contain the velocity operator between the impurity potential operators generated by the same impurity center, also turn out to be negligible (the contribution can be neglected after averaging the expression over the impurity coordinate). This makes it possible to replace the average of the product of delta functions over the distribution of impurity centers in expression (3) by the product of the averages:

$$\langle \hat{v}_i \delta(\hat{H} - E) \hat{v}_j \delta(\hat{H} - E) \rangle_{\text{imp}} = \hat{v}_i \langle \delta(\hat{H} - E) \rangle_{\text{imp}} \hat{v}_j \langle \delta(\hat{H} - E) \rangle_{\text{imp}}.$$

The latter can be represented in the form  $\langle \delta(\hat{H} - E) \rangle_{\text{imp}} = i/2\pi (\hat{G}_V^+(E) - \hat{G}_V^-(E))$ , where

$$\hat{G}_V^{\pm}(E) = \left\langle \frac{1}{E - \hat{H} \pm i\delta} \right\rangle_{\text{imp}} = \frac{1}{E - \hat{\varepsilon} - \hat{\Sigma}^{\pm}(E)} \quad (4)$$

is the total Green’s function, the quantity  $\hat{\Sigma}^{\pm}(E) = \langle \hat{\Sigma}_R^{\pm}(E) \rangle_{\text{imp}}$  by definition (see Refs. 19 and 20) is the self-energy part averaged over the impurity distribution,

$$\hat{\Sigma}_R^{\pm}(E) = \hat{V}_R + \hat{V}_R \hat{G}_V^{\pm}(E) \hat{V}_R + \dots, \quad (5)$$

$\hat{V}_R$  is the potential operator of an individual impurity center at the point  $\mathbf{R}$ , and

$$\hat{G}^{\pm}(E) = \frac{1}{E - \hat{\varepsilon} \pm i\delta} \quad (6)$$

is the Green’s function without the impurity taken into account.

In evaluating the series (5) we can use the analogy with the expression for the  $\hat{T}^{\pm}$  operator:

$$\hat{T}_R^{\pm}(E) = \hat{V}_R + \hat{V}_R \hat{G}^{\pm}(E) \hat{V}_R + \dots \quad (7)$$

It follows from the explicit form of the Green’s function (4), (6) that the relation  $\hat{G}_V^{\pm}(E) = \hat{G}^{\pm}(E - \hat{\Sigma}^{\pm}(E))$ . Then  $\hat{\Sigma}_R^{\pm}(E)$  can be expressed through the equation

$$\hat{\Sigma}_R^{\pm}(E) = \hat{T}_R^{\pm}(E - \Sigma^{\pm}(E)). \quad (8)$$

In solving equation (8) we can assume that the operator  $\hat{\Sigma}^{\pm}(E)$  is diagonal:  $\hat{\Sigma}^{\pm}(E) = \Sigma^{\pm}(E) \hat{I}$ , where  $\hat{I}$  is the unit operator.

For the operator  $\hat{T}^{\pm}(E)$  the expression proposed in Ref. 21 is valid (with a correction for the form of the spectrum (1)):

$$T_{\nu\mu}^{\pm}(E) = t^{\pm}(E) \varphi_{\nu}^*(\mathbf{R}) \varphi_{\mu}(\mathbf{R}), \quad (9)$$

$$t^{\pm}(E, \mathbf{R}) = \frac{S}{1 - S G_q^{\pm}(E, Z)},$$

where  $\mathbf{R} = (X, Y, Z)$  is the coordinate of the impurity center,

$$\varphi_{\nu}(\mathbf{r}) = \frac{1}{2\pi\hbar} \sqrt{\frac{2}{L_z}} e^{i\hbar P_x x} e^{i\hbar P_y y} \sin\left(\frac{\pi n z}{L_z}\right),$$

$$\nu = (n, P_x, P_y), \quad (10)$$

is the eigenfunction of the unperturbed energy operator  $\hat{\varepsilon}$ ,

$$S = \int V(\mathbf{r}) \psi_0(\mathbf{r}) d^3\mathbf{r} \quad (11)$$

is the renormalized strength of the impurity (see Refs. 16 and 21),  $\psi_0(\mathbf{r})$  is determined by Dyson’s equation

$$\psi_0(\mathbf{r}) = 1 + \int G_{\text{cl}}(\mathbf{r}, \mathbf{r}') V(\mathbf{r}') \psi_0(\mathbf{r}') d^3\mathbf{r}',$$

$V(\mathbf{r})$  is the impurity potential, and  $G_q^{\pm}(E, Z)$  is the Green’s function after subtraction of the singular contribution  $G_{\text{cl}}(\mathbf{r}, \mathbf{r}') \sim 1/|\mathbf{r} - \mathbf{r}'|$  and is given by the following relation in the coordinate representation:

$$G^{\pm}(E; \mathbf{r}, \mathbf{r}') = \sum_{\nu} \frac{\varphi_{\nu}(\mathbf{r}) \varphi_{\nu}^*(\mathbf{r}')}{E - \varepsilon_{\nu} \pm i\delta} = G_{\text{cl}}(\mathbf{r} - \mathbf{r}') + G_q^{\pm}(E, Z) \quad (12)$$

( $G_{\text{cl}}(\mathbf{r}, \mathbf{r}')$  is equal to the real part of the Green’s function  $G^{\pm}(E; \mathbf{r}, \mathbf{r}')$  (6) without the quantization taken into account, and  $L_z \rightarrow \infty$ ; Ref. 21). In the case under consideration, that of a short-range impurity, the dependence of  $G_q^{\pm}(E; \mathbf{r}, \mathbf{r}')$  on  $|\mathbf{r} - \mathbf{r}'|$  can be neglected, but in contrast to the case of magnetic quantization considered in Ref. 21, there is dependence on the  $z$  component of the coordinate  $\mathbf{r} = \mathbf{r}'$ .

It follows from the definition of the Green’s function that

$$G_q^\pm(E, z) = G_e^\pm(E, z) + G_h^\pm(E, z), \quad (13)$$

where the subscripts  $e$  and  $h$  indicate whether the summation in (12) is done only over the electron valley or only over the hole valley. Taking into account the explicit form of the energy spectrum (1) and the definition of the Green's function, we can write  $G_h^\pm(E, z) = -G_e^\mp(\varepsilon_{OL} - E, z)|_{e \rightarrow h}$ . After substituting  $\varphi_\nu(\mathbf{r})$  in the explicit form (10) into expression (12) and using Poisson's formula [the divergent contribution of the zeroth harmonic is renormalized by  $G_{cl}(\mathbf{r} - \mathbf{r}')$ ], we obtain

$$G_e^\pm(E, z) = \mp \frac{2\pi^2 i m_e}{L_z (2\pi\hbar)^2} \sqrt{\frac{E}{\alpha_e}} \left[ 1 \mp 2 \sum_{k=1}^{\infty} \frac{i}{\sqrt{E} \beta_0^e} e^{\pm i \beta_0^e \sqrt{E}} \right. \\ \left. \pm \sum_{k=-\infty}^{\infty} \frac{i}{\sqrt{E} |\beta_z^e|} e^{\pm i |\beta_z^e| \sqrt{E}} \right], \quad (14)$$

where  $\beta_0^{e,h} = 2\pi k / \sqrt{\alpha_{e,h}}$ ,  $\beta_z^{e,h} = (2\pi / \sqrt{\alpha_{e,h}})(k + z/L_z)$ .

In making the transition to  $\Sigma^\pm(E)$  we must perform the average of  $\hat{T}_R^\pm(E)$  over the  $Z$  coordinate of the impurity center. The series in formula (14) contains a singularity with respect to  $z$ , and therefore in the expansion in the small parameter  $1/n_F \sim \sqrt{\alpha_e/\varepsilon_F}$ ,  $\sqrt{\alpha_h/(\varepsilon_{OL} - \varepsilon_F)} \ll 1$  ( $n_F$  is the index of the highest-numbered subband that overlaps the Fermi surface) it is more convenient to leave the singular part in the denominator of expression (9). The off-diagonal matrix elements of the operator  $T_{\mu\nu}^\pm(E)$  after averaging over  $Z$  are smaller by a factor of  $1/n_F$  than the diagonal ones for both the monotonic part and oscillatory part, and they can be neglected. In solving Eq. (8) it is sufficient to use the approximation  $\hat{\Sigma}_R^\pm(E) \approx \hat{T}_R^\pm(E - \Sigma_{cl}^\pm)$ , where  $\Sigma_{cl}^\pm$  is the nonquantum, monotonic part of  $\Sigma^\pm(E)$ , since the last terms of the expansion are small in the parameter  $\hbar/\tau\varepsilon_F$ ,  $\hbar/\tau(\varepsilon_{OL} - \varepsilon_F) \ll 1$ . In the equation for the conductivity,  $\text{Re} \Sigma_{cl}$  can be taken into account by a simple renormalization of  $\varepsilon_F$ :

$$\varepsilon_F \Rightarrow \tilde{\varepsilon}_F = \varepsilon_F - \text{Re} \Sigma_{cl}^\pm(\tilde{\varepsilon}_F) \quad (15)$$

(as is shown in Refs. 20 and 22). From here on,  $\varepsilon_F$  will be understood to mean the latter, renormalized quantity. The  $\text{Im} \Sigma_{cl}^\pm$  in the argument of the  $T$  matrix leads to the Dingle factor.

Let us consider expression (3) for  $T=0$  with allowance for the renormalization (15) of  $\varepsilon_F$ . After some transformations the impurity potential will appear in expression (3) only in the combination

$$\frac{1}{\tau(E)} = \frac{i}{\hbar} [\Sigma^+(E) - \Sigma^-(E)], \quad (16)$$

which plays the role of the relaxation time (in the framework of the approximations made, this is the same as the method of the quantum kinetic equation<sup>22</sup>).

Conductivity oscillations not due to scattering on impurities (i.e., oscillations that survive when  $\tau(E)$  is replaced by a constant in the expression for the conductivity) which arise in evaluating the trace  $\text{Tr}$  in formula (3), have the same frequency as in (16) but a substantially smaller amplitude, which, after calculation of (3) is smaller in order of magnitude by a factor of  $1/n_F^2$  than the monotonic part of the conductivity, and they can be neglected. The reason for their

smallness is that the velocity operator  $\hat{v}_x$  appears under the  $\text{Tr}$  sign in expression (3), and its eigenvalues are close to zero in the vicinity of the points  $\mathbf{p} = (0, 0, \pm p_F)$  which determine the oscillation frequency. Therefore expression (3) can be replaced by its classical analog, the quantization being taken into account only in the evaluation of expression (16). At  $T=0$  the conductivity will be proportional to the relaxation time:

$$\sigma_{xx} \sim \tau(\varepsilon_F). \quad (17)$$

To make the resulting expression for the conductivity less awkward, we shall restrict discussion to the limiting case of a weak impurity with a scattering amplitude much less than the de Broglie wavelength at the Fermi level. Then

$$\frac{1}{\tau(E)} = \frac{1}{\tau_0} \left( 1 + \sum_{k=1}^{\infty} R_k^e(E) + \sum_{k=1}^{\infty} R_k^h(E) \right), \quad (18)$$

where

$$R_k^e(E) = \frac{1}{2\pi k} \sqrt{\frac{\alpha_e}{E}} \exp\left(-\frac{\pi k \hbar}{2\tau_0 \sqrt{\alpha_e E}}\right) \sin\left(2\pi k \sqrt{\frac{E}{\alpha_e}}\right), \quad (19)$$

$$R_k^h(E) = \frac{1}{2\pi k} \sqrt{\frac{\alpha_h}{\varepsilon_{OL} - E}} \\ \times \exp\left(-\frac{\pi k \hbar}{2\tau_0 \sqrt{\alpha_h(\varepsilon_{OL} - E)}}\right) \sin\left(2\pi k \sqrt{\frac{\varepsilon_{OL} - E}{\alpha_h}}\right).$$

It follows from the assumptions made earlier that  $\sum_{k=1}^{\infty} R_k^{e,h} \sim 1/n_F \ll 1$ , i.e., the oscillatory part of the relaxation time is much smaller than the classical part  $\tau_0$ . Thus the expression for the conductivity can be written in the form

$$\sigma_{xx} = \sigma^{cl} \left( 1 + \sum_{k=1}^{\infty} R_k^e(\varepsilon_F) + \sum_{k=1}^{\infty} R_k^h(\varepsilon_F) \right)^{-1} \\ \approx \sigma^{cl} \left( 1 - \sum_{k=1}^{\infty} R_k^e(\varepsilon_F) - \sum_{k=1}^{\infty} R_k^h(\varepsilon_F) \right. \\ \left. + 2 \sum_{k,l=1}^{\infty} R_k^e(\varepsilon_F) R_l^h(\varepsilon_F) \right), \quad (20)$$

where in the terms of second-order smallness in  $1/n_F$  we have kept only the contributions responsible for the QHTOs;  $\sigma^{cl}$  is the classical part of the conductivity  $\sigma_{xx}$ .

#### 4. CONDUCTIVITY AT FINITE TEMPERATURE

The Fermi-Dirac distribution function appears in expression (3) for the conductivity only in the combination

$$\frac{1}{T} f^0(E) (1 - f^0(E)) = -\frac{\partial f^0(E)}{\partial E}.$$

Thus we can write

$$\sigma_{xx} = \int \left( -\frac{\partial f^0(E)}{\partial E} \right) F(E) dE, \quad (21)$$

where  $F(E)$  is a function of  $E$  that is independent of  $T$ . At  $T=0$

$$\sigma_{xx}|_{T=0} = F(\varepsilon_F), \quad (22)$$

which makes it possible to obtain the explicit form of  $F(E)$  from the dependence of the conductivity on  $\varepsilon_F$  (20). After that it remains only to substitute it into expression (21), the evaluation of which will give the temperature dependence of the conductivity. The derivation given here is analogous to the calculation of the temperature dependence of the thermodynamic potential  $\Omega$  in Ref. 11.

The nonclassical part of the conductivity at  $T=0$  in formula (20) behaves as an oscillatory function of  $\varepsilon_F$ . At a certain value  $\varepsilon_F = E_{k,l}$

$$E_{k,l} = \varepsilon_{OL} \frac{\alpha_h k^2}{\alpha_h k^2 + \alpha_e l^2}, \quad (23)$$

the condition  $k\Delta E_h^F = l\Delta E_e^F$  is satisfied, where  $k$  and  $l$  are integers. Here

$$\Delta E_e^F = 2\sqrt{\varepsilon_F \alpha_e}, \quad \Delta E_h^F = 2\sqrt{(\varepsilon_{OL} - \varepsilon_F) \alpha_h} \quad (24)$$

is the distance between adjacent subbands with positions close to the Fermi level in the electron and hole valleys, respectively. That is, at  $\varepsilon_F = E_{k,l}$  the frequency of the conductivity oscillations (for one of the harmonics) as a function of  $\varepsilon_F$  goes to zero, which leads to a softer than exponential temperature dependence. In the calculation of the temperature dependence of the conductivity we shall distinguish two limiting cases: the energy level  $E_{k,l}$  lies far outside the limits of the temperature smearing of the Fermi level, making it possible to neglect the unequal spacing of the energy spectrum, and the opposite limiting case with  $|E_{k,l} - \varepsilon_F| \ll T$ , when the unequal spacing determines the region of energy values that gives the main contribution to the integral (21). Substituting the expression (20) for the conductivity into the integral (21), we obtain

$$\sigma_{xx} = \sigma^{cl} + \sigma^h + \sigma^e + \sum_{k,l=1}^{\infty} (\sigma_{k,l}^- + \sigma_{k,l}^+), \quad (25)$$

where  $\sigma^{cl}$  is the monotonic part of the conductivity,

$$\begin{aligned} \sigma^e = & -\sigma^{cl} \sum_{k=1}^{\infty} \frac{1}{2\pi k} \sqrt{\frac{\alpha_e}{\varepsilon_F}} \Psi \left( \frac{2\pi^2 k T}{\Delta E_e^F} \right) \\ & \times \exp \left( -\frac{\pi k \hbar}{\tau_0 \Delta E_e^F} \right) \sin \left( 2\pi k \sqrt{\frac{\varepsilon_F}{\alpha_e}} \right), \end{aligned} \quad (26)$$

$$\begin{aligned} \sigma^h = & -\sigma^{cl} \sum_{k=1}^{\infty} \frac{1}{2\pi k} \sqrt{\frac{\alpha_h}{\varepsilon_{OL} - \varepsilon_F}} \Psi \left( \frac{2\pi^2 k T}{\Delta E_h^F} \right) \\ & \times \exp \left( -\frac{\pi k \hbar}{\tau_0 \Delta E_h^F} \right) \sin \left( 2\pi k \sqrt{\frac{\varepsilon_{OL} - \varepsilon_F}{\alpha_h}} \right). \end{aligned} \quad (27)$$

Expressions (26) and (27) agree with the results of Refs. 6 and 7:

$$\begin{aligned} \sigma_{k,l}^{\pm} = & \mp \sigma^{cl} \frac{1}{4\pi^2 k l} \sqrt{\frac{\alpha_e}{\varepsilon_F}} \sqrt{\frac{\alpha_h}{\varepsilon_{OL} - \varepsilon_F}} \Psi \left( 2\pi^2 T \left( \frac{k}{\Delta E_e^F} \right. \right. \\ & \left. \left. \mp \frac{l}{\Delta E_h^F} \right) \right) \exp \left( -\frac{\pi k \hbar}{\tau_0} \left( \frac{k}{\Delta E_e^F} + \frac{l}{\Delta E_h^F} \right) \right) \\ & \times \cos \left( 2\pi \sqrt{\frac{\varepsilon_F}{\alpha_e}} \pm 2\pi l \sqrt{\frac{\varepsilon_{OL} - \varepsilon_F}{\alpha_h}} \right), \end{aligned} \quad (28)$$

where

$$\Psi(x) = \frac{x}{\sinh x}.$$

Expressions (26)–(28) for  $\sigma^{e,h}$  and  $\sigma_{k,l}^-$  are given for any  $T$ , the contribution  $\sigma_{k,l}^+$  is determined by expression (28) if at least one of the following two conditions is met:

$$T \ll |E_{k,l} - \varepsilon_F|, \quad (29)$$

$$T \ll \min\{\alpha_e^{1/4} E_{k,l}^{3/4}, \alpha_h^{1/4} (\varepsilon_{OL} - E_{k,l})^{3/4}\}. \quad (30)$$

Condition (30) describes the region near  $T=0$  in which the temperature dependence of  $\sigma_{k,l}^+$  can be neglected.

In the opposite case, when the condition

$$T \gg |E_{k,l} - \varepsilon_F| \quad (31)$$

holds along with at least one of the inequalities

$$T \gg \alpha_e^{1/4} E_{k,l}^{3/4}, \quad T \gg \alpha_h^{1/4} (\varepsilon_{OL} - E_{k,l})^{3/4}, \quad (32)$$

the expression for  $\sigma_{k,l}^+$  will have the form

$$\begin{aligned} \sigma_{k,l}^+ = & -\sigma^{cl} \frac{1}{8\pi^2 k l T} \sqrt{\frac{\alpha_e}{E_{k,l}}} \sqrt{\frac{\alpha_h}{\varepsilon_{OL} - E_{k,l}}} \\ & \times \frac{1}{\sqrt{k/\sqrt{\alpha_e E_{k,l}^3} + l/\sqrt{\alpha_h (\varepsilon_{OL} - E_{k,l})^3}}} \\ & \exp \left( -\frac{\pi \hbar}{2\tau_0} \left( \frac{k}{\sqrt{\alpha_e E_{k,l}}} + \frac{l}{\sqrt{\alpha_h (\varepsilon_{OL} - E_{k,l})}} \right) \right) \\ & \times \cos \left( 2\pi k \sqrt{\frac{E_{k,l}}{\alpha_e}} + 2\pi l \sqrt{\frac{\varepsilon_{OL} - E_{k,l}}{\alpha_h}} - \frac{\pi}{4} \right). \end{aligned} \quad (33)$$

The oscillation frequency of the contributions (26)–(28), as in the case  $T=0$ , is determined by  $\tau(\varepsilon_F)$  (but the coefficients of proportionality in formula (17) for  $T \neq 0$  will be different for different harmonics). The oscillation frequency of contribution (33) is determined by  $\tau(E_{k,l})$  and is not necessarily a combination of the frequencies of the low-temperature contributions (26) and (27).

In formulas (25)–(28) and (33) and in conditions (29) and (31) one can restrict consideration to the case of small  $k, l$ , since the quantum oscillations are exponentially suppressed by the Dingle factor as these indices increase.

## CONCLUSIONS

We have shown that the presence of an elastic short-range impurity can give rise to high-temperature oscillations.

Unlike the case of valleys with the same type of charge carrier, when QHTOs would be expected only at values of  $\Delta E$  at the Fermi level which are close to multiples, in the case of two types of charge carriers the high-temperature character of the oscillations is manifested at any values of  $\Delta E_e$  and  $\Delta E_h$ , reaching a maximum amplitude at values which are multiples. Indeed, the presence of electron and hole valleys leads to a difference in the signs in the temperature factor and in the argument of the cosine in formula (28), i.e., the contribution  $\sigma_{k,l}^+$  will be simultaneously higher-temperature and higher-frequency in comparison with the contributions (26) and (27) (as in the case of magnetic quantization; see Refs. 13–15). In the case of two groups of

charge carriers of the same type those signs will be the same, and the high-temperature oscillations will be low-frequency (see Ref. 16 for the case of magnetic quantization, where the role of the two groups of charge carriers is played by regions near the extremal cross sections).

Because of the unequal spacing of the size-quantized energy spectrum, the condition of multiples,  $l\Delta E_e = k\Delta E_h$ , can be satisfied only in regions of momentum space which are characterized by an energy  $E_{k,l}$ . The latter, together with  $\varepsilon_F$ , can determine the frequency and amplitude of the size oscillations and the character of the temperature dependence of the conductivity. Here the frequency of the QHTOs need not be a combination of the frequencies of the low-temperature modes of oscillation.

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