ELECTRONIC PROPERTIES OF METALS AND ALLOYS

On the quantum oscillations of the sound attenuation coefficient in layered conductors

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The attenuation of a transverse sound wave in a layered conductor with a quasi-two-dimensional dispersion relation of the charge carriers in a quantizing magnetic field is considered. The oscillatory dependence of the sound attenuation coefficient on the inverse magnetic field is analyzed, and the role of Joule losses in the absorption of energy from the sound wave by electrons is ascertained for different orientations of the magnetic field with respect to the plane of the layers. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1645182]

The specifics of a quasi-two-dimensional dispersion relation of the charge carriers in a layered conductor are manifested in peculiar effects in the propagation of sound waves at low temperatures in a high magnetic field **H**, when the mean free time τ of the charge carriers is considerably longer than the period of gyration $2\pi/\Omega$ of an electron along a closed orbit in the magnetic field.

In a layered conductor a longitudinal sound wave is very weakly attenuated if the wave vector **k** and the vector **H** are directed along the normal to the layers. The high acoustic transparency of the conductor in such an experimental geometry is due to the fact that the Joule losses are insignificant, and the energy losses due to renormalization of the energy of the charge carriers (a deformation mechanism of absorption) are proportional to the square of the small quasi-two-dimensionality parameter η of the electron energy spectrum.[1,2] If there is even a small deviation of the magnetic field or wave vector from the normal to the layers the role of the Joule losses increases substantially.

Unlike the case of longitudinal sound, in the propagation of sound waves with the transverse polarization the Joule losses are substantial, over a wide range of magnetic field, for any orientation of the vectors **k** and **H** with respect to the layers.

If the temperature smearing T of the Fermi distribution function of the charge carriers is much less than the distance $\Delta\varepsilon=\hbar\Omega$ between the quantized Landau energy levels, then all of the thermodynamic and kinetic characteristics of the conductor, including the sound attenuation coefficient Γ (Refs. 3 and 4), oscillate with variation of $1/H$. The periods of these oscillations are determined by the areas of the extremal cross sections of the Fermi surface, and the amplitudes contain information about the effective cyclotron masses of the electrons on the extremal cross sections.

Let us consider the attenuation of a transverse sound wave in a layered conductor in a quantizing magnetic field in the case when

$$T \ll \hbar\Omega \ll \eta\mu, \tag{1}$$

where μ is the chemical potential of the electrons.

For the sake of brevity in the calculations we take the dispersion relation of the charge carriers in the form

$$\varepsilon(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m} - \eta v_0 \frac{\hbar}{a} \cos\frac{ap_z}{\hbar}. \tag{2}$$

Here **p** and m are the quasimomentum and mass of the electron, $v_0=(2\varepsilon_F/m)^{1/2}$ is its characteristic velocity along the layers, a is the distance between layers, and the quasi-two-dimensionality parameter η can be less than unity.

Although the dependence of the energy of the charge carriers on their quasimomentum in organic layered conductors is more complicated, the use of a model dispersion relation of the form (2) permits a complete explanation of the dependence of the sound attenuation coefficient on the value of the magnetic field and the orientation of the vectors **k** and **H**. Generalization to the case of a quasi-two-dimensional spectrum of arbitrary form does not present any difficulty and leads only to a refinement of numerical factors of the order of unity in the expression for Γ.

A sound wave propagating in a conductor leads to renormalization of the charge carrier energy:[5]

$$\delta\varepsilon = \lambda_{ik}(\mathbf{p}) u_{ik}. \tag{3}$$

Here u_{ik} is the strain tensor, and λ_{ik} are the components of the deformation potential tensor, taken with allowance for conservation of the number of charge carriers.

The quasi-two-dimensional character of the charge-carrier spectrum is reflected in anisotropy of the deformation potential. The deformation interaction of electrons with a sound wave is weakened for sound waves propagating along the normal to the layers or polarized along it. If the tensor components λ_{ik} with $i,k \neq z$ are of the order of magnitude of the Fermi energy, then the components of the deformation potential for which at least one of the indices is equal to z can be written in the form[6]

$$\lambda_{ik} = \eta Ł_{ik} \varepsilon_F \cos \frac{ap_z}{\hbar}, \tag{4}$$

where $Ł_{ik}$ is a number of the order of unity.

Besides the deformation interaction with the sound wave, electrons also are acted on by the electromagnetic wave generated by the sound.[7,8] In a reference frame tied to the vibrating crystal lattice, the electric field of this wave has the form

$$\tilde{\mathbf{E}} = \mathbf{E} - \frac{i\omega}{c}[\mathbf{u}\times\mathbf{H}] + \frac{m\mathbf{u}\omega^2}{e}, \tag{5}$$

where ω is the frequency of the wave, $\mathbf{u}$ is the displacement vector of the sites of the crystal lattice, e is the charge of the electron, and c is the speed of light. The electric field $\mathbf{E}$ satisfies the Maxwell equations

$$\text{curl curl } \mathbf{E} = \frac{4\pi i\omega}{c^2}\mathbf{j} + \frac{\omega^2}{c^2}\mathbf{E} \tag{6}$$

and the condition of continuity of the electric current in the conductor:

$$\text{div } \mathbf{j} = 0. \tag{7}$$

In a magnetic field $\mathbf{H} = (0, H\sin\theta, H\cos\theta)$ deviating from the normal to the layers by an angle θ, the cross section of the Fermi surface on the plane $p_H \equiv \mathbf{p}\cdot\mathbf{H}/H = \text{const}$ are closed and do not contain points of self-intersection if

$$|\theta| < \text{arctg } 1/\eta . \tag{8}$$

In this case the electron energy levels can be found with the aid of the quasiclassical quantization condition

$$S(\varepsilon, p_H) = \frac{2\pi e\hbar H}{c}\left(n + \frac{1}{2}\right), \tag{9}$$

where $S(\varepsilon, p_H)$ is the area bounded by the electron trajectory. It is easily seen that in the case of the dispersion relation (2) the energy levels take the form

$$\varepsilon_n = \left(n + \frac{1}{2}\right) h\Omega_0 \cos\theta \sqrt{1 + \eta \frac{v_0 am}{\hbar} \tan^2\theta \cos\zeta} - \eta \frac{v_0\hbar}{a}\cos\zeta - \eta^2 \frac{m v_0^2 \tan^2\theta \sin^2\zeta}{2[1 + \eta(v_0 am/\hbar)\tan^2\theta\cos\zeta]}, \tag{10}$$

where $\zeta = ap_H/(\hbar\cos\theta)$, $\Omega_0 = eH/mc$.

At temperatures low compared to the Debye temperature the sound attenuation in a conducting crystal is determined mainly by the interaction of the acoustic wave with conduction electrons. In the quasi-classical approximation the sound energy absorption coefficient Γ can be written in the form[2]

$$\Gamma = \frac{2}{\rho u^2 \omega^2 s} \frac{2eH}{c(2\pi\hbar)^2} \sum_n \int dp_H \left(-\frac{\partial f_0}{\partial \varepsilon_n}\right) \frac{\overline{|\psi|^2}}{\tau} . \tag{11}$$

Here ρ is the density of the crystal, s is the speed of sound, τ is the mean free time of the charge carriers, f_0 is the Fermi distribution function, and the overbar denotes averaging over the time of motion t of the electron along the quasiclassical closed orbit in the magnetic field. The function ψ, which takes into account the excitation of the electron system by the sound wave, can be written in the form

$$\psi = \int_{-\infty}^{t} dt' \, [e\mathbf{v}\cdot\tilde{\mathbf{E}} - i\omega\lambda_{ij}u_{ij}] \times \exp\{i\mathbf{k}\cdot[\mathbf{r}(t') - \mathbf{r}(t)] + \nu(t'-t)\}, \tag{12}$$

where $\nu = i\omega + 1/\tau$.

Let us consider a sound wave with a displacement vector $\mathbf{u} = (u,0,0)$, propagating in the direction normal to the layers. Using formulas (4), (11), and (12) and also the equations of motion for a charge in a magnetic field,

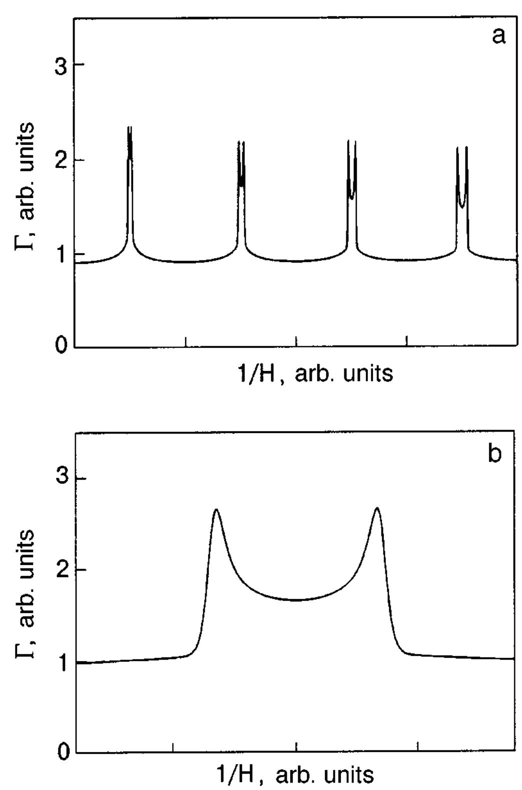

FIG. 1. The dependence of Γ on $1/H$ for $\theta = 0$, $\eta = 10^{-2}$, $\beta\gamma < 1$ in arbitrary units. The figures have different horizontal scales.

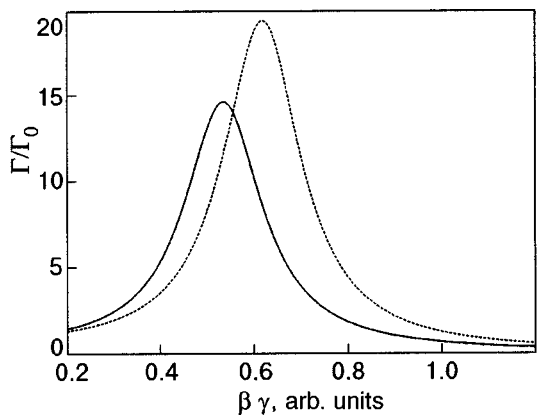

FIG. 2. Curves of $\Gamma_{\rm mon}$ for different values of the angle θ.

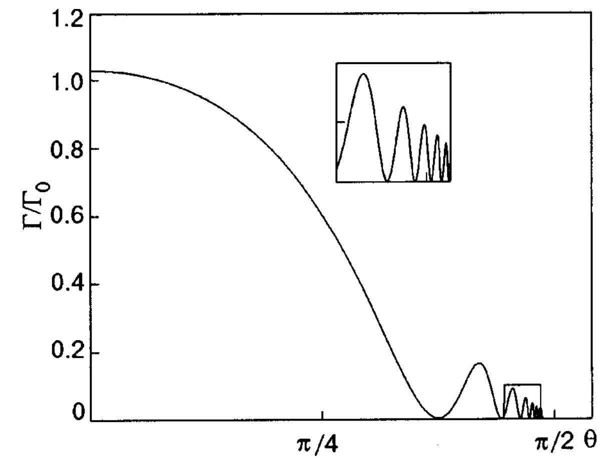

FIG. 3. $\Gamma_{\rm mon}$ versus angle θ for $\beta\gamma\gg 1$, in relative units.

$$\frac{\partial p_x}{\partial t}=\frac{eH}{c}(v_y\cos\theta-v_z\sin\theta),$$

$$\frac{\partial p_y}{\partial t}=-\frac{eH}{c}v_x\cos\theta,\quad \frac{\partial p_z}{\partial t}=\frac{eH}{c}v_x\sin\theta\frac{\partial p_z}{\partial t}, \tag{13}$$

one is readily convinced that in the region of sound frequencies in which the inequality $kl\eta\ll 1$ holds, the sound absorption coefficient can be written in the form

$$\Gamma=\frac{2}{\rho u^2\omega^2 s}\frac{2eH\tau}{c(2\pi\hbar)^2}\sum_n\int dp_H\left(-\frac{\partial f_0}{\partial\varepsilon_n}\right)$$
$$\times\left\{\left[\eta k\omega u L_{xz}\varepsilon_F J_0\left(\frac{amv_0}{\hbar}\tan\theta\right)\cos\zeta\right]^2\right.$$
$$\left.+\frac{(e\gamma v_0)^2}{2\cos^2\theta}(|\tilde E_x|^2+|\tilde E_y|^2)\right\}, \tag{14}$$

where J_0 is the Bessel function, and the parameter $\gamma=1/(\Omega_0\tau)\ll 1$.

With the use of Maxwell's equations (6), (7) it is not hard to find the electric field in the conductor if it is assumed that

$$\cos\theta\gg\gamma. \tag{15}$$

This condition, like inequality (8), excludes from consideration a small region of angles θ near $\theta=\pm\pi/2$ where the magnetic field is almost parallel to the layers.

In the leading approximation in the small parameter η the components of the electric field of the electromagnetic wave have the form

$$\tilde E_x=\frac{i\omega}{c}uH\cos^2\theta\frac{i\beta\gamma}{\cos^2\theta-(\beta\gamma)^2-2i\beta\gamma^2},$$

$$\tilde E_y=-\frac{i\omega}{c}uH\cos\theta\frac{\cos^2\theta-i\beta\gamma^2}{\cos^2\theta-(\beta\gamma)^2-2i\beta\gamma^2}, \tag{16}$$

where $\beta=(s\omega_p/c\omega)^2\omega\tau$, and ω_p is the plasma frequency.

Using the Poisson summation formula and changing from integration over n to integration over energy with the aid of formula (10), we obtain the following expression for the oscillatory (in $1/H$) part of the sound energy absorption coefficient:

$$\Gamma_{\rm osc}=\frac{\Gamma_0}{kl}\left(\frac{\hbar\Omega_0\cos\theta}{\eta\mu}\right)^{\frac12}\left[\left(\frac{(kl\eta)^2}{2}\right)L_{xz}^2J_0^2\left(\frac{amv_0}{\hbar}\tan\theta\right)+F(\gamma,\theta)\right]$$
$$\times\sum_N\frac{(-1)^N}{\sqrt{N}}\Psi(N\Lambda)\left[\cos\left(\frac{NcS_1}{2eH\hbar}-\pi/4\right)+\cos\left(\frac{NcS_2}{2eH\hbar}+\pi/4\right)\right]. \tag{17}$$

Here $\Gamma_0=2mN_cv_0\omega/\rho s^2$, N_c is the electron density, $l=v_0\tau$, $\Phi(z)=z/\sinh z$, $\Lambda=2\pi^2T/\hbar\Omega_0\cos\theta$, and the extremal values of the critical area of the Fermi surface on a plane $p_H=$const have the form

$$S_{1,2}=\frac{2\pi m}{\cos\theta}\frac{\mu\pm\eta v_0\hbar/a}{\sqrt{1\mp\eta(amv_0/h)\tan^2\theta}}. \tag{18}$$

In a quasi-two-dimensional conductor these values do not differ strongly from each other, and the oscillations therefore have a double-peak (doublet) form (Fig. 1).

The terms containing the factor L_{xz} in formula (17) describe the absorption of energy from the sound wave due to renormalization of the electron spectrum in a vibrating lattice (deformation mechanism). The Joule losses are determined by the function $F(\gamma,\theta)$, which has the form

$$F(\gamma,\theta)=\cos^2\theta\frac{\cos^2\theta+\beta^2\gamma^2}{[\cos^2\theta-(\beta\gamma)^2]^2+\beta^2\gamma^4}. \tag{19}$$

Analogous terms are also contained in the smoothly varying (with magnetic field) part of the absorption coefficient:

$$\Gamma_{\rm mon}=\frac{\Gamma_0}{kl}\left[\frac{(kl\eta)^2}{2}L_{xz}^2J_0^2\left(\frac{amv_0}{\hbar}\tan\theta\right)+F(\gamma,\theta)\right]. \quad (20)$$

The density of charge carriers in the organic conductors now under intensive study are comparable to the density of conduction electrons in ordinary metals, so that the parameter β can be much greater than unity. At a sufficiently high magnetic field ($\beta\gamma<\cos\theta$) the induction mechanism of sound-wave attenuation is the main one, and a peak should be observed on the absorption curve at $\beta\gamma=\cos\theta$ due to the excitation of a helicoidal wave in the conductor (Fig. 2). If $\beta\gamma$ is much greater than unity, then the Joule losses are small ($F(\gamma,\theta)=\cos^2\theta/\beta^2\gamma^2$) and there can be competition between the induction and deformation mechanisms of absorption of the sound energy by electrons. At sufficiently large angles of deviation of the magnetic field from the normal to the layers the coefficient Γ varies periodically with the angle θ. In the region where $(\beta\gamma)^{-1}\simeq kl\eta$ the amplitude of these oscillations is comparable to the monotonically varying (with angle) part of the absorption coefficient (Fig. 3).

The effects considered above, which are specific to quasi-two-dimensional conductivities, are completely observable in the region of ultrasonic frequencies $\omega\sim10^8$ s^{-1} in magnetic fields of the order of tens of tesla.

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