Eigenmodes of the electromagnetic field in the presence of a magnetic domain structure

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The spectrum of weakly damped eigenmodes of the electromagnetic field in metals in a quantizing magnetic field are determined under conditions such that a magnetic domain structure exists. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1542469]

At low temperatures the thermodynamic and kinetic characteristics of a metal placed in a quantizing magnetic field $\mathbf{H}=(0,0,H_0)$ have an oscillatory dependence on the inverse magnetic field. The cause of these oscillations is the presence of features of the density of states of the charge carriers due to the energy quantization in the magnetic field. Here the charges are actually acted upon by a field averaged over regions of the order of the Larmor radius, i.e., a magnetic induction $\mathbf{B}$. As long as the magnetic susceptibility χ is small, the difference between $\mathbf{B}$ and $\mathbf{H}$ can be neglected. If the distance between energy levels $\Delta\varepsilon\cong\hbar\Omega$ of the charge carriers in the magnetic field is much larger than the carrier temperature T and the level width $\hbar/\tau$ but much smaller than the Fermi energy ε_F, i.e., $\hbar/\tau,\ T\ll\hbar\Omega\ll\varepsilon_F$, the oscillatory part of the magnetic susceptibility can reach values of the order of unity, and the magnetization $\mathbf{M}(\mathbf{B})$ and the magnetic field $\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}(\mathbf{B})$ become functions of the magnetic induction. Here $\hbar$, Ω, and τ are Planck's constant, the cyclotron frequency, and the mean free time of the conduction electrons, respectively. In this case the problem of taking the magnetism of the medium into account is a self-consistent problem even in conductors that do not have magnetic ordering. If $\chi>1/4\pi$ the state of the system becomes unstable, and the sample separates into alternating domains with different values of the magnetic induction.[1,2]

In this paper we investigate the weakly damped eigenmodes of the electromagnetic field in uncompensated metals under conditions such that the distribution of the magnetic induction has a stationary domain structure. The alternating electromagnetic field in the metal is determined by the system of Maxwell's equations

$$\text{curl}\ \mathbf{B}=\frac{4\pi}{c}\mathbf{J},\ \ \text{curl}\ \mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\ \ \text{div}\,\mathbf{B}=0, \tag{1}$$

where c is the speed of light in vacuum, $\mathbf{J}=\mathbf{j}+\mathbf{j}'$ is the total current density, consisting of the conduction current density $\mathbf{j}$ due to the electric field $\mathbf{E}$ and the magnetization current density $\mathbf{j}'=c\ \text{curl}\,\mathbf{M}$ induced by the magnetic field.

In the case of weak temporal and spatial dispersion

$$\omega\ll\Omega,\quad kr_0\ll1,\quad k_z v_F\tau\ll1,$$

$$\kappa^2\equiv|1-4\pi\chi(\mathbf{B}_0)|\ll1, \tag{2}$$

where r_0 is the radius of curvature of the orbit of the charge carriers in the uniform field $\mathbf{B}_0=(0,0,B_0)$, v_F is their Fermi velocity, and ω and $\mathbf{k}$ are the frequency and wave vector of the alternating field $\mathbf{B}(y,z,t)$. The integral expressions for the current density and magnetization can be reduced to local form, i.e., they can be written in the form of an expansion in powers of the alternating electric and magnetic fields and their derivatives. For $\kappa^2\equiv|1-4\pi\chi(\mathbf{B}_0)|\ll1$ the linear term of the expansion of the magnetic field $\mathbf{H}$ in powers of $\mathbf{B}(\mathbf{r},t)$ can turn out to be of the same order of magnitude as the nonlinear terms, and the wave processes become substantially nonlinear. For small-amplitude waves it is sufficient to take into account only the nonlinear correction to the magnetization, which is proportional to the third power of $\mathbf{B}$.[3,4] In the expression for the current density one can stop at the linear approximation in the electric field $\mathbf{E}$ and neglect the gradient terms, which are proportional to powers of the small parameter $(kr_0)^2$, and the quantum oscillatory correction, which is proportional to $(\hbar\Omega/\varepsilon_F)^{1/2}$. The current density $\mathbf{j}'$ induced by the magnetic field is determined by the magnetization component M_z, since the vector $\mathbf{M}$ is directed predominantly along $\mathbf{B}_0$. The expression for $\mathbf{j}'=(j'_x,0,0)$ can be written in the form[3–5]

$$j'_x=c(\text{curl}\ \ \mathbf{M})_x=c\frac{\partial M_z}{\partial y}=c\chi(\mathbf{B}_0)\frac{\partial B_z}{\partial y}-4\pi c\beta\frac{\partial B_z^3}{\partial y}$$
$$+4\pi\alpha c r_0^2\frac{\partial^3 B_z}{\partial y^3}, \tag{3}$$

where $\beta=\zeta(\varepsilon_F/\hbar\Omega B_0)^2$, and α and ζ are numerical coefficients of the order of unity which depend on the concrete form of the dispersion relation for the charge carriers.

In the stationary case in the absence of electric field the solution of system (1) for $\chi(B_0)>1/4\pi$ has the form

$$B_1(y)=b_0\frac{\mu}{\sqrt{1+\mu^2}}\,\text{sn}\left(\frac{y}{\delta\sqrt{1+\mu^2}},\,\mu\right) \tag{4}$$

and describes a periodic domain structure with period $Y=4\delta\sqrt{1+\mu^2}K(\mu)$ and domain-wall thickness $\delta=\sqrt{4\pi\alpha r_0/\kappa}$. Here

$$b_0=(\kappa^2/2\pi\beta)^{1/2}\approx\kappa B_0(\hbar\Omega/\varepsilon_F),$$

$$K(\mu)=\int_0^1 dt[(1-t^2)(1-\mu^2t^2)]^{-1/2}\equiv K$$

is a complete elliptic integral of the first kind. The modulus μ of the Jacobi elliptic function sn determines the period Y and is found from the condition that the total thermodynamic potential, including the surface energy at the boundaries of the domains, be minimized with respect to Y. In a case of more practical importance, when the linear dimensions L of the sample are significantly larger than the Larmor radius of the electron, the estimate $Y\sim\sqrt{\kappa^2r_0L}$ is valid.[6] Without loss of generality one can assume that the domain sizes are large compared to δ, i.e., $Y\gg\delta$, or

$$K\gg\pi. \tag{5}$$

Then it is easily noted that μ is close to unity, since the asymptotic expression $K\approx-2\ln(1-\mu^2)$ holds for $K\gg1$.

We set $B_z(y,z,t)=B_1(y)+B^{\sim}(y,z,t)$, where $B^{\sim}(y,z,t)=b(y)e^{-i\omega t+ik_zz}$ is a small space–time perturbation. Linearizing the system of Maxwell's equations (1) with respect to $\mathbf{B}^{\sim}(y,z,t)$ and eliminating the electric field $\mathbf{E}$, we obtain the following equation for the time-dependent field $\mathbf{B}^{\sim}(y,z,t)$:

$$\frac{\partial\mathbf{B}^{\sim}}{\partial t}=-\frac{c^2}{4\pi}\operatorname{curl}\ (\hat{\rho}\ \operatorname{curl}\ \mathbf{H}^{\sim}). \tag{6}$$

Here

$$(\hat{\rho}\ \operatorname{curl}\ \mathbf{H}^{\sim})_i=\rho_{ij}(\operatorname{curl}\ \mathbf{H}^{\sim})_j,\quad H_x^{\sim}=B_x^{\sim},\quad H_y^{\sim}=B_y^{\sim},$$

$$H_z^{\sim}=-\kappa^2B_z^{\sim}+12\pi\beta B_1^2(y)B_z^{\sim}-4\pi\alpha r_0^2\frac{\partial^2B_z^{\sim}}{\partial y^2}.$$

The resistivity tensor can be written in the form of a sum of symmetric and antisymmetric parts: $\rho_{ij}\equiv\rho_{ij}^{(s)}+\rho_{ij}^{(a)}$. The components $\rho_{ij}^{(s)}$ are of the same order of magnitude and tend toward constant values for $B_0\to\infty$. We shall assume that the tensor $\rho_{ij}^{(s)}$ is reduced to its principal axes. Generally speaking, this is valid only in the case when the magnetic field is directed along an axis of symmetry of the crystal. However, taking the off-diagonal components of the resistivity tensor into account does not lead to a qualitative change in the wave spectrum but only gives rise to additional terms in the wave damping decrement which do not alter its order of magnitude.

In the leading approximation in powers of the small parameter $(\Omega\tau)^{-1}$ the diagonal components of the resistivity tensor have the values $\rho_{xx}=\beta_1\rho_0(1-i\omega\tau)$, $\rho_{yy}=\beta_2\rho_0(1-i\omega\tau)$, and $\rho_{zz}=\beta_3\rho_0$. Here $\rho_0=\sigma_0^{-1}$, $\sigma_0\cong\omega_p^2\tau/4\pi$ is the static electrical conductivity of the metal in the absence of magnetic field, ω_p is the frequency of plasma oscillations of the charge carriers, and β_1, β_2, and β_3 are dimensionless coefficients of the order of unity which depend on the concrete form of the dispersion relation of the charge carriers and for simplicity will be assumed equal to unity. In the expression for the antisymmetric part of the resistivity tensor $\rho_{ij}^{(a)}$ it is sufficient to keep only the leading, Hall components $\rho_{xy}=-\rho_{yx}=B_0/ce(n_e-n_h)$, where n_e and n_h are the electron and hole densities, and e is the absolute value of the electron charge.

Under these conditions the system of equations (6) takes the form

$$\frac{\partial B_x^{\sim}}{\partial t}=-\frac{c^2\rho_{xy}}{4\pi}\frac{\partial}{\partial z}\left(\frac{\partial H_z^{\sim}}{\partial y}-\frac{\partial B_y^{\sim}}{\partial z}\right)+\frac{c^2\rho_0}{4\pi}\left(\frac{\partial^2}{\partial y^2}+(1-i\omega\tau)\frac{\partial^2}{\partial z^2}\right)B_x^{\sim},$$

$$\frac{\partial B_y^{\sim}}{\partial t}=-\frac{c^2\rho_{xy}}{4\pi}\frac{\partial^2B_x^{\sim}}{\partial z^2}-\frac{c^2\rho_0}{4\pi}(1-i\omega\tau)\frac{\partial}{\partial z}\left(\frac{\partial H_z^{\sim}}{\partial y}-\frac{\partial B_y^{\sim}}{\partial z}\right),$$

$$\frac{\partial B_z^{\sim}}{\partial t}=\frac{c^2\rho_{xy}}{4\pi}\frac{\partial^2B_x^{\sim}}{\partial z\partial y}+\frac{c^2\rho_0}{4\pi}(1-i\omega\tau)\frac{\partial}{\partial y}\left(\frac{\partial H_z^{\sim}}{\partial y}-\frac{\partial B_y^{\sim}}{\partial z}\right). \tag{7}$$

Eliminating $B_x^{\sim}$ and $B_y^{\sim}$ from these equations and neglecting terms proportional to $(\Omega\tau)^{-2}$, we obtain the following equation for $b(y)$:

$$\left[k_z^2-i\gamma(1-i\omega\tau)\omega\left(\frac{4\pi}{c^2|\rho_{xy}|}\right)\right]\left[\kappa^2\frac{\partial^2b(y)}{\partial y^2}-12\pi\beta\frac{\partial^2}{\partial y^2}(b(y)B_1^2(y))+4\pi\alpha r_0^2\frac{\partial^4b(y)}{\partial y^4}\right]$$
$$=-i\gamma\omega\left(\frac{4\pi}{c^2|\rho_{xy}|}\right)\frac{\partial^2b(y)}{\partial y^2}+\left[\left(\frac{4\pi}{c^2|\rho_{xy}|}\right)^2\omega^2-k_z^4+2i\gamma(1-i\omega\tau)\omega k_z^2\left(\frac{4\pi}{c^2|\rho_{xy}|}\right)\right]b(y). \tag{8}$$

Here

$$\gamma=(\sigma_0|\rho_{xy}|)^{-1}\approx\left|\frac{n_e-n_h}{n_e+n_h}\right|(\Omega\tau)^{-1}\sim(\Omega\tau)^{-1}\ll1.$$

This equation determines the amplitude and frequency of the eigenmodes of the electromagnetic field in the presence of a periodic domain structure.

The case when the expression in square brackets on the right-hand side of Eq. (8) equals zero corresponds to a wave with frequency

$$\omega=\frac{k^2cB_0}{4\pi e|n_e-n_h|}(1-i\gamma), \tag{9}$$

propagating along the direction of the external magnetic field. In this case Eq. (8) goes over to a Lamé equation, and its solution is expressed in theta functions.[7]

In the limiting case $\gamma\ll\kappa^2$ the solution of this equation has the form

$$b(y)=\Lambda\ \operatorname{cn}\left(\frac{y}{\delta\sqrt{1+\mu^2}},\mu\right)\operatorname{dn}\left(\frac{y}{\delta\sqrt{1+\mu^2}},\mu\right), \tag{10}$$

where cn and dn are Jacobi elliptic functions. By virtue of inequality (5) the function $b(y)$ is substantially nonzero only in the region of a domain wall, i.e., in the vicinity of the points $y_n=2nK\delta\sqrt{1+\mu^2}$, where n is an integer. In the region $|y=y_n|\gg\delta$ the time-dependent field $\mathbf{B}^{\sim}(\mathbf{r},t)$ is a helicoid wave propagating along the direction of $\mathbf{B}_0$. If dissipative effects are neglected, the remaining components of the magnetic field have the values

$$\begin{pmatrix} B_x^{\sim} \\ B_y^{\sim} \end{pmatrix} = -k\delta A(1+\mu^2)\begin{pmatrix} 1 \\ i \end{pmatrix} \mathrm{sn}\left(\frac{y}{\delta\sqrt{1+\mu^2}}, \mu \right) e^{-i\omega t + ikz}. \tag{11}$$

We consider the case of arbitrary propagation direction of the wave. We introduce a new unknown function $u(y)$ such that $b(y) = d^2u(y)/dy^2$. The equation for this function can be written as

$$u^{(4)}(\xi) + \left[-6\mu^2 \mathrm{sn}^2(\xi,\mu) + (1+\mu^2)\left(1 + i\frac{\gamma}{\kappa^2}\frac{V}{\eta_z^2} \right) \right] \times u''(\xi) = (1+\mu^2)^2 W u(\xi), \tag{12}$$

where

$$W = \left\{ \frac{V^2 - \eta_z^4}{\eta_z^2}\left[1 + i\gamma(1 - i\omega_0\tau V)\frac{V}{\eta_z^2} \right] + 2i\gamma(1 - i\omega_0\tau V)V \right\},$$

$$\eta_z = \frac{k_z\delta}{\kappa}, \quad V = \frac{\omega}{\omega_0}, \quad \omega_0 = \frac{cB_0\kappa^2}{4\pi e|n_e - n_h|\delta^2} \sim \frac{c^2\Omega\kappa^2}{\omega_p^2\delta^2}.$$

When condition (5) holds and the variable ξ lies in the interval

$$(2m-1)K \leqslant \xi \leqslant (2m+1)K$$

the elliptic sine can be replaced by the hyperbolic tangent: $\mathrm{sn}(\xi,1) = \tanh\xi$. Assuming $\mu = 1$ in Eq. (12), we obtain

$$u_m^{(4)}(\xi_m) + \left(\frac{6}{\cosh^2 \xi_m} - 4 + 2i\nu \right) u_m''(\xi_m) = 4Wu_m(\xi_m). \tag{13}$$

Here $\xi_m \equiv \xi - 2mK$, where m is an integer, $-K \leqslant \xi_m \leqslant K$, and $\nu = (\gamma/\kappa^2)(V/\eta_z^2)$.

In the region $-K \leqslant \xi_m \leqslant 0$ the solution of this equation can be sought in the form of a series in powers of $e^{2\xi_m}$:

$$u_m^{(-)}(\xi_m, \lambda) = e^{2\lambda\xi_m} \sum_{n=0}^{\infty} a_n(\lambda) e^{2n\xi_m}, \tag{14}$$

where λ is a parameter which is not a negative integer.

Substituting expression (14) into Eq. (13) and collecting the coefficients of equal powers of $e^{2\xi_m}$, we obtain an infinite system of linear equations for the unknowns $a_n(\lambda)$:

$$\begin{aligned} &\Phi(0)a_0 = 0, \\ &2\Psi(0)a_0 + \Phi(1)a_1 = 0, \\ &\Phi(0)a_0 + 2\Psi(1)a_1 + \Phi(2)a_2 = 0, \\ &\Phi(n-2)a_{n-2} + 2\Psi(n-1)a_{n-1} + \Phi(n)a_n = 0, \quad n \geqslant 2, \end{aligned} \tag{15}$$

where

$$\Phi(n) \equiv (n+\lambda)^4 - \left(1 - \frac{i\nu}{2} \right)(n+\lambda)^2 - \frac{W}{4},$$

$$\Psi(n) \equiv (n+\lambda)^4 + 2\left(1 - \frac{i\nu}{4} \right)(n+\lambda)^2 - \frac{W}{4}.$$

In the case $0 \leqslant \xi_m \leqslant K$ the solution of equation (13) can be written in the form of a series in powers of $e^{2\xi_m}$:

$$u_m^{(+)}(\xi_m, \lambda) = e^{-2\lambda\xi_m} \sum_{n=0}^{\infty} a_n(\lambda) e^{-2n\xi_m} \tag{16}$$

with the same coefficients $a_n(\lambda)$ that satisfy the system of equations (15), where a_0 can be specified arbitrarily and the remaining coefficients are found from the recursion relations

$$a_1 = -2a_0\frac{\Psi(0)}{\Phi(1)}, \quad a_2 = -2a_1\frac{\Psi(1)}{\Phi(2)}, \ldots$$

$$a_n = -\frac{a_{n-2}\Phi(n-2) + 2a_{n-1}\Psi(n-1)}{\Phi(n)}. \tag{17}$$

A simple numerical analysis shows that for $n \to \infty$ the coefficients a_n have the following properties:

$$a_n \to 0, \left| \frac{a_{n+1}}{a_n} \right| \to 1 - 0, \ \ \mathrm{sgn}\,\mathrm{Re}\frac{a_{n+1}}{a_n} = -1,$$

$$\mathrm{sgn}\,\mathrm{Im}\frac{a_{n+1}}{a_n} = -1.$$

The first equation of system (15) implies a discrete relation between λ and V and η_z:

$$\Phi(0) \equiv \lambda^4 - \left(1 - \frac{i\nu}{2} \right)\lambda^2 - \frac{W}{4} = 0. \tag{18}$$

The four roots of this equation,

$$\begin{aligned} \lambda_{1,2} &= \pm\sqrt{\frac{1}{2}}\left(1 - \frac{i\nu}{2} - \sqrt{\left(1 - \frac{i\nu}{2} \right)^2 + W} \right)^{1/2}, \\ \lambda_{3,4} &= \pm\sqrt{\frac{1}{2}}\left(1 - \frac{i\nu}{2} + \sqrt{\left(1 - \frac{i\nu}{2} \right)^2 + W} \right)^{1/2} \end{aligned} \tag{19}$$

together with expressions (14) and (16) determine the four linearly independent solutions of equation (13):

$$u_m(\xi_m) = \begin{cases} \sum_{i=1}^{4} A_i u_m^{(-)}(\xi_m, \lambda_i), & -K \leqslant \xi_m \leqslant 0, \\ \sum_{i=1}^{4} C_i u_m^{(+)}(\xi_m, \lambda_i), & 0 \leqslant \xi_m \leqslant K, \end{cases} \tag{20}$$

where

$$\begin{aligned} u_m^{(-)}(\xi_m, \lambda_i) &= \mathrm{e}^{2\lambda_i\xi_m}\left(1 + \sum_{n=1}^{\infty} a_n(\lambda_i)\mathrm{e}^{2n\xi_m} \right), \\ u_m^{(+)}(\xi_m, \lambda_i) &= \mathrm{e}^{-2\lambda_i\xi_m}\left(1 + \sum_{n=1}^{\infty} a_n(\lambda_i)\mathrm{e}^{-2n\xi_m} \right), \end{aligned} \tag{21}$$

$a_0(\lambda_i) = 1$, and A_i and C_i are constants. Series (21) converges absolutely in the entire domain of definition except at the point $\xi_m = 0$, where it converges conditionally. It follows from formulas (21) that $u_m^{(+)}(0,\lambda_i) = u_m^{(-)}(0,\lambda_i)$.

The functions (20) form a fundamental system of solutions of the differential equation (13). In the interval $(2m+1)K \leqslant \xi \leqslant (2m+3)K$ (or $-K \leqslant \xi_{m+1} \leqslant K$) the solution of equation (13) should be sought in the form

$$u_{m+1}(\xi_{m+1},\lambda_i) = C u_m(\xi_m - 2K,\lambda_i), \tag{22}$$

where C is a constant.

If dissipative effects are neglected completely, i.e., for $\gamma \to 0$, the real values $W > 0$ correspond to imaginary λ_1, λ_2 and real λ_3, λ_4. The wave processes correspond to the solutions $u(\xi_m,\lambda_1)$ and $u(\xi_m,\lambda_2)$. Assuming $\lambda_{1,2} = \pm i\eta_y/2$, where η_y is real, we find from Eq. (21) in the limit $\gamma \to 0$ that the frequency of the eigenmodes of the electromagnetic field has the value

$$\omega = \omega_0 \eta_z \sqrt{\eta_z^2 + \eta_y^2 + \frac{\eta_y^4}{4}}. \tag{23}$$

Let us construct a solution of equation (12) in the interval $0 \leqslant \xi_m \leqslant 2K$ in the form a traveling wave. In a neighborhood of the point $\xi_m = K$ the sum in expressions (21) has order of magnitude $O(\mathrm{e}^{-2K})$. Splicing the asymptotic expressions $u_m^{(+)}(\xi_m, -i\eta_y)$ and $u_{m+1}^{(-)}(\xi_{m+1}, i\eta_y)$ for $\xi_m \to K$ and using relation (22), we obtain

$$u(\xi_m) = u_m^{(+)}(\xi_m, -i\eta_y) = C_2 \mathrm{e}^{i\eta_y \xi_m} = u_{m+1}^{(-)}(\xi_{m+1}, i\eta_y) = C u_m^{(-)}(\xi_m - 2K, i\eta_y) = C A_1 \mathrm{e}^{i\eta_y(\xi_m - 2K)}. \tag{24}$$

Equating the coefficients of $\mathrm{e}^{i\eta_y \xi_m}$, we find $C_2 = C A_1 \mathrm{e}^{-2i\eta_y K}$. Summing the two asymptotic expressions $u_m^{(+)}(\xi_m)$ and $u_{m+1}^{(-)}(\xi_{m+1})$ and then subtracting off their common part (24), we obtain a solution of equation (12) which is valid on the interval $0 \leqslant \xi_m \leqslant 2K$:

$$u(\xi_m) = C_2 \mathrm{e}^{i\eta_y \xi_m}\left(1 + \sum_{n=1}^{\infty} a_n(-i\eta_y)\mathrm{e}^{-2n\xi_m} + \sum_{n=1}^{\infty} a_n(i\eta_y)\mathrm{e}^{2n(\xi_m - 2K)}\right). \tag{25}$$

It follows from relation (22) that the multiplicative factor C has the value

$$C = \frac{u(2K)}{u(0)} = \exp\left[2i\,\arg\left(1 + \sum_{n=1}^{\infty} a_n(i\eta_y)\right) + 2i\eta_y K\right] \equiv \mathrm{e}^{2iKs}. \tag{26}$$

The solution of equation (12) can be written in the form

$$u(\xi) = \mathrm{e}^{is\xi} F(\xi), \tag{27}$$

where $F(\xi)$ is a periodic function with period $2K$, and the dimensionless wave number $s \equiv \sqrt{2} k_y \delta$ has the value

$$s = \eta_y + \frac{\arg(1 + \Sigma_{n=1}^{\infty} a_n(i\eta_y))}{K}. \tag{28}$$

The complex conjugate of function (25) is also a solution of equation (12).

Relation (23) implies the folowing dispersion relation of the traveling wave:

$$\omega = \frac{cB_0}{4\pi e|n_e - n_h|}\left[k_z \sqrt{k_z^2 + \frac{\kappa^2 \eta_y^2(k_y)}{\delta^2}\left(1 + \frac{\eta_y^2(k_y)}{4}\right)}\right], \tag{29}$$

where η_y is determined as a function of k_y by expression (28).

In the case of weak spatial dispersion $k_z v_F \ll \tau^{-1}$, the damping is due solely to the scattering of electrons, $\mathrm{Im}\,\omega \sim \gamma\omega$. When the opposite inequality holds, $\tau^{-1} \ll k_z v_F \ll \Omega$, the eigenmode spectrum remains the same, while the expression for the damping decrement acquires additional terms due to Čerenkov absorption of the electromagnetic field by electrons moving in-phase with the wave. In that case the quantization of the energy levels of the electrons has a substantial influence on the damping of the wave.[8]

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