On the propagation of acoustic waves in quasi-two-dimensional conductors in a quantizing magnetic field

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The damping of acoustic waves propagating perpendicular to the layers of a quasi-two-dimensional conductor is analyzed for the case of low temperatures, at which the energy quantization of the conduction electrons leads to an oscillatory dependence of the acoustic damping coefficient on the inverse magnetic field. The acoustic damping decrement is found for different orientations of the magnetic field with respect to the layers. It is shown that that a layered conductor is most transparent for an acoustic wave in the case when the magnetic field is perpendicular to the layers. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1596595]

Acoustoelectronic effects in degenerate conductors placed in a sufficiently high magnetic field **H** are extremely sensitive to the form of the energy spectrum of the charge carriers.[1–3] The experimental study of these effects in metals in the case when the gyration frequency Ω of the electrons in the magnetic field is much higher than their collision frequency $1/\tau$ has permitted the complete recovery of the shape of the Fermi surface, the main characteristic of the electron energy spectrum.

At sufficiently low temperatures T, when the distance between electron quantum energy levels $\Delta\varepsilon=\hbar\Omega$ is significantly greater than the temperature smearing of the Fermi distribution function of the charge carriers, $f_0(\varepsilon)$, the acoustic damping decrement Γ undergoes resonance oscillations with variation of the inverse value of the high magnetic field $(\Omega\tau\gg 1)$.

In degenerate conductors having a layered structure the electron energy spectrum is substantially anisotropic and, as a rule, is quasi-two-dimensional. The energy ε of the charge carriers in quasi-two-dimensional conductors depends weakly on the momentum projection p_z onto the normal **n** to the layers.

The specifics of the quasi-two-dimensional electron energy spectrum of layered conductors are manifested in an enhancement of quantum oscillation effects in comparison with ordinary metals, since a rather large number of charge carriers with the Fermi energy ε_F are involved in their formation. At the same time, the low electronic conductivity of layered conductors along the normal to the layers leads to acoustic transparency for waves propagating perpendicular to the layers.[4,5] In this connection let us consider the propagation of an acoustic wave in the easiest direction for it, i.e., along the normal to the layers of a quasi-two-dimensional conductor placed in a magnetic field $\mathbf{H}=(0,H\sin\theta,H\cos\theta)$ inclined at an angle θ to the wave vector **k** and the normal **n**.

At low temperatures the absorption of energy from sound waves in a degenerate conductor is mainly due to the interaction of the charge carriers with the wave and is determined by the dissipative function of the electrons,

$$Q=T\frac{dS}{dt}, \tag{1}$$

where S is the entropy density of the conduction electrons, which is related to the nonequilibrium density matrix $\hat{f}$ by the relation[6,7]

$$S=\mathrm{tr}\{\hat{f}\ln\hat{f}+(1-\hat{f})\ln(1-\hat{f})\}. \tag{2}$$

The summation in (2) is over all variables specifying the state of the conduction electrons except for the spin variables.

The density matrix $\hat{f}$ must be determined with the aid of the kinetic equation

$$\frac{\partial\hat{f}}{\partial t}+\hat{\mathbf{v}}\frac{\partial\hat{f}}{\partial\mathbf{r}}+[\hat{H}_0+\hat{H}_1,\hat{f}]=\hat{W}_{\mathrm{coll}}\{\hat{f}\}, \tag{3}$$

where $\hat{W}_{\mathrm{coll}}(\hat{f})$ is the collision operator of the charge carriers, which describes their scattering by impurity atoms and vibrations of the crystal lattice, i.e., phonons; $\hat{H}_0$ is the Hamiltonian of the conduction electrons in the magnetic field, and $\hat{\mathbf{v}}$ is their velocity operator, and $\hat{H}_1$ is a correction to the unperturbed Hamiltonian $\hat{H}_0$ to take into account the perturbation of the electron system by the acoustic wave.

In a vibrating lattice the electron energy spectrum is sensitive to the strain of the crystal, and in the linear approximation in the small displacement **u** of the lattice ions the energy of the conduction electrons acquires an additional amount $\delta\varepsilon=\lambda_{ik}(\mathbf{p})u_{ik}$, where $u_{ik}=(1/2)(\partial u_i/\partial x_k+\partial u_k/\partial x_i)$ is the strain tensor, and λ_{ik} is the deformation potential tensor.[8] It is natural to assume that the energy spectrum of the charge carriers remains highly anisotropic after renormalization by the sound wave. The components of the deformation potential tensor in the plane of the layers is of the same order of magnitude as the Fermi energy of the electrons, while the components for which one or both of the indices is z are significantly smaller. It follows from conservation of the number of charge carriers that each of the tensor components λ_{ik} averaged over all states of the conduction electrons is equal to zero.

In addition to renormalization of the energy of the charge carriers a sound wave generates an accompanying electromagnetic wave. The electric field of this wave in a reference frame moving with the vibrating crystal lattice with a velocity $\dot{\mathbf{u}}$ has the form

$$\tilde{\mathbf{E}}=\mathbf{E}+\frac{1}{c}(\dot{\mathbf{u}}\times\mathbf{H})-\frac{m\hat{\mathbf{u}}}{e}, \tag{5}$$

where $\mathbf{E}$ is the electric field in the nonmoving laboratory reference frame, which must be determined from Maxwell's equations

$$\text{curl curl } \mathbf{E}=-\frac{4\pi}{c^2}\frac{\partial\mathbf{j}}{\partial t}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}, \quad \operatorname{div}\mathbf{E}=4\pi\rho',$$

supplemented by the constitutive relations linking the current $\mathbf{j}$ to the electric field of the wave. Here c is the speed of light in vacuum, and ρ' is the uncompensated charge density, which is asymptotically vanishing in the expansion in powers of $1/N_e$ in conductors with a high density of conduction electrons N_e. To the same accuracy the charge conservation law has the form

$$\operatorname{div}\mathbf{j}=0. \tag{6}$$

The damping of a low-amplitude sound wave can be analyzed with the aid of the solution of the kinetic equation (3), linearized with respect to the deviation of the density matrix from the equilibrium $\hat{f}_0$, and the entropy production

$$\frac{dS}{dt}=\operatorname{tr}\hat{W}_{\text{coll}}(\hat{f})\ln\frac{1-\hat{f}}{\hat{f}} \tag{7}$$

is a quadratic function of $\hat{f}_1=\hat{f}-\hat{f}_0$ and can be represented in the form

$$\frac{dS}{dt}=-\operatorname{tr}\hat{W}_{\text{coll}}(\hat{f}_1)\frac{\hat{f}_1}{\hat{f}_0(1-\hat{f}_0)}. \tag{8}$$

The diagonal matrix elements of the equilibrium density matrix $\hat{f}_0$ are equal to the Fermi distribution function of the charge carriers $f_{0nn}=f_0(\varepsilon_n(p_H))$, where $\varepsilon_n(p_H)$ are the eigenvalues of the Hamiltonian $\hat{H}_0$, and $p_H=\mathbf{p}\cdot\mathbf{H}/H$.

In a magnetic field the kinematic momentum $\mathbf{p}$ in the expression for the energy $\varepsilon(\mathbf{p})$ should be replaced by $\hat{\mathbf{P}}-(e/c)\mathbf{A}$, where $\hat{\mathbf{P}}=-(i/\hbar)(\partial/\partial\mathbf{r})$ is the generalized momentum of the electron and $\mathbf{A}$ is the vector potential. If the latter is chosen in the Landau gauge, $\mathbf{A}=(0,Hx\cos\theta,-Hx\sin\theta)$, then the Hamiltonian

$$\hat{H}_0=\varepsilon\left(\hat{P}_x,\hat{P}_y-\frac{e}{c}Hx\cos\theta,\hat{P}_z+\frac{e}{c}Hx\sin\theta\right) \tag{9}$$

will depend on only one coordinate, x. In this gauge for the vector potential the solution of the Schrödinger equation

$$\hat{H}_0\Psi=\varepsilon\Psi, \tag{10}$$

which essentially contains only one differential operator, $\hat{P}_x=-i\hbar(\partial/\partial x)$, should be sought in the form

$$\Psi(x,y,z)=\exp(iP_y y/\hbar+iP_z z/\hbar)\psi(x). \tag{11}$$

The energy of the electrons on closed orbits in the magnetic field will depend on the generalized momenta P_y and P_z, which are "good" quantum numbers, and on the discrete quantum number $n=0,1,2,3,\ldots$.

In the summation in (2) over all the electron states, which are specified by the quantum number n and the generalized momenta P_y and P_z, it is more convenient to use combinations of P_y and P_z in the form an integral of the motion: $p_H=P_y\sin\theta+P_z\cos\theta$. Here is is necessary to specify an additional quantum number, e.g., P_y, which in the quasi-classical approximation determines the position of the center of the electron orbit in the magnetic field. For $T\gg\hbar\Omega$ one usually uses the time of motion of the charge in the magnetic field, t_H, instead of P_y as the additional variable along with ε and p_H, in accordance with the equation

$$\frac{\partial p_x}{\partial t_H}=\frac{eH}{c}(v_y\cos\theta-v_z\sin\theta),\ \frac{\partial p_y}{\partial t_H}=-\frac{eH}{c}v_x\cos\theta,$$
$$\frac{\partial p_z}{\partial t_H}=\frac{eH}{c}v_x\sin\theta. \tag{12}$$

In the quasi-classical approximation, when the main role in the electronic absorption of sound waves is played by the charge carriers at energy levels with large values of n, the wave function of the electrons can be found under the most general assumptions about the form of the Hamiltonian. However, in certain particular cases one can find the energy spectrum and the wave function of the conduction electrons for arbitrary values of the high magnetic field, including the ultraquantum limit. As an example of such a case we consider the simplest quasi-two-dimensional electron energy spectrum:

$$\varepsilon(\mathbf{p})=\frac{p_x^2+p_y^2}{2m}-\eta v_0\frac{\hbar}{a}\cos\frac{ap_z}{h}. \tag{13}$$

Here a is the distance between layers, $v_0=(2\varepsilon_F/m)^{1/2}$ is the characteristic velocity along the layers for the electrons with the Fermi energy ε_F, m is the mass of an electron, and $\eta\ll1$ is the quasi-two-dimensionality parameter of the charge-carrier spectrum.

Substituting (11) into Eq. (10), we easily find that for angles θ that are not too large, specifically for $\eta\tan\theta<1$, in the leading approximation in the small parameter aeH/cmv_0 the electron energy levels have the form

$$\varepsilon_n=\left(n+\frac{1}{2}\right)\hbar\Omega\sqrt{1+\eta\frac{v_0am}{\hbar}\tan^2\theta\cos\zeta}$$
$$-\eta\frac{v_0\hbar}{a}\cos\zeta-\eta^2\frac{mv_0^2\tan^2\theta\sin^2\zeta}{2[1+\eta(v_0am/\hbar)\tan^2\theta\cos\zeta]}, \tag{14}$$

where $\zeta=ap_H/(\hbar\cos\theta)$ and $\Omega=eH/(mc\cos\theta)$. If the spin splitting is not taken into account, ε_n depends only on two variables: the continuously varying ζ, and the discrete quantum number n, which enumerates the electron energy levels in the magnetic field.

The kinetic equation linearized with respect to the weak perturbation of the charge carriers by the acoustic wave has the form

$$\left[-i\omega+\frac{i}{\hbar}(\varepsilon_n-\varepsilon_m)\right]f_{1nm}+i\mathbf{k}\cdot\mathbf{v}_{nl}f_{1lm}-\{\hat{W}_{\rm coll}(\hat{f}_1)\}_{nm}$$
$$=\frac{f_0(\varepsilon_m)-f_0(\varepsilon_n)}{\varepsilon_m-\varepsilon_n}(e\tilde{\mathbf{E}}\cdot\mathbf{v}+\omega\lambda_{ij}u_ik_j)_{nm},\qquad(15)$$

where $\mathbf{k}$ is the wave vector of the acoustic wave.

Using the solution of the kinetic equation, one can calculate the dissipative function and, dividing it by the acoustic energy flux density, obtain the damping coefficient for the sound wave:

$$\Gamma=\frac{T}{\rho u^2\omega^2 s/2}\frac{dS}{dt}.\qquad(16)$$

Here ρ is the density of the crystal, and s is the sound velocity.

We consider the propagation of a linearly polarized longitudinal wave $\mathbf{u}=(0,0,u)$ along the normal to the layers of the conductor in the case when the following inequality holds:

$$T\ll\hbar\Omega\ll\eta\mu,\qquad(17)$$

where μ is the chemical potential.

In the quasi-classical approximation the entropy production in the electron system can be written in the form

$$\frac{dS}{dt}=-\frac{2eH}{c(2\pi\hbar)^2}\sum_{n,m}\int dp_H\hat{W}_{\rm coll}(\hat{f}_1)^{nm}$$
$$\times\frac{f_1^{nm}}{f_0(\varepsilon_n)[1-f_0(\varepsilon_m)]}.\qquad(18)$$

The diagonal matrix elements of the operators $\hat{f}_1$ and $\hat{W}_{\rm coll}(\hat{f}_1)$ are quantities averaged over the different phases of the quasi-classical electron trajectory $\varphi=\Omega t_H$. In the case of closed electron orbits the off-diagonal matrix elements v_i^{nm} of the electron velocity operator are proportional to periodic functions of the form $\cos(n-m)\varphi$.

If collisions of electrons with phonons are extremely rare, and the conduction electrons are scattered mainly by impurity atoms, then the dissipative processes in the system of charge carriers can be taken into account with the aid of the relaxation-time (τ) approximation for the collision integral. Applying the Poisson equation to expression (18) and changing from integration over n to integration over ε, we can write the oscillatory (in the magnetic field) part of the acoustic absorption coefficient in the form

$$\Gamma^{\rm osc}=\frac{4\pi}{\rho u^2\omega^2 s}\frac{2eH}{c(2\pi\hbar)^3}2\,{\rm Re}\sum_{N=1}^{\infty}\int d\varepsilon\left(-\frac{\partial f_0(\varepsilon)}{\partial\varepsilon}\right)$$
$$\times\int dp_H\exp[2\pi iNn(\varepsilon,p_H)]\frac{1}{2\pi\Omega\tau}$$
$$\times\int_0^{2\pi}d\varphi\left|\frac{1}{\Omega}\int_{-\infty}^{\varphi}d\varphi'g(\varphi')\right.$$
$$\left.\times\exp\left\{2ikz(\varphi')-i\frac{\omega}{\Omega}\varphi'+\frac{\varphi'}{\Omega\tau}\right\}\right|^2.\qquad(19)$$

Here $g(\varphi)=e\tilde{\mathbf{E}}\cdot\mathbf{v}+\lambda_{zz}u\omega k$. The component of the deformation potential can be described by the expression

$$\lambda_{zz}=\eta\lambda\cos\frac{ap_z}{\hbar},\quad\lambda=\frac{mv_0^2}{2}.\qquad(20)$$

With the aid of Maxwell's equations (5) and formulas (4) one is readily convinced that in the case of a strong external magnetic field ($\Omega\tau\gg1$) and for $kv_0\tau\eta\ll1$ and $\omega\tau\ll1$ the components of the field of the electromagnetic wave generated by the sound wave have the form

$$\tilde{E}_x=\frac{i\omega}{c}uH\sin\theta\frac{1-i\beta\gamma^2}{1-(\beta\gamma)^2-2i\beta\gamma^2},$$
$$\tilde{E}_y=-\frac{i\omega}{c}uH\sin\theta\frac{i\beta\gamma^2}{1-(\beta\gamma)^2-2i\beta\gamma^2}.\qquad(21)$$

Here $\gamma=1/(\Omega\tau)\ll1$, and the parameter $\beta=(s\omega_p/c\omega)^2\omega\tau$ can be quite large if the plasma frequency ω_p has a value comparable to the typical value for an ordinary metal. This assumption is based on the fact that the conductivity in the plane of the layers of organic conductors is of the same order as that of good metals. The formulas given for the electromagnetic field of the wave do not take into account the Shubnikov–de Haas oscillations of the conductivity, the amplitudes of which are a factor of $(\eta\mu/\hbar\Omega)^{1/2}$ smaller than the part of the conductivity that varies monotonically with the magnetic field.

Substituting expressions (20) and (21) into formula (19), we obtain

$$\Gamma^{\rm osc}=\frac{2m^3v_0^2}{\rho s\tau a(2\pi\hbar)^2}{\rm Re}\sum_{N=1}^{\infty}\int d\varepsilon\left(-\frac{\partial f_0(\varepsilon)}{\partial\varepsilon}\right)$$
$$\times\int_0^{2\pi}d\zeta\exp[2\pi iNn(\varepsilon,\zeta)]$$
$$\times\left\{\frac{(kv_0\tau\eta)^2}{2}J_0^2\left(\frac{amv_0}{\hbar}\tan\theta\right)\cos^2\zeta+F(\gamma)\tan^2\theta\right\}.\qquad(22)$$

Here J_0 is the Bessel function, and

$$F(\gamma)=\frac{1+\beta^2\gamma^2+\beta^2\gamma^4}{[1-(\beta\gamma)^2]^2+\beta^2\gamma^4}.$$

The second term in curly brackets in formula (22) determines the Joule losses due to the electromagnetic fields excited by the sound wave. The function $F(\gamma)$ is of the order of unity over a wide range of magnetic fields, and only under conditions of resonance coupling of the acoustic and electromagnetic waves, when the wavelength of the helicoidal wave excited by the sound is comparable to the wavelength of the acoustic wave, i.e., $\beta\gamma=1$, does the function $F(\gamma)$ become of the order of $\gamma^{-2}\gg1$. In the case considered here, the Joule losses are much greater than the absorption due to the renormalization of the electron energy directly on account of the deformation of the crystal, when the external magnetic field deviates from the normal to the layers by an angle $\theta\gg kl\eta$.

Performing the integration over ζ and ε in (22), we obtain

$$\Gamma^{\text{osc}}=\frac{\Gamma_0}{kl}\sum_{N=1}(-1)^N\Phi(N\Lambda)\cos\left(\frac{2\pi N\mu}{\hbar\Omega}\right)\times\left\{\left(\frac{k/\eta}{2}\right)^2 J_0^2\left(\frac{amv_0}{\hbar}\tan\theta\right)[J_0(2\pi N\chi)-J_2(2\pi N\chi)]+F(\gamma)J_0(2\pi N\chi)\tan^2\theta\right\}, \quad (23)$$

where $\Gamma_0=mN_c v_0\omega/4\pi\rho s^2$, N_c is the density of charge carriers in the conductor, $l=v_0\tau$, μ is the chemical potential, $\Phi(z)=z/\sinh z$, $\Lambda=2\pi^2T/\hbar\Omega$, and $\chi=(\eta v_0/\hbar\Omega)[(\hbar/a)+(ma/2\hbar)\tan\theta]$ is equal in order of magnitude to $\eta\mu/\hbar\Omega$.

At temperatures that are not too low, when $\Lambda\simeq 1$, the amplitude Γ^{osc} is smaller by a factor of $(\eta\mu/\hbar\Omega)^{1/2}$ than the part of the acoustic damping coefficients that varies smoothly with magnetic field,

$$\Gamma^{\text{mon}}\simeq\frac{\Gamma_0}{kl}\left[\left(\frac{k/\eta}{2}\right)^2 J_0^2\left(\frac{amv_0}{\hbar}\tan\theta\right)+F(\gamma)\tan^2\theta\right]. \quad (24)$$

In the quasi-classical approximation, when $\eta\mu\gg\hbar\Omega$, the following asymptotic expression is valid for Γ^{osc}:

$$\Gamma^{\text{osc}}\simeq\frac{\Gamma_0}{kl}\sum_{N=1}\frac{(-1)^N\Phi(N\Lambda)}{(N\chi)^{1/2}}\cos\left(\frac{2\pi N\mu}{\hbar\Omega}\right)\times\cos\left(2\pi N\chi-\frac{\pi}{4}\right)\left[\left(\frac{kl\eta}{2}\right)^2 J_0^2\left(\frac{amv_0}{\hbar}\tan\theta\right)+F(\gamma)\tan^2\theta\right]. \quad (25)$$

The use of the rather simple model (13) of the dispersion relation for conduction electrons in the calculation permits a correct description of the character of the propagation of sound waves in a quantizing magnetic field (Fig. 1).

In the quasi-classical approximation it is not difficult to generalize the results obtained to the case of a quasi-two-dimensional electron energy spectrum of arbitrary form. As in the case of the dispersion relation (13), a longitudinal sound wave propagates a considerable distance into the interior of the sample along the normal to the layers if the magnetic field deviates from the normal by a small angle $\theta<kl\eta$.[9]

If the Fermi surface for such an orientation of the magnetic field has only two different extremal values of its cross section on a plane $p_H=\text{const}$, viz., $S_{\min}$ and $S_{\max}$, then the ratio $\Gamma_{\text{osc}}/\Gamma_{\text{mon}}$ can be written in the form

$$\frac{\Gamma_{\text{osc}}}{\Gamma_{\text{mon}}}\simeq\sqrt{\frac{\hbar\Omega}{\eta\mu}}\sum_{N=1}\frac{(-1)^N}{\sqrt{N}}\Psi(N\Lambda)\times\cos\left(\frac{Nc(S_{\max}+S_{\min})}{2eH\hbar}-\pi N\right)\times\cos\left[\frac{Nc(S_{\max}-S_{\min})}{2eH\hbar}-\frac{\pi}{4}\right]. \quad (26)$$

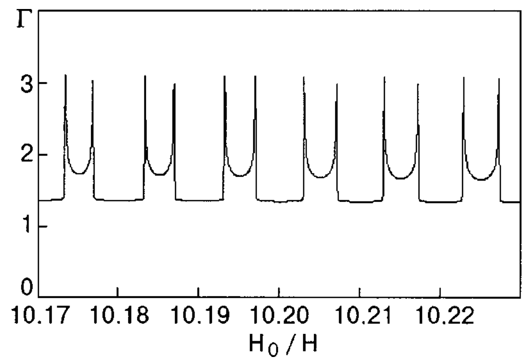

FIG. 1. Acoustic absorption coefficient in a layered conductor as a function of H_0/H ($H_0=\eta\mu mc/e\hbar$) in relative units for $\eta=0.01$, $T/\eta\mu=5\times10^{-3}$, $\theta=0$.

This case of anomalous acoustic transparency does not take place for transverse polarization of the sound wave, $\mathbf{u}\perp\mathbf{k}$, the damping of which is determined mainly by the Joule losses for any orientation of the magnetic field.

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