Propagation of electromagnetic waves in the electron Fermi liquid of a quasi-two-dimensional conductor under strong spatial dispersion

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We have analyzed propagation of electromagnetic waves in a Fermi liquid of charge carriers in quasi-two-dimensional layered conductors placed in a magnetic field. It is shown that high-frequency collective modes, which are absent in a gas of charge carriers, can be observed even at low intensity of the Fermi-liquid interaction. The spectrum of the weakly damping waves has been obtained.

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At low temperatures allowance for the Fermi-liquid correlation of conduction electrons in metals results in appearance of high-frequency collective modes that are absent in a gas of charge carriers. Experimental observation of these waves in ordinary quasi-isotropic metals is faced with difficulties. The spectrum of the longitudinal wave lies near the plasma frequency ω_p, which is extremely high. In the absence of an external magnetic field the ratio of the frequency of the transverse collective mode to ω_p is of the order of 10^{-2}, but the existence of this wave needs a sufficiently strong electron-electron interaction.[1] In the presence of a high external magnetic field the other types of weakly attenuating electromagnetic waves, such as helicoidal, magnetohydrodynamics, and cyclotron waves, occur. The spectrum of their high-frequency branches depends essentially on the constants of the Fermi-liquid interaction. The influence of the Fermi-liquid correlation of charge carriers on the wave processes in quasi-isotropic metals has been investigated by many authors. The results of a considerable part of these studies have been reported in the monograph in Ref. 2

Below we consider the propagation of electromagnetic waves in layered conductors placed in an external magnetic field with the induction $\mathbf{B}_0=(0,0,B_0)$ orthogonal to the conducting layers. We also analyze the influence of the Fermi-liquid interaction on the spectrum of cyclotron waves under the conditions of strong spatial dispersion. In layered conductors the conditions for observation of the Fermi-liquid waves, propagating along the magnetic field direction, are much more favorable than in quasi-isotropic metals. The specifics of the quasi-two-dimensional electron energy spectrum of the layered conductors gives rise to peculiar Fermi-liquid collective modes whose spectrum corresponds to moderately high frequencies.

Recently an entire series of conducting crystals that possess a layered structure with a pronounced anisotropy of electrical conductivity of the metal type has been synthesized. These are organic conductors of the family of tetrathiafulvalene salts, dichalcogenides of transition metals, graphite and its intercalates, etc. Galvanomagnetic phenomena and quantum oscillatory effects in these compounds have been studied intensely in many laboratories (see, for example, Refs. 3–5 and citations therein). The results of these investigations suggest that kinetic and electrodynamics properties of the layered conductors at low temperatures can be described making use of the concept of quasiparticles, analogous to conduction electrons in metals. Energy of charge carriers $\varepsilon(\mathbf{p})$ in layered conductors depends weakly on the momentum projection $p_z=\mathbf{p}\cdot\mathbf{n}$ on the normal $\mathbf{n}$ to the layer plane and can be represented in the form

$$\varepsilon(\mathbf{p}) = \varepsilon_0(p_x,p_y) + \sum_{n=1}^{\infty} \varepsilon_n(p_x,p_y,\eta)\cos\left(\frac{np_z}{p_0}\right). \tag{1}$$

Here $\varepsilon_{n+1}(p_x,p_y,\eta)\ll\varepsilon_n(p_x,p_y,\eta)$, ε_F is the Fermi energy, $\hbar/p_0$ is the distance between layers, and $\hbar$ is the Planck constant. The parameter η characterizes the anisotropy of the charge carriers energy spectrum, and η^2 is about the ratio of the conductivity across the layers to the in-plane conductivity in the absence of a magnetic field.

Kinetic properties of the system of fermions should be described by means of the kinetic equation for the density matrix $\hat{\rho}$. In the quasiclassical case when $\hbar\Omega\lesssim T\ll\eta\varepsilon_F$ the quantization of the charge carrier energy in the magnetic field does not affect essentially the magnetization $\mathbf{M}$ (Ω is the cyclotron frequency of charge carriers, T is the temperature). Under these conditions the density matrix can be presented as an operator in the space of spin variables and as a function depending on coordinates and momentum. The equation for single-particle density matrix has the form

$$\frac{\partial\hat{\rho}}{\partial t} - \frac{i}{\hbar}[\hat{\varepsilon},\hat{\rho}]_\sigma + \frac{1}{2}\{\hat{\varepsilon},\hat{\rho}\} + \frac{1}{2}\{\hat{\rho},\hat{\varepsilon}\} + e\mathbf{E}\cdot\frac{\partial\hat{\rho}}{\partial\mathbf{p}} + \frac{1}{2}\frac{e}{c}\left(\left[\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}\times\mathbf{B}\right]\cdot\frac{\partial\hat{\rho}}{\partial\mathbf{p}} + \frac{\partial\hat{\rho}}{\partial\mathbf{p}}\cdot\left[\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}\times\mathbf{B}\right]\right) = \hat{I}_{\text{coll}}. \tag{2}$$

Here $[\hat{\varepsilon},\hat{\rho}]_\sigma$ is the commutator for matrices in the space of spin variables, $\{\hat{\varepsilon},\hat{\rho}\}$ is the classical Poisson bracket, $\hat{I}_{\text{coll}}$ is the collision operator, e is the electron charge, c is the velocity of light, $\mathbf{E}$ is electric field, $\mathbf{B}=\mathbf{B}_0+\mathbf{B}^{\sim}(\mathbf{r},t)$, $\mathbf{B}^{\sim}(\mathbf{r},t)$ is a field of wave, $\hat{\varepsilon}=\varepsilon(\mathbf{p})\delta_{\alpha\beta}-\mu_0\hat{\boldsymbol{\sigma}}\cdot\mathbf{B}+\delta\hat{\varepsilon}(\mathbf{p},\mathbf{r},t)$, $\delta_{\alpha\beta}$ is the Kronecker symbol, and μ_0 is the magnetic momentum of an electron. The interaction between electrons results in the correction to their energy

$$\delta\hat{\varepsilon}(\mathbf{p},\mathbf{r},t) = \mathrm{Tr}_{\sigma'} \int \frac{d^3p'}{(2\pi\hbar)^3}\Lambda(\mathbf{p},\hat{\boldsymbol{\sigma}},\mathbf{p}',\hat{\boldsymbol{\sigma}}')\,\delta\hat{\rho}(\mathbf{p},\mathbf{r},\hat{\boldsymbol{\sigma}}',t) \tag{3}$$

which can be described with the aid of the correlation function[6,7]

$$\Lambda(\mathbf{p},\hat{\boldsymbol{\sigma}},\mathbf{p}',\hat{\boldsymbol{\sigma}}') = L(\mathbf{p},\mathbf{p}') + S(\mathbf{p},\mathbf{p}')\hat{\boldsymbol{\sigma}}\hat{\boldsymbol{\sigma}}' \tag{4}$$

where $\delta\hat{\rho}$ is nonequilibrium correction the density matrix, and $\boldsymbol{\sigma}$ are Pauli matrices. The term depending on the operators of spin on the right-hand part of Eq. (4) correspond to the exchange interaction between electrons. Since the in-plane interaction between charge carriers exceeds substantially the interaction between quasiparticles belonging to different layers, then both the energy and the Landau correlation function can be expanded into asymptotic series about η, the leading term being not dependent on p_z. This suggestion simplifies essentially the kinetic equation and makes it possible to obtain its solution for rather general form of the correlation function.

To find the electromagnetic field of the wave it is necessary to solve the Maxwell equations

$$\mathrm{rot}\,\mathbf{B}^{\sim} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \quad \mathrm{div}\,\mathbf{B}^{\sim} = 0, \quad \mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}^{\sim}}{\partial t}, \tag{5}$$

supplemented by the equation linking the current density $\mathbf{j}$ to the perturbation of density matrix of electron subsystem

$$\mathbf{j}(\mathbf{r},t) = e\,\mathrm{Tr}_\sigma \int \frac{d^3p}{(2\pi\hbar)^3}\hat{\rho}(\mathbf{p},\mathbf{r},\hat{\boldsymbol{\sigma}},t)\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}} + c\mu_0 \mathrm{rot}\,\mathrm{Tr}_\sigma \int \frac{d^3p}{(2\pi\hbar)^3}\hat{\boldsymbol{\sigma}}\hat{\rho}(\mathbf{p},\mathbf{r},\hat{\boldsymbol{\sigma}},t). \tag{6}$$

Instead of Eq. (2) for $\hat{\rho}$, it will be more convenient to use the set of four equations for the distribution function $f(\mathbf{r},\mathbf{p},t) = \mathrm{Tr}_\sigma\hat{\rho}$ and for the spin density $\mathbf{g}(\mathbf{r},\mathbf{p},t) = \mathrm{Tr}_\sigma(\hat{\boldsymbol{\sigma}}\hat{\rho})$. To obtain the first equation one must apply the operation of taking the trace over spin variables Tr_σ to Eq. (2). Three others are the result of applying Tr_σ to the initial equation multiplied by $\hat{\boldsymbol{\sigma}}$. The function $\mathbf{g}(\mathbf{r},\mathbf{p},t)$ and the second term on the right-hand part of Eq. (6) describe paramagnetic spin waves predicted by Silin[8] and observed in isotropic metals by Schultz and Dunifer.[9] Below we shall neglect small oscillations of the spin density and consider electromagnetic modes induced by the distribution function perturbation.

Present the distribution function f in the form $f(\mathbf{r},\mathbf{p},t) = f_0(\varepsilon) - \psi(\mathbf{r},\mathbf{p},t)\,\partial f_0/\partial\varepsilon$, where $f_0(\varepsilon)$ is the equilibrium Fermi function. The nonequilibrium correction should be determined by solving the linearized kinetic equation

$$\frac{\partial\psi}{\partial t} + \left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}} + \frac{e}{c}(\mathbf{v}\times\mathbf{B}_0)\frac{\partial}{\partial\mathbf{p}}\right)\Phi = e\mathbf{v}\mathbf{E} + I_{\mathrm{coll}}(\Phi). \tag{7}$$

Here $\Phi \equiv \psi + \langle L\psi\rangle$,

$$\langle L\psi\rangle \equiv \int \frac{2d^3p'}{(2\pi\hbar)^3}\left(-\frac{\partial f_0(\varepsilon')}{\partial\varepsilon'}\right)L(\mathbf{p},\mathbf{p}')\,\psi(\mathbf{r},\mathbf{p}',t),$$

and $I_{\mathrm{coll}} = \mathrm{Tr}_\sigma(\hat{I}_{\mathrm{coll}})$ is the collision integral.

The wave process can be regarded as harmonic and the coordinate and time dependencies of the fields $\mathbf{E},\mathbf{B}^{\sim}$ and of the function ψ can be represented as $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$. Regardless of the weak current of spin magnetization in Eq. (6), the current density in a conductor can be written in the form

$$j_i = e\int \frac{2d^3p}{(2\pi\hbar)^3}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)v_i\Phi = \sigma_{ij}(\omega,\mathbf{k})E_j, \tag{8}$$

where $\sigma_{ij}(\omega,\mathbf{k})$ is the electrical conductivity tensor, $\mathbf{v} = \partial\varepsilon(\mathbf{p})/\partial\mathbf{p}$. Substituting $\mathbf{E},\mathbf{B}^{\sim}$ in the form of harmonic wave into the Maxwell equations (5), we obtain the dispersion equation

$$\det\left[k^2\delta_{ij} - k_ik_j - \frac{\omega^2}{c^2}\varepsilon_{ij}(\omega,\mathbf{k})\right] = 0, \tag{9}$$

which determines the spectrum $\omega(\mathbf{k})$ of oscillations of electromagnetic field in a conductor. Here $\varepsilon_{ij}(\omega,\mathbf{k}) = \delta_{ij} + (4\pi i/\omega)\sigma_{ij}(\omega,\mathbf{k})$ is the dielectric penetrability tensor. We shall use the reference frame where the wave vector $\mathbf{k} = (k_x, 0, k_z)$ is oriented in the xz plane. For frequencies much less than the plasma frequency ω_p the first term in the expression for ε_{ij} can be neglected.

In the lowest-order approximation about the small parameter η the function $L(\mathbf{p},\mathbf{p}')$ does not depend on p_z and can be presented as

$$L(\mathbf{p},\mathbf{p}') = \sum_{n=-\infty}^{\infty} L_n(\varepsilon)e^{in(\varphi-\varphi')}. \tag{10}$$

We have chosen the integrals of motion of an electron in a magnetic field ε and p_z and the phase of its velocity $\varphi = \Omega t_1$ as variables in the $\mathbf{p}$ space. Here $t_1 = (c/eB_0)\int dl/v_\perp$ is the time of motion along the trajectory $\varepsilon(\mathbf{p}) = \varepsilon_F$, $p_z = \mathrm{const}$, $dl = (dp_x^2 + dp_y^2)^{1/2}$, and $v_\perp$ is the velocity component orthogonal to the external magnetic field.[1] Because of the symmetry of function $\Lambda(\mathbf{p},\hat{\boldsymbol{\sigma}},\mathbf{p}',\hat{\boldsymbol{\sigma}}')$ with respect to its arguments, the coefficients in Eq. (10) satisfy the condition $L_n = L_{-n}$. Allowing for next-order terms of the expansion for the correlation function about η does not lead to the noticeable correction of the results.

Expand the functions Φ and ψ into a Fourier series with respect to φ:

$$\Phi = \sum_{n=-\infty}^{\infty}\Phi_n(\varepsilon,p_z)e^{in\varphi}, \quad \psi = \sum_{n=-\infty}^{\infty}\psi_n(\varepsilon,p_z)e^{in\varphi}.$$

Equalizing coefficients at $\exp(in\varphi)$ in the equality $\Phi = \psi + \langle L\psi\rangle$ and using the relationship (10), we obtain

$$\psi = \Phi - \sum_{n=-\infty}^{\infty}\lambda_n\overline{\Phi}_n e^{in\varphi}. \tag{11}$$

Here

$$\overline{\Phi}_n = \frac{1}{(2\pi)^2}\int_0^{2\pi} d\varphi\, e^{-in\varphi}\int_{-\pi}^{\pi} d\theta\,\Phi(\varepsilon_F,\theta,\varphi) \equiv \langle e^{-in\varphi}\Phi\rangle_{\theta,\varphi} \tag{12}$$

is the Fourier coefficient for the function $\langle\Phi\rangle_\theta$, $\theta = p_z/p_0$, $\langle\cdots\rangle_\theta = (1/2\pi)\int_{-\pi}^{\pi} d\theta\cdots$, $\lambda_n = L_n^{\sim}/(1+L_n^{\sim})$, $L_n^{\sim} = \nu(\varepsilon_F)L_n$, and $\nu(\varepsilon_F)$ is the density of states at the Fermi level. Substituting Eq. (11) into Eq. (7) and using the equality $\partial/\partial t_1 = (e/c)(\mathbf{v}\times\mathbf{B}_0)\cdot\partial/\partial\mathbf{p}$ we find the kinetic equation for the perturbation of the renormalized distribution function for electrons with the quasi-two-dimensional dispersion law (1):

$$\frac{\partial\Phi}{\partial\varphi} - \frac{i}{\Omega}(\omega - \mathbf{k}\cdot\mathbf{v})\Phi = \frac{e\mathbf{v}\cdot\mathbf{E}}{\Omega} - \frac{i\omega}{\Omega}\sum_{n=-\infty}^{\infty}\lambda_n\overline{\Phi}_n(\varepsilon)e^{in\varphi} + \frac{1}{\Omega}I_{\mathrm{coll}}(\Phi). \tag{13}$$

The collision integral in the τ approximation can be written as $I_{\mathrm{coll}}(\Phi) = -\tau^{-1}\Phi$ (τ is the relaxation time for the momentum). Below we consider wave processes in the range of frequencies satisfying the condition

$$\omega_p \gg \omega \gg \tau^{-1}, \tag{14}$$

where the asymptotic expression for the spectrum of collective modes does not depend essentially on the concrete form of the collision integral.

For the sake of brevity in the calculations we confine ourselves to account of the two terms in Eq. (1). Neglecting the anisotropy in the layer plane we write the charge carrier dispersion law in the form

$$\varepsilon(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m} - \varepsilon_0\cos\frac{p_z}{p_0}. \tag{15}$$

Here $\varepsilon_0 = \eta v_F p_0$, $v_F^2 \equiv 2\varepsilon_F/m$, and m is the effective mass. In a magnetic field directed along the normal to the layers, the components of electron velocity are given by

$$v_x = v_\perp\cos\varphi,\quad v_y = -v_\perp\sin\varphi, \tag{16}$$

$$v_z = \eta v_F\sin\frac{p_z}{p_0},\quad v_\perp = v_F\sqrt{1+\epsilon\cos\frac{p_z}{p_0}},$$

where $\epsilon = \varepsilon_0/\varepsilon_F$. In this case the cyclotron frequency in the main order approximation with respect to η and plasma frequency are, respectively, $\Omega = |e|B_0/mc$, and $\omega_p^2 = 4\pi e^2 n_0/m$ (n_0 is charge carrier density).

As a result of simple calculations Eq. (13) can be transformed into

$$\Phi = \int_{-\infty}^{\varphi} d\varphi'\exp\left(-\frac{i}{\Omega}(\tilde{\omega} - k_z v_z)(\varphi' - \varphi) + i\frac{k_x v_\perp}{\Omega}(\sin\varphi' - \sin\varphi)\right) \times\left(\frac{e\mathbf{v}\cdot\mathbf{E}}{\Omega} - i\frac{\omega}{\Omega}\sum_{p=-\infty}^{\infty}\lambda_p\overline{\Phi}_p e^{ip\varphi'}\right), \tag{17}$$

where $\tilde{\omega} = \omega + i\tau^{-1}$. Equations (8), (9), and (17) describe eigenmodes of the electromagnetic field in the Fermi liquid of electrons with the dispersion law (15).

In the case when the wave vector $\mathbf{k} = (0,0,k)$ coincides with the magnetic field direction, the dispersion equation (9) brakes down into three equations,

$$\varepsilon_{zz} = 0, \tag{18}$$

$$\varepsilon_{xx} \pm i\varepsilon_{xy} = \left(\frac{kc}{\omega}\right)^2 \tag{19}$$

[we have taken into account the equations $\varepsilon_{xx} = \varepsilon_{yy}$, $\varepsilon_{xz} = \varepsilon_{yz} = 0$ because of the isotropy in the layer plane of the electron dispersion law (15)]. Equation (18) describes purely longitudinal oscillations and coincides exactly with the dispersion equation for longitudinal oscillations of the charged Fermi liquid in the absence of a magnetic field. The other two equations describe transverse electromagnetic waves with different polarizations. Circularly polarized components of the electric field $E_\pm = E_x \pm iE_y$ of these waves and of the electric current density $j_\pm = j_x \pm ij_y$ are connected by the simple relationship

$$j_\pm = (\sigma_{xx} \mp i\sigma_{xy})E_\pm. \tag{20}$$

Making use of expression (8) we have

$$j_\pm = e\nu(\varepsilon_F)\langle(v_x \pm iv_y)\Phi\rangle_{\theta,\varphi} = ev_F\nu(\varepsilon_F)\overline{\Phi}_{\pm1}, \tag{21}$$

where $\overline{\Phi}_{\pm1}$ is determined by the formula (12), and $\nu(\varepsilon_F) = mp_0/\pi\hbar^3$ is the density of states of electrons with the dispersion law (15). The coefficients $\overline{\Phi}_{\pm1}$ can be readily found from Eq. (17) being transformed to the form

$$\Phi = \frac{i}{2}\frac{ev_\perp E_+ e^{i\varphi}}{\tilde{\omega} - kv_z - \Omega} + \frac{i}{2}\frac{ev_\perp E_- e^{-i\varphi}}{\tilde{\omega} - kv_z + \Omega} + \omega\sum_{n=-\infty}^{\infty}\lambda_n\frac{\overline{\Phi}_n e^{in\varphi}}{\tilde{\omega} - kv_z - n\Omega}. \tag{22}$$

With the aid of relations (20) and (21) it is easy to determine $\sigma_{xx} \mp i\sigma_{xy}$ and represent the dispersion equation (19) in the collisionless limit ($\tau \to \infty$) as

$$\left(\frac{kc}{\omega}\right)^2 = 1 - \frac{\omega_p^2}{\omega}\left[\mathrm{sgn}(\omega \mp \Omega)\sqrt{(\omega \mp \Omega)^2 - (\eta kv_F)^2} - \lambda_1\omega\right]^{-1}. \tag{23}$$

At frequencies much lower than the plasma frequency, the solution of Eq. (24) is given by

$$\omega^\pm = \frac{\sqrt{(\eta kv_F)^2 - [(\eta kv_F)^2 - \Omega^2](\lambda_1 - \omega_p^2/k^2c^2)^2} \pm \Omega}{1 - (\lambda_1 - \omega_p^2/k^2c^2)^2}. \tag{24}$$

At $\lambda_1 > 0$ this formula describes two branches of oscillations. The low-frequency branch ($\omega < \Omega, k < \omega_p/c\sqrt{\lambda_1}$) in the limiting case when $\eta kv_F \ll \Omega$ represents a helicoidal wave with frequency $\omega = \Omega k^2c^2/\omega_p^2$, which propagates along an external magnetic field direction. The other, a high-frequency wave ($\omega > \Omega$), results from the Fermi-liquid interaction between charge carriers and exists if the following conditions hold:

$$(\omega \mp \Omega)^2 - (\lambda_1 \omega)^2 < (\eta k v_F)^2 < (\omega \mp \Omega)^2, \quad k > \frac{\omega_p}{c\sqrt{\lambda_1}}. \tag{25}$$

The inequalities (25) show that the frequency takes real values at real values of the wave vector. The inequality $(\eta k v_F)^2 < (\omega \mp \Omega)^2$ is a usual condition of the absence of Landau's collisionless attenuation in a magnetic field. When this condition is satisfied the attenuation rate of the wave is determined by collision processes and is proportional to τ^{-1}. The values $k = k_{\min} = \omega_p / c\sqrt{\lambda_1}$, $\omega^{\pm}(k_{\min}) = \eta \omega_p v_F / c\sqrt{\lambda_1} \pm \Omega$ correspond to the edge of the wave spectrum. The dependence of limiting values for the wave vector and the frequency on λ_1 shows that the waves under consideration can exist at however weak a Fermi-liquid interaction. It suffices that $\lambda_1 > 0$, but the conditions for the excitation of these modes become more difficult with decreasing of λ_1.

Proceeding to the limit $\Omega \to 0$ in Eq. (24), we obtain the spectrum of the Fermi-liquid wave in the absence of a magnetic field,

$$\omega = \frac{\eta k v_F}{\sqrt{1 - (\lambda_1 - \omega_p^2/k^2 c^2)^2}}. \tag{26}$$

The magnetic field lifts the degeneracy from the spectrum, which leads to the appearance of two waves with different polarizations, the limiting value for the frequency ω^- being decreased. It is easily seen from Eq. (23) that at $\mathbf{B}_0 = \mathbf{0}$ the value of $k_{\min}$ remains unchanged.

As follows from Eqs. (9) and (15), the spectrum of the electromagnetic waves with wave vector $\mathbf{k} = (k, 0, 0)$ orthogonal to an external magnetic field is determined by the equations

$$\frac{k^2 c^2}{\omega^2} = \varepsilon_{zz}, \tag{27}$$

$$\frac{k^2 c^2}{\omega^2} \varepsilon_{xx} = \varepsilon_{xx} \varepsilon_{yy} + \varepsilon_{xy}^2. \tag{28}$$

The first of these equations describes the wave with electric field vector directed along the vector $\mathbf{B}_0$. In order to obtain the spectrum of this wave, the conductivity component σ_{zz} should be determined. Multiplying Eq. (17) by v_z and averaging over the Fermi surface, we obtain

$$\begin{aligned}\sigma_{zz} &= e \eta v_F \nu(\varepsilon_F) \langle \sin \theta \Phi \rangle_{\theta \cdot \varphi} / E_z \\ &= \frac{e^2 \eta^2 v_F^2 \nu(\varepsilon_F)}{i\Omega} \langle \sin^2 \theta f_0(\theta) \rangle_\theta \\ &= -i \frac{\eta^2 \omega_p^2}{4\pi\Omega} \sum_{m=-\infty}^{\infty} \frac{\langle \sin^2 \theta J_m^2(\xi) \rangle_\theta}{m - \tilde{\omega}/\Omega}.\end{aligned} \tag{29}$$

Here

$$\begin{aligned} f_0(\theta) = -\frac{1}{2\pi i}(1 - e^{2\pi i \tilde{\omega}/\Omega})^{-1} \int_0^{2\pi} \int_0^{2\pi} d\varphi\, d\varphi_1 \\ \times \exp\left(iR(\varphi, \varphi_1) + i\frac{\tilde{\omega}}{\Omega}\varphi_1 \right),\end{aligned} \tag{30}$$

$R(\varphi, \varphi_1) = -\xi \sin \varphi + \xi \sin(\varphi - \varphi_1)$, $\xi = k v_\perp / \Omega$, and $J_m(\xi)$ is the Bessel function.

In the main approximation in η, the Fermi-liquid interaction does not affect the σ_{zz} component of the conductivity tensor. This is the case when Fermi-liquid effects manifest themselves if the dependence of the Landau correlation function $L(\mathbf{p}, \mathbf{p}')$ on η is taken into account.

In the case when $k\xi \gg 1$ the exponents in the integrand in expression (30) are rapidly oscillating functions and the asymptotic expression for $f_0(\theta)$ can be obtained by means of the stationary phase method. As a result, the dispersion equation takes the form

$$\frac{k^2 c^2}{\omega^2} = 1 - \eta^2 \frac{\omega_p^2}{\Omega \omega} \frac{1}{\xi_0} \left(\cot \frac{\pi \tilde{\omega}}{\Omega} + \frac{2J_1(\epsilon \xi_0)}{\epsilon \xi_0} \frac{\sin 2\xi_0}{\sin(\pi \tilde{\omega}/\Omega)} \right), \tag{31}$$

where $\xi_0 = k r_0 \equiv k v_F / \Omega$, and r_0 is the Larmour radius of electron. Under the condition when $(n^2/\xi_0^3)(\omega_p v_F/\Omega_c)^2 \ll 1$, it follows from Eq. (31) that the spectrum of the cyclotron waves is

$$\begin{aligned}\omega(k) = n\Omega \Bigg\{ 1 - \frac{\eta^2}{\pi \xi_0^3} \left(\frac{\omega_p v_F}{\Omega c} \right)^2 \\ \times \left(1 + (-1)^n \frac{2J_1(\epsilon \xi_0)}{\epsilon \xi_0} \sin 2\xi_0 \right) \Bigg\}, \quad n = 1, 2, 3, \ldots .\end{aligned} \tag{32}$$

Equation (28) corresponds to the wave with the vector $\mathbf{E}$ orthogonal to an external magnetic field. The components of current density are determined by the expressions

$$j_x = e\nu(\varepsilon_F) \langle v_x \Phi \rangle_{\theta,\varphi} = \frac{1}{2} e v_F \nu(\varepsilon_F)(\overline{\Phi}_1 + \overline{\Phi}_{-1}) = \sigma_{xx} E_x + \sigma_{xy} E_y,$$

$$j_y = e\nu(\varepsilon_F) \langle v_y \Phi \rangle_{\theta,\varphi} = \frac{1}{2i} e v_F \nu(\varepsilon_F)(\overline{\Phi}_1 - \overline{\Phi}_{-1}) = \sigma_{yx} E_x + \sigma_{yy} E_y. \tag{33}$$

After multiplying Eq. (17) by $e^{-in\varphi}$ and then integrating over θ and φ we obtain the infinitesimal set of linear equations for $\overline{\Phi}_n$:

$$\sum_{p=-\infty}^{\infty} \left(\delta_{np} + \frac{\omega}{\Omega} \lambda_p f_{n,p} \right) \overline{\Phi}_p = -i \frac{e v_F}{2\Omega} E_+ f_{n,1} - i \frac{e v_F}{2\Omega} E_- f_{n,-1}, \tag{34}$$

where

$$f_{n,p} = -\frac{1}{2\pi i}(1 - e^{2\pi i(\tilde{\omega}/\Omega)})^{-1} \times \left\langle \int_0^{2\pi}\int_0^{2\pi} d\varphi d\varphi_1 e^{iR(\varphi,\varphi_1) - i(n-p)\varphi + i(\tilde{\omega}/\Omega - p)\varphi_1} \right\rangle_\theta = \sum_{m=-\infty}^{\infty} \frac{\langle J_{m-n}(\xi) J_{m-p}(\xi)\rangle_\theta}{m - \frac{\tilde{\omega}}{\Omega}}.$$

If $kr_0 \gg 1$, the asymptotic expression for the coefficients $f_{n,p}$ can be evaluated by means of the stationary phase method. As a result, for $n,p \ll kr_0$ we have

$$f_{n,p} = -\frac{1}{\xi_0}\left(\cot\frac{\pi\tilde{\omega}}{\Omega}\cos\frac{\pi}{2}(n-p) + J_0(\epsilon\xi_0)\frac{\sin(2\xi_0 + (\pi/2)(n+p))}{\sin(\pi\tilde{\omega}/\Omega)}\right). \tag{35}$$

The coefficients of the Fourier series for the smooth function $L(\mathbf{p},\mathbf{p}')$ decrease significantly with an increase of their number, so we will restrict ourselves to a finite number of the terms. Determine $\overline{\Phi}_{\pm 1}$ using Eq. (34) and find the conductivity tensor components from Eq. (33). At $kr_0 \gg 1$ the components of conductivity tensor σ_{xx}, σ_{xy} are much smaller than σ_{yy} and Eq. (28) takes the form

$$\frac{k^2c^2}{\omega^2} = \varepsilon_{yy}. \tag{36}$$

In the frame of the model that allows for the zeroth and first Fourier harmonics of the Landau function

$$L(\mathbf{p},\mathbf{p}') = L_0 + 2L_1\cos(\varphi - \varphi'),$$

the dispersion equation becomes

$$1 - \lambda_0\frac{\omega}{\Omega}\frac{1}{\xi_0}\left(\cot\frac{\pi\tilde{\omega}}{\Omega} + J_0(\epsilon\xi_0)\frac{\sin 2\xi_0}{\sin(\pi\tilde{\omega}/\Omega)}\right) = 2\frac{\omega}{\Omega}\frac{1}{\xi_0}\left[\left(\frac{\omega_p v_F}{\Omega c\xi_0}\right)^2 - \lambda_1\right] \times \left(-\cot\frac{\pi\tilde{\omega}}{\Omega} + J_0(\epsilon\xi_0)\frac{\sin 2\xi_0}{\sin(\pi\tilde{\omega}/\Omega)} + \frac{\lambda_0}{\xi_0}\frac{\omega}{\Omega}\frac{\cos^2(\pi\tilde{\omega}/\Omega) - J_0^2(\epsilon\xi_0)}{\sin^2(\pi\tilde{\omega}/\Omega)}\right). \tag{37}$$

The solution of Eq. (37) may be written as

$$\omega = n\Omega + \Delta\omega, \quad 0 < \Delta\omega < \Omega, \tag{38}$$

where $n\Omega$ is the frequency corresponding to the cyclotron resonance. In the case when

$$\frac{1}{\xi_0^3}\left(\frac{\omega_p v_F}{\Omega c}\right)^2 \gg 1,$$

which can be realized in conductors whose charge carrier density is about one electron per an atom, the left-hand part of Eq. (37) may be neglected. Then in the range of the wave numbers where the inequality $1 - |J_0(\epsilon\xi_0)\sin(2\xi_0)| \gg \xi_0^{-1}$ is satisfied, the spectrum for cyclotron waves can be found in the analytical form

$$\Delta\omega = \Delta\omega_0 + \Delta\omega_1,$$

$$\Delta\omega_0 = \Omega\left(\frac{1}{2} - \frac{(-1)^n}{\pi}\arcsin[J_0(\epsilon\xi_0)\sin 2\xi_0]\right),$$

$$\Delta\omega_1 = \frac{\Omega\lambda_0}{\pi\xi_0}\left(n + \frac{\Delta\omega_0}{\Omega}\right)\frac{J_0^2(\epsilon\xi_0)\cos^2 2\xi_0}{1 - J_0^2(\epsilon\xi_0)\sin^2 2\xi_0}. \tag{39}$$

In the case when

$$\frac{1}{\xi_0^3}\left(\frac{\omega_p v_F}{\Omega c}\right)^2 \ll 1,$$

the solution of (36) can be represented in the form of Eq. (38) with $|\Delta\omega| \ll \Omega$ in whole range where $\xi_0 \gg n$ and

$$\Delta\omega = \frac{n\Omega}{\pi\xi_0}\left(a \pm \sqrt{a^2 + 2(\gamma - \lambda_1)\lambda_0[1 - J_0^2(\epsilon\xi_0)]}\right),$$

$$a = \frac{1}{2}\{\lambda_0 - 2(\gamma - \lambda_1) + (-1)^n[\lambda_0 + 2(\gamma - \lambda_1)] \times J_0(\epsilon\xi_0)\sin 2\xi_0\}, \quad \gamma = \left(\frac{\omega_p v_F}{\Omega c\xi_0}\right)^2. \tag{40}$$

As appears from the formulas (39) and (40), under the strong spatial dispersion conditions the frequencies of the cyclotron waves are oscillating functions of the wave number. It should be noted that in layered conductors the collective mode with the frequencies near the cyclotron resonance frequencies can exist at $k_z v_z \sim \eta k_z v_F \ll \Omega$. Because of the small value of the parameter η, this condition is satisfied in a wide range of k_z even in not very strong magnetic fields.

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