ISSN 1063-7761, Journal of Experimental and Theoretical Physics, 2007, Vol. 105, No. 1, pp. 165–169. © Pleiades Publishing, Inc., 2007.
Original Russian Text © V.G. Peschansky, D.I. Stepanenko, 2007, published in Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, 2007, Vol. 132, No. 1, pp. 188–192.

Spin Waves in the Fermi Liquid of Quasi-Two-Dimensional Conductors

V. G. Peschansky and D. I. Stepanenko

Verkin Physicotechnical Institute of Low Temperatures, National Academy of Sciences of Ukraine, Kharkov, 61103 Ukraine
e-mail: vpeschansky@ilt.kharkov.ua

Abstract—The spin waves in layered conductors with a charge carrier dispersion law that admits of open trajectories in momentum space are investigated in terms of the Fermi-liquid theory.

PACS numbers: 72.15.Nj, 75.30.Fv
DOI: 10.1134/S1063776107070369

In the presence of a strong magnetic field, various weakly damped collective Bose modes, most of which have counterparts in the gas approximation, can exist in the plasma of interacting electrons of normal metals at low temperatures. The spin waves predicted by Silin [1] and detected experimentally in alkali metals by Schultz and Dunifer [2] are the type of excitations attributable to correlation effects that is characteristic only of the electron Fermi liquid.

In degenerate conductors with a quasi-two-dimensional charge carrier energy spectrum, the domain of existence of spin waves is wider than that in metals [3, 4]. In layered conductors with metal-type conductivity, the propagation of spin waves for any direction of the wave vector is possible even under conditions of strong spatial dispersion [4]. This is because the drift velocity of conduction electrons is a quantity of the second order of smallness in low-dimensionality parameter η of the electron energy spectrum at certain orientations of the magnetic field deflected significantly from the layers. As a result, there is no Landau damping and the spin density oscillation decays in the relaxation time in the system of electrons and their spins, i.e., it is undamped in the collisionless limit. Previously, we considered the spin waves in the case where the constant-energy surfaces are in the form of a corrugated cylinder with an isotropic spectrum in the plane of the layers. Experimental studies of galvanomagnetic effects in layered structures [5, 6], in particular, organic conductors from the family of tetrathiafulvalene salts, show that the Fermi surface (FS) can consist of topologically different elements, for example, in the form of corrugated planes.

In this paper, we investigate the spin waves in layered conductors with an anisotropic charge carrier dispersion law that admits of open trajectories in **p** space.

Let us consider the following charge carrier dispersion law:

$$\varepsilon(\mathbf{p}) = \varepsilon_0 + A\cos\frac{p_z}{p_1} + B\cos\frac{p_y}{p_2} + \tilde{C}\cos\frac{p_z}{p_0}. \qquad (1)$$

We will assume the constants A, B, and $\tilde{C}$ to be positive, with $\tilde{C} = \eta(AB)^{1/2}$ being of the order of $\eta\varepsilon_F$, ε_0 is a constant, $\varepsilon_F - \varepsilon_0 \equiv \varepsilon_1 > 0$, and ε_F is the Fermi energy. The parameters $p_1 = \hbar/a_1$, $p_2 = \hbar/a_2$, and $p_0 = \hbar/a_0$ are uniquely related to the main lattice periods a_1, a_2, and a_0; a_0 is the interlayer spacing; and $\hbar$ is the Planck constant. At $A = B$, all open constant-energy surfaces are in the form of a corrugated cylinder with anisotropy in the plane of the layers. If A differs markedly from B, then open constant-energy surfaces in the form of corrugated planes are possible.

In layered conductors in a magnetic field deflected from the layers, $\mathbf{B}_0 = (B_0\sin\vartheta, 0, B_0\cos\vartheta)$, at $\eta\tan\vartheta \ll 1$, all FS sections by the $p_B = (\mathbf{p}\cdot\mathbf{B}_0)/B_0 =$ const plane are almost indistinguishable and the electron cyclotron frequency in the main approximation in η does not depend on p_B. If the angle ϑ is close to $\pi/2$, then the closed sections are highly elongated and the electron does not make a complete turn in such an orbit in momentum space in the mean free time τ. At $\tan\vartheta \geq 1/\eta$, some of these electron orbits are self-crossing ones and the period of electron motion $T(p_B)$ in such an orbit diverges logarithmically.

In a magnetic field oriented in the plane of the layers, i.e., when $\vartheta = \pi/2$, the overwhelming majority of FS sections by the $p_B = p_x =$ const plane are open and only a small number of sections near the FS reference point are closed. For such a magnetic field orientation, all charge carriers drift in the plane of the layers and the fan of all possible electron drift directions fills the entire xy plane.

In the main approximation in small parameter $\eta\tan\vartheta \ll 1$, the term containing the electron velocity along the normal to the layers may be discarded in the equations of quasi-particle motion

$$\frac{dp_x}{dt} = \frac{eB_0}{c}\cos\vartheta v_y,$$

$$\frac{dp_y}{dt} = \frac{eB_0}{c}(-v_x\cos\vartheta + v_z\sin\vartheta), \quad (2)$$

$$p_z = \frac{p_B}{\cos\vartheta} - p_x\tan\vartheta.$$

As a result, system (2) is reduced to the equation of a physical pendulum for linear combinations of momentum components $p_x^{(0)}/p_1 \pm p_y^{(0)}/p_2$. Here, e is the electron charge and c is the speed of light. Assuming, for definiteness, that $B \geq A$, after simple transformations we obtain

$$p_{x,y}^{(0)}(t, \kappa_0) = p_{1,2}\left\{\operatorname{am}\left(\kappa_0\lambda(\kappa_0)\Omega(t+t_0), \frac{1}{\kappa_0}\right) \pm \operatorname{am}\left(\kappa_0\lambda(\kappa_0)\Omega(t+t_0+C), \frac{1}{\kappa_0}\right)\right\}, \quad (3)$$

where

$$\lambda(\kappa_0) = \frac{2K(\kappa_0)}{\pi}, \quad \kappa_0^2 = \frac{(A+B)^2 - \varepsilon_1^2}{4AB},$$

$$\operatorname{am}\left(\kappa\lambda\Omega t, \frac{1}{\kappa}\right)$$

is the amplitude of the elliptic integral, $K(\kappa_0)$ is the first elliptic integral,

$$\Omega = \frac{\pi|e|B_0\sqrt{AB}}{2K(\kappa_0)cp_1p_2}\cos\vartheta$$

is the cyclotron frequency in the zeroth approximation in η,

$$C = \frac{1}{\lambda\Omega}\int\limits_{\varepsilon_1/(A+B)}^{1}\frac{dz}{\sqrt{(1-z^2)(z^2-1+\kappa_0^2)}}.$$

The constant t_0 defining the initial phase can be chosen from the condition $p_x^{(0)}(0, \kappa_0) = 0$, then $t_0 = -C/2$.

If the energy spectrum has such parameters A, B, and ε_1 that $\kappa_0 < 1$, then the amplitude of the elliptic integral is a bounded function,

$$\operatorname{am}\left(\kappa_0\lambda\Omega t, \frac{1}{\kappa_0}\right) = \arcsin\kappa_0\operatorname{sn}(\lambda\Omega t, \kappa_0)$$

and the electron trajectories are closed. In what follows, sn, cn, and dn stand for the Jacobi elliptic functions.

Although the system of equations (2) is integrable in quadratures, its solutions cannot be represented in explicit form. In the case of finite motion, the averaging method [7] can be used to find the asymptotic solution of system (2). Let us pass from the variables p_x and p_y to the variables ψ and κ using the substitution

$$p_x = p_x^{(0)}(\psi, \kappa), \quad p_y = p_y^{(0)}(\psi, \kappa). \quad (4)$$

The parameter ψ defines the oscillation phase, while κ is an analogue of the amplitude in the quasi-linear theory. Instead of the system of equations (2), we will obtain a system with a rapidly rotating phase:

$$\frac{d\kappa}{dt} = \eta\frac{\alpha\Omega}{\Delta(\kappa)}\sin\left(\alpha\frac{p_x^{(0)}(\psi,\kappa)}{p_1} - \beta\right) \times \frac{1}{p_1}\frac{\partial p_x^{(0)}(\psi,\kappa)}{\partial\psi} \equiv \eta X(\psi,\kappa),$$

$$\frac{1}{\Omega}\frac{d\psi}{dt} = 1 - \eta\frac{\alpha}{\Delta(\kappa)}\sin\left(\alpha\frac{p_x^{(0)}(\psi,\kappa)}{p_1} - \beta\right) \times \frac{1}{p_1}\frac{\partial p_x^{(0)}(\psi,\kappa)}{\partial\kappa} \equiv 1 + \eta Y(\psi,\kappa), \quad (5)$$

where

$$\alpha = \frac{p_1}{p_0}\tan\vartheta, \quad \beta = \frac{p_B}{p_0\cos\vartheta}, \quad \eta = \frac{\tilde{C}}{\sqrt{AB}},$$

$$\Delta(\kappa) = -\frac{2\nu\kappa}{\sqrt{1-\nu^2\kappa^2}}, \quad \nu = \frac{2\sqrt{AB}}{A+B}.$$

In accordance with the asymptotic methods of nonlinear mechanics [7], the solutions of system (5) can be represented in the first approximation in η as

$$\kappa = \bar{\kappa} + \eta v(\varphi, \bar{\kappa}), \quad \psi = \varphi + \eta u(\varphi, \bar{\kappa}), \quad (6)$$

where the functions v and u are defined by

$$v(\varphi, \bar{\kappa}) = \int^{\bar{\varphi}} d\varphi X(\varphi, \bar{\kappa}) = -\frac{\eta}{\Delta(\bar{\kappa})}\cos\left(\alpha\frac{p_x^{(0)}(\varphi,\kappa)}{p_1} - \beta\right), \quad (7)$$

$$u(\varphi, \bar{\kappa}) = \int^{\bar{\varphi}} d\varphi(Y(\varphi,\bar{\kappa}) - \langle Y(\varphi,\bar{\kappa})\rangle_\varphi),$$

while the “averaged” variables $\bar{\kappa}$ and φ satisfy the equations

$$\frac{d\bar{\kappa}}{dt} = \eta\langle X(\varphi, \bar{\kappa})\rangle_\varphi = 0,$$
$$\frac{1}{\Omega}\frac{d\varphi}{dt} = 1 + \eta\langle Y(\varphi, \bar{\kappa})\rangle_\varphi. \quad (8)$$

Here,

$$\langle \ldots \rangle_\varphi = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \ldots.$$

The solutions of the system of equations (8)

$$\bar{\kappa} = \kappa_0 + \Delta\kappa = \text{const},$$
$$\varphi = \Omega(1 + \eta\langle Y(\varphi, \bar{\kappa})\rangle_\varphi)t \equiv \Omega_B(\beta)t \quad (9)$$

together with Eqs. (3), (4), (6), and (7) define the electron trajectory $\mathbf{p}(t)$ with the dispersion law (1) in the first approximation in η, where $\Delta\kappa \propto \eta$ can be found from the equation $\varepsilon(\mathbf{p}(t)) = \varepsilon_F$ at $t = 0$. The electron velocity components can be easily determined from the equations of motion (2) by differentiating the corresponding momentum components.

If $\kappa_0 > 1$, then the FS is a corrugated plane and the electron trajectory becomes open, but the quasi-momentum $p_y(t)$ in the direction perpendicular to the magnetic field remains a bounded periodic function of time. To find the asymptotic of the solutions to the equations of motion for $\kappa_0 > 1$, we should introduce a new variable, $\tilde{p}_x(t) \equiv p_x(t) - 2p_1\Omega t$, and apply the above operations to the functions $\tilde{p}_x(t)$ and $p_y(t)$.

When the quantization of the charge carrier energy levels is unimportant, $\hbar\Omega < T \ll \eta\varepsilon_F$ (T is the temperature), the Fermi-liquid interaction can be described by the Landau–Silin correlation function [8, 9]

$$L(\mathbf{p}, \hat{\boldsymbol{\sigma}}, \mathbf{p}', \hat{\boldsymbol{\sigma}}') = N(\mathbf{p}, \mathbf{p}') + S(\mathbf{p}, \mathbf{p}')\hat{\boldsymbol{\sigma}}\hat{\boldsymbol{\sigma}}',$$

where $\hat{\boldsymbol{\sigma}}$ are the Pauli matrices.

As follows from Eqs. (2) and (3), in quasi-two-dimensional conductors at $\eta\tan\vartheta \ll 1$, not only the charge carrier energy in the single-electron approximation $\varepsilon(\mathbf{p})$, but also the additional energy related to the electron–electron interaction effects depend weakly on the momentum component along the magnetic field p_B. This means that the Landau correlation function can be expanded into an asymptotic series in powers of η. In the zeroth approximation in small parameter η, the functions $N(\mathbf{p}, \mathbf{p}')$ and $S(\mathbf{p}, \mathbf{p}')$ do not depend on p_B and can be represented as the series

$$N(\mathbf{p}, \mathbf{p}') = \sum_{n=-\infty}^{\infty} N_n(\varepsilon_F)e^{i(\varphi - \varphi')},$$
$$S(\mathbf{p}, \mathbf{p}') = \sum_{n=-\infty}^{\infty} S_n(\varepsilon_F)e^{i(\varphi - \varphi')}. \quad (10)$$

The paramagnetic spin waves are high-frequency oscillations of the spin density $\mathbf{g}(\mathbf{p}, \mathbf{r}, t) = \mathrm{Sp}_\sigma \hat{\boldsymbol{\sigma}}\hat{\rho}(\mathbf{p}, \mathbf{r}, \boldsymbol{\sigma}, t)$ at $\omega \gg \tau_1^{-1}$ and τ_2^{-1}, where τ_1 and τ_2 are the momentum and spin density relaxation times, respectively, and $\hat{\rho}(\mathbf{p}, \mathbf{r}, \boldsymbol{\sigma}, t)$ is the density matrix. For small deviations from the equilibrium state, the function $\mathbf{g}(\mathbf{p}, \mathbf{r}, t)$ can be represented as a sum of the equilibrium part

$$\mathbf{g}_0(\varepsilon) = -\mu\mathbf{B}_0\frac{\partial f_0(\varepsilon)}{\partial\varepsilon}$$

and the small nonequilibrium addition

$$\delta\mathbf{g}(\mathbf{p}, \mathbf{r}, t) = -\boldsymbol{\Xi}(\mathbf{p}, \mathbf{r}, t)\frac{\partial f_0(\varepsilon)}{\partial\varepsilon},$$

where $f_0(\varepsilon)$ is the Fermi function, $\mu = \mu_0/(1 + S_0^{\sim})$, μ_0 is the magnetic moment of a conduction electron, $S_n^{\sim} = \nu(\varepsilon_F)S_n$, and $\varphi(\varepsilon_F)$ is the state density at the Fermi level.

The components $\Phi^{(\pm)} = \Phi_{x_1} \pm i\Phi_y$ of the nonequilibrium addition to the spin density renormalized by the Fermi-liquid interaction,

$$\boldsymbol{\Phi}(\mathbf{p}, \mathbf{r}, t) = \boldsymbol{\Xi}(\mathbf{p}, \mathbf{r}, t) + \int\frac{d^3p'}{(2\pi\hbar)^3}\left(-\frac{\partial f_0(\varepsilon')}{\partial\varepsilon'}\right)$$
$$\times S(\mathbf{p}, \mathbf{p}')\boldsymbol{\Xi}(\mathbf{p}', \mathbf{r}, t) \equiv \boldsymbol{\Xi} + \langle S\boldsymbol{\Xi}\rangle, \quad (11)$$

which we assume to be proportional to $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, satisfy the integral equations [4]

$$\Phi^{(\pm)} = \int_{-\infty}^{\varphi} d\varphi'$$
$$\times \exp\left(\frac{i}{\Omega_B}\int_{\varphi'}^{\varphi} d\varphi''(\tilde{\omega} \mp \Omega_s - \mathbf{k}\cdot\mathbf{v}(\varphi'', p_B))\right)$$
$$\times\left(i\frac{\mu_0}{\Omega_B}(\mathbf{k}\cdot\mathbf{v}(\varphi', p_B) - \Omega_s)B_{\pm}^{-}\right.$$
$$\left. - i\frac{\omega}{\Omega_B}\sum_{p=-\infty}^{\infty}\lambda_p\bar{\Phi}_p^{(\pm)}e^{ip\varphi'}\right). \quad (12)$$

Here, $\Phi_{x_1} = \Phi_x\cos\vartheta - \Phi_z\sin\vartheta$, the x_1 axis is perpendicular to the y axis and to the vector $\mathbf{B}_0$,

$$\bar{\Phi}_p^{(\pm)} \equiv \langle e^{-ip\varphi}\Phi^{(\pm)}\rangle/\langle 1\rangle, \quad \lambda_p = \frac{S_p^{\sim}}{1+S_p^{\sim}}, \quad \tilde{\omega} = \omega + i0,$$

$B_{\pm}^{\sim} = B_{x_1}^{\sim} \pm iB_y^{\sim}$ are the circularly polarized components of the variable magnetic field, $\Omega_s = \omega_s/(1 + S_0^{\sim})$, and $\omega_s = -2\mu_0 B_0/\hbar$ is the spin paramagnetic resonance frequency.

Since the Fourier series coefficients for the smooth function $S(\mathbf{p}, \mathbf{p}')$ decrease rapidly with their increasing number, it will suffice to retain a finite number of terms in the series in Eq. (12). Multiplying this equation by $e^{-in\varphi}$ and integrating it over the variables φ and $\beta = p_B/(p_0\cos\vartheta)$ yields a system of linear algebraic equations for the coefficients $\bar{\Phi}_p^{(\pm)}$. The spin density eigenmode frequency can be determined, to terms proportional to the static magnetic susceptibility $\chi_0 \sim \nu(\varepsilon_F)\mu_0^2$, from the equation

$$\det\left|\delta_{np} - \lambda_p\frac{\omega}{\Omega}\langle f_{np}(\beta)\rangle_\beta\right| = 0, \tag{13}$$

where

$$f_{np}(\beta) = \frac{1}{2\pi i}\frac{\int_0^{2\pi}\int_0^{2\pi} d\varphi d\varphi_1 \exp\left\{i(p-n)\varphi - ip\varphi_1 - \frac{i}{\Omega_B}\int_{\varphi-\varphi_1}^{\varphi} d\varphi(\tilde{\omega} \mp \Omega_s - \mathbf{k}\cdot\mathbf{v}(\varphi, p_B))\right\}}{1 - \exp\left\{\frac{i}{\Omega_B}\int_0^{2\pi} d\varphi(\tilde{\omega} \mp \Omega_s - \mathbf{k}\cdot\mathbf{v}(\varphi, p_B))\right\}},$$

$$\langle\ldots\rangle_\beta = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\beta\ldots,$$

δ_{np} is the Kronecker delta.

The condition for the absence of collisionless absorption for spin waves is reduced to satisfying the inequality

$$\omega \mp \Omega_s - n\Omega - \langle\mathbf{k}\cdot\mathbf{v}\rangle_\varphi > 0. \tag{14}$$

Outside the domain of ω and $\mathbf{k}$ corresponding to condition (14), the functions $f_{np}(\beta)$ have a pole and the dispersion equation after the integration over p_B acquires an imaginary part responsible for the strong wave absorption.

Because of peculiarities of the electron energy spectrum, the component of the electron drift velocity $\mathbf{v_D} = \langle\mathbf{v}\rangle_\varphi$ along the direction $\mathbf{k}$,

$$\begin{aligned}\mathbf{k}\cdot\mathbf{v_D} &= \langle\mathbf{k}\cdot\mathbf{v}\rangle_\varphi = -\frac{ck_x}{eB_0\cos\vartheta}\left\langle\frac{dp_y}{dt}\right\rangle_\varphi \\ &+ \frac{ck_y}{eB_0\cos\vartheta}\left\langle\frac{dp_x}{dt}\right\rangle_\varphi + (k_x\tan\vartheta + k_z)\langle v_z\rangle_\varphi\end{aligned} \tag{15}$$

may turn out to be negligible even if $kr_0 \gg 1$, where $v_0 = \sqrt{AB}/p_0$ is the characteristic electron velocity in the plane of the layers, $r_0 = v_0/\Omega$. Under these conditions, there exist solutions to the dispersion equation (11) near the resonance:

$$\omega = n_1\Omega \pm \Omega_s + \frac{n_1\Omega \pm \Omega_s}{\pi k r_0}\gamma_i, \quad n_1 = 1, 2, \ldots, \tag{16}$$

where γ_i are the roots of the equation

$$\det\left|\delta_{np} - (-1)^{n_1}\lambda_p\gamma_i^{-1}\langle I_{np}(\beta)\rangle_\beta\right| = 0, \tag{17}$$

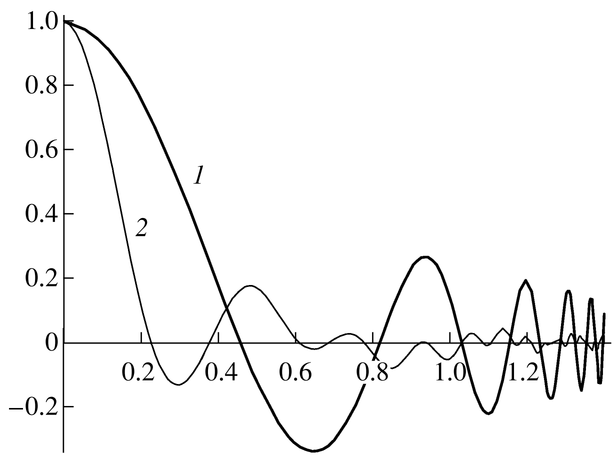
$$\begin{aligned}I_{np}(\beta) &= \frac{kv_0}{\sqrt{|\mathbf{k}\cdot\mathbf{v}'_\varphi(\delta)\mathbf{k}\cdot\mathbf{v}'_\varphi(\delta+\pi)|}} \\ &\times\left\{\cos\left[(n-p)\frac{\pi}{2}\right] + (-1)^{p+1}\right. \\ &\times\left.\sin\left[\frac{1}{\Omega_B}\int_\delta^{\pi+\delta} d\varphi\mathbf{k}\cdot\mathbf{v} - (n-p)\frac{\pi}{2}\right]\right\},\end{aligned} \tag{18}$$

$\mathbf{v}'_\varphi(\varphi) \equiv \partial\mathbf{v}(\varphi)/\partial\varphi$; δ and $\delta + \pi$ are the roots of the equation $\mathbf{k}\cdot\mathbf{v}(\varphi) = 0$.

When it will suffice to retain the zeroth and first harmonics in the Fourier expansion (10) of the correlation function, Eq. (17) is a quadratic equation for γ_i. In quasi-isotropic conductors at $\eta = 1$, the existence of spin waves near the resonance under conditions of strong spatial dispersion is possible only for $\mathbf{k} \perp \mathbf{B}_0$.

At $\kappa_0 < 1$, the drift velocity is an oscillating function of $\alpha = (p_1/p_0)\tan\vartheta$:

$$\mathbf{k}\cdot\mathbf{v_D} = -\eta v_0(k_x\tan\vartheta + k_z)F(\vartheta)\sin\beta, \tag{19}$$



Dependence of the function $F(\vartheta)$ on ϑ: curve *1*, at $A = B$, $\kappa = 0.9$, and $p_1/p_0 = 3$; curve *2*, at $B/A = 2.547$, $\kappa_0^{-1} = 0.9$, $p_1/p_0 = 3$, and $\varepsilon_1/(A + B) = 0.4$.

where

$$F(\vartheta) = \langle \cos(\alpha p_x^{(0)}(\varphi, \bar{\kappa})/p_1) \rangle_\varphi.$$

For certain angles ϑ between the magnetic field $\mathbf{B}_0$ and the normal to the layers, $F(\vartheta) = 0$, the velocity $\mathbf{v_D}$ is a quantity of the second order of smallness in η, and there is virtually no Landau damping. For these $\mathbf{B}_0$ orientations, the existence of collective modes is possible even if $\eta k v_0 \geq \Omega$ and $k v_0/\Omega \gg 1$ for arbitrary directions of the wave vector $\mathbf{k}$.

Solutions to a dispersion equation of form (16) also exist at $\kappa_0 > 1$. When $\mathbf{k} = (k_x, 0, k_z)$, the mean value of $\mathbf{k} \cdot \mathbf{v}$ is given by Eq. (19). In this case, the spectrum of spin waves near the resonance is defined by Eqs. (16)–(18), in which the electron velocity oscillation frequency is meant by Ω.

The plots of $F(\vartheta)$ at $\kappa_0 < 1$ and $\kappa_0 > 1$ are shown in the figure for typical parameters.

The excitation conditions for spin waves with frequencies close to the resonance frequencies (16) turn out to be more favorable than those in quasi-isotropic metals with similar momentum and spin density relaxation times, since almost all of the charge carriers with the Fermi energy are involved in their formation.

REFERENCES

1. V. P. Silin, Zh. Éksp. Teor. Fiz. **35**, 1243 (1958) [Sov. Phys. JETP **8**, 870 (1959)].
2. S. Schultz and G. Dunifer, Phys. Rev. Lett. **18**, 283 (1967).
3. V. G. Peschanskiĭ and D. I. Stepanenko, Pis'ma Zh. Éksp. Teor. Fiz. **78**, 772 (2003) [JETP Lett. **78**, 322 (2003)].
4. O. V. Kirichenko, V. G. Peschanskiĭ, and D. I. Stepanenko, Zh. Éksp. Teor. Fiz. **126**, 1435 (2004) [JETP **99**, 1253 (2004)].
5. J. Singleton, Rep. Progr. Phys. **63**, 1111 (2000).
6. M. V. Kartsovnik, Chem. Rev. **104**, 5737 (2004).
7. N. N. Bogolyubov and Yu. A. Mitropol'skiĭ, *Asymptotic Methods in the Theory of Nonlinear Oscillations*, 4th ed. (Nauka, Moscow, 1974; Gordon and Breach, New York, 1962).
8. L. D. Landau, Zh. Éksp. Teor. Fiz. **30**, 1058 (1956) [Sov. Phys. JETP **3**, 920 (1956)].
9. V. P. Silin, Zh. Éksp. Teor. Fiz. **33**, 495 (1957) [Sov. Phys. JETP **6**, 387 (1958)].

Translated by V. Astakhov