

Nonlocal mixing of supercurrents in Josephson ballistic point contacts

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(Received 18 February 2003; published 8 May 2003)

We study coherent current states in the mesoscopic superconducting weak link simultaneously subjected to the order parameter phase difference ϕ on the contact and to the tangential to the junction interface superfluid velocity \mathbf{v}_s in the banks. The Josephson current-phase relation $I_J(\phi)$ controlled by the external transport current $I_T(\mathbf{v}_s)$ is obtained. At ϕ close to π the nonlocal nature of the Josephson phase-dependent current results in the appearance of two vortexlike states in the vicinity of the contact.

DOI: 10.1103/PhysRevB.67.172504

PACS number(s): 74.45.+c, 74.50.+r

The superfluid flow of Cooper pairs in a superconductor is related to the space dependence of the phase χ of the order parameter. In a (quasi)homogeneous current state the supercurrent density \mathbf{j} locally depends on the superfluid velocity $\mathbf{v}_s = \hbar/2m \nabla \chi(\mathbf{r})$. Such a state is realized in narrow films or wires.¹ In the case in which the phase χ strongly varies in the scale of superconducting coherence length ξ_0 the relation between the current density $\mathbf{j}(\mathbf{r})$ and $\chi(\mathbf{r})$ becomes nonlocal. This situation (opposite to the homogeneous current state) is realized in Josephson weak links (for a review, see Ref. 2), e.g., in superconducting point contacts—microconstrictions between two bulk superconductors (banks). The Josephson current nonlocally depends on $\chi(\mathbf{r})$ and is determined (parametrized) by the total phase difference ϕ across the weak link. The current-phase relation $\mathbf{j}(\phi)$ for ballistic point contact was obtained in Ref. 3. The nonlocal nature of Josephson current in mesoscopic junctions was demonstrated by Heida *et al.*⁴ and studied in theoretical papers.⁵

The Josephson weak link could be considered as a “mixer” of two superconducting macroscopic quantum states in the banks. The result of the mixing is the phase-dependent current carrying state with current flowing from one bank to another. The properties of this state depend on the properties of the states of the banks. For example, in a Josephson junction between unconventional (*d*-wave) superconductors the surface current, tangential to the contact interface, appears simultaneously with Josephson current (see, e.g., Ref. 6).

In this paper we study coherent current states in the Josephson weak link between conventional superconductors, whose banks are in the homogeneous current states. The questions we raise are the following: How are two superconducting current carrying states in the banks coherently mixed by a mesoscopic Josephson junction, or in other words, what is the result of the interplay between transport current $j_T(\mathbf{v}_s)$ flowing parallel to the junction interface and nonlocal Josephson current $j_J(\phi)$? How are the Josephson properties of the system influenced by the external controlling transport current? We have found that the distribution of the current in a region of nonlocal mixing strongly depends on the global phase difference ϕ between banks and for $\phi = \pi$ the distribution contains the vortexlike states. The current-phase relation $j_J(\phi)$ at ϕ near π essentially depends on the superfluid ve-

locity in the banks \mathbf{v}_s , in particular, the absolute value of the derivative $dj_J/d\phi$ at $\phi = \pi$ is suppressed by the transport current.

We consider the Josephson weak link with direct conductivity—a microbridge between thin superconducting films. The bridge sizes, length L and width $2a$, are assumed to be smaller than the coherence length ξ_0 . In this case even for temperature T near the critical temperature T_c the local description based on the Ginzburg-Landau approach is not applicable. To describe the coherent current states in the system we use the quasiclassical Eilenberger equations,⁷ which are valid for temperatures $0 < T < T_c$ and for arbitrary relation between the contact size and coherence length ξ_0 . On the other hand, we assume that a and L are much larger than the Fermi wavelength λ_F . The electron mean free path is supposed to be much larger than ξ_0 .

Suppose the homogeneous transport current I_T with a superfluid velocity \mathbf{v}_s flows in the banks of the contact. The situation with controlled phase difference ϕ and preset current I_T may be realized if the microbridge is incorporated in a cylindrical thin film (Fig. 1). Let the radius of the cylinder be less than the London penetration depth and larger than the coherence length. In this case the phase difference ϕ is governed by the external magnetic flux Φ , $\phi = (2e/\hbar c)\Phi$, and the external transport current I_T flowing along the cylinder is homogeneously distributed far from the microconstriction.

The Eilenberger equations for the ξ -integrated Green's functions have the form⁷

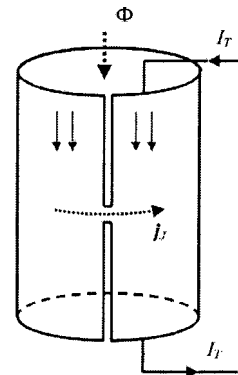


FIG. 1. Scheme of the realization of the microbridge with transport current in the banks and the controlled phase difference.

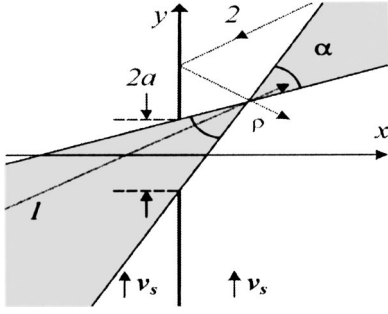


FIG. 2. Model of the microbridge as the slit in the thin insulating partition.

$$\mathbf{v}_F \frac{\partial}{\partial \mathbf{r}} \hat{G}_\omega(\mathbf{v}_F, \mathbf{r}) + [\omega \hat{\tau}_3 + \hat{\Delta}(\mathbf{v}_F, \mathbf{r}), \hat{G}_\omega(\mathbf{v}_F, \mathbf{r})] = 0, \quad (1)$$

where

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^\dagger & 0 \end{pmatrix}, \quad \hat{G}_\omega(\mathbf{v}_F, \mathbf{r}) = \begin{pmatrix} g_\omega & f_\omega \\ f_\omega^\dagger & -g_\omega \end{pmatrix}, \quad (2)$$

$\hat{\tau}_3$ is the Pauli matrix, Δ is the superconducting order parameter, and $\hat{G}_\omega(\mathbf{v}_F, \mathbf{r})$ is the matrix Green's function, which depends on the electron velocity on the Fermi surface \mathbf{v}_F , the coordinate \mathbf{r} , and the Matsubara frequency $\omega = (2n + 1)\pi T$, with n being an integer number.

The order parameter Δ is determined by the self-consistency equation

$$\Delta(\mathbf{r}) = \pi \lambda T \sum_\omega \langle f_\omega(\mathbf{v}_F, \mathbf{r}) \rangle_{\mathbf{v}_F}. \quad (3)$$

Solution of the matrix Eq. (1) together with Eq. (3) determines the current density $\mathbf{j}(\mathbf{r})$ in the system,

$$\mathbf{j}(\mathbf{r}) = -2\pi i e N(0) T \sum_\omega \langle \mathbf{v}_F g_\omega(\mathbf{v}_F, \mathbf{r}) \rangle_{\mathbf{v}_F}. \quad (4)$$

Here λ is the BCS coupling constant, $N(0)$ is the density of states at the Fermi surface, and $\langle \dots \rangle_{\mathbf{v}_F}$ is the averaging over directions of the velocity \mathbf{v}_F .

If the film thickness w is much smaller than ξ_0 , the spatial distributions of $\Delta(\mathbf{r})$ and $\mathbf{j}(\mathbf{r})$ depend only on coordinates in the plane of the film and the Eilenberger equations (1) reduce to the two-dimensional ones. We solve these equations in the model of the microbridge as a slit in a thin impenetrable partition ($L=0$) at $x=0$ between two half planes $x \leq 0$ (Fig. 2). The equations (1) for Green's function $\hat{G}_\omega(\mathbf{v}_F, x, y)$ have to be supplemented by the continuity condition at the slit ($x=0, |y| < a$) and by the condition of the specular reflection at the line ($x=0, |y| \geq a$). Far from the constriction the Green's functions must satisfy the conditions, which describe the homogeneous current parallel to the y axis.

As was shown in Ref. 3 in the zero approximation on the small parameter $a/\xi_0 \ll 1$ for a self-consistent calculation of the superconducting current it is not necessary to find the spatial dependence $\Delta(x, y)$. In the same approximation the superfluid velocity \mathbf{v}_s does not depend on coordinates. The

spatial variation of Δ and \mathbf{v}_s is essential at the distances ρ from the contact $\rho \approx a$. The Green's functions are varied at the distances of order ξ_0 , and in the main approximation on the parameter $a/\xi_0 \ll 1$ they are defined by the values of $\Delta(\mathbf{v}_s)$ and \mathbf{v}_s in the banks of the contact. For Δ and \mathbf{v}_s being constants at each half plane an analytical solution of Eilenberger equations can be found by the method of integration along quasiclassical trajectories. Under the condition $a/\xi_0 \ll 1$ such a solution is self-consistent. In any point $\rho = (x, y)$ all ballistic trajectories can be classified as transit trajectories (marked by a "1" in Fig. 2), for which $\mathbf{v}_F \in \alpha(\rho)$ [$\alpha(\rho)$ being the angle at which the slit is seen from the point ρ] and nontransit trajectories $\mathbf{v}_F \notin \alpha(\rho)$ (marked by a "2" in Fig. 2). For transit trajectories the Green's functions satisfy the boundary conditions in both the banks. For the nontransit trajectories they satisfy to a specular reflection condition at the partition and the conditions in the left or right bank. Making use of the solution of Eilenberger equations, we obtain the following expression for the current density (4) at the slit:

$$\begin{aligned} \mathbf{j}(x=0, |y| < a, \phi, \mathbf{v}_s) \\ = 4\pi |e| N(0) v_F T \\ \times \sum_{\omega > 0} \left\langle \hat{\mathbf{v}} \text{Im} \frac{i\Omega \sin \frac{\phi}{2} - \eta \tilde{\omega} \cos \frac{\phi}{2}}{\eta \Omega \cos \frac{\phi}{2} - i \tilde{\omega} \sin \frac{\phi}{2}} \right\rangle_{\hat{\mathbf{v}}}, \quad (5) \end{aligned}$$

where $\Omega = \sqrt{\tilde{\omega}^2 + \Delta^2}$, $\tilde{\omega} = \omega + i \mathbf{p}_F \mathbf{v}_s$, $\hat{\mathbf{v}} = \mathbf{v}_F / v_F$ is the unit vector, and $\eta = \text{sign}(v_x)$. We should require $\text{Re } \Omega > 0$, which fixes the sign of the square root to be $\text{sign}(\mathbf{p}_F \mathbf{v}_s)$. Under the condition $a/\xi_0 \ll 1$ the current density at the slit does not depend on the y coordinate, and the total current through the contact (Josephson current) is equal to $I_J = 2awj_x$ ($x=0, |y| < a, \phi, \mathbf{v}_s$).

For $\mathbf{v}_s = 0$ the component of the current (5) tangential to the contact $j_y \equiv 0$ and for the Josephson current density $j_J \equiv j_x$ we have the result obtained in Ref. 3. In the general case $\mathbf{v}_s \neq 0$ the current (5) has both j_J and j_y components. The tangential current j_y depends on the phase ϕ and is not equal to the transport current density \mathbf{j}_T in the banks. In particular, at ϕ near π it goes in the direction opposite the external transport current (see below).

To describe the influence of the transport current in the banks on the Josephson current we introduce the dimensionless parameter $q = v_s p_F / \Delta_0$ [$\Delta_0 = \Delta(T=0, v_s=0)$]. The value of q is varied in the range $0 < q < q_c$. The critical value q_c corresponds to the critical current density in the homogeneous current state. At zero temperature $q_c = 1$, and the gap Δ does not depend on q .⁸ In Fig. 3 we plot the Josephson current $I_J(\phi)$ at temperature $T = 0.1T_c$ for different values of q . The presence of the tangential transport current in the banks suppresses the value of the critical Josephson current and essentially changes the derivative $dI_J/d\phi$ at $\phi = \pi$. We emphasize that the dependence of the Josephson current I_J

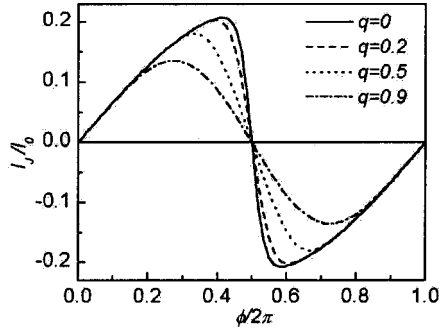


FIG. 3. Josephson current I_J versus phase ϕ . $I_0 = 2aw \cdot 4\pi|e|N(0)v_F T_c$.

(ϕ) on q , which is shown in Fig. 3, does not relate to the suppression of the gap by the transport current, which is negligible for such low temperatures.

The derivative $dI_J/d\phi$ at $\phi = \pi$ determines the kinetic inductance of the Josephson junction,² which is relevant, e.g., for the operation of a superconducting quantum interference device. The expression for the derivative of the Josephson current I_J at $\phi = \pi$ has the form

$$\frac{dI_J}{d\phi}\Big|_{\phi=\pi} = -2aw \frac{|e|N(0)\Delta^2}{mv_s} f\left(\frac{v_s p_F}{\pi T}\right), \quad (6)$$

where the function $f(x)$ is plotted in Fig. 4. The derivative $dI_J/d\phi(\phi = \pi)$ is inversely proportional to v_s at $T \ll p_F v_s$ and it is inversely proportional to T at $T \gg p_F v_s$.

By using the Green's functions along transit and nontransit trajectories, calculated in the main approximation on the small parameter a/ξ_0 , we can find the spatial distributions of the order parameter and the current density in the contact (see Ref. 3). The numerically calculated current-density distributions for different values of phase ϕ and the temperature $T = 0.1T_c$ are shown in Figs. 5 and 6. For small values of the phase difference ϕ between banks, the current density $\mathbf{j}(\rho)$ is just the vector sum of the homogeneous transport current density in the banks $\mathbf{j}_T(\mathbf{v}_s)$ and the conventional Josephson current $\mathbf{j}_J(\phi, \rho, v_s = 0)$ (Fig. 5). For ϕ near π the constructive interference of supercurrents takes place. At $\phi = \pi$ there are no Josephson current, $j_J = 0$, and the current is distributed in such a way that there are two antisymmetric "vortices" close to the contact region (Fig. 6). Far from the constriction (at

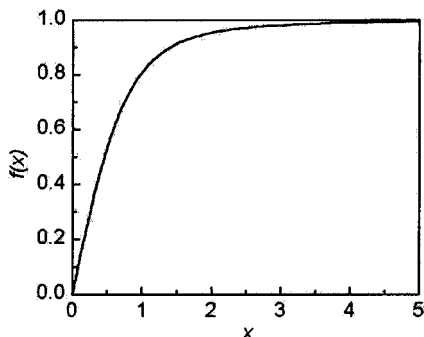


FIG. 4. The function $f(x)$ from expression (6).

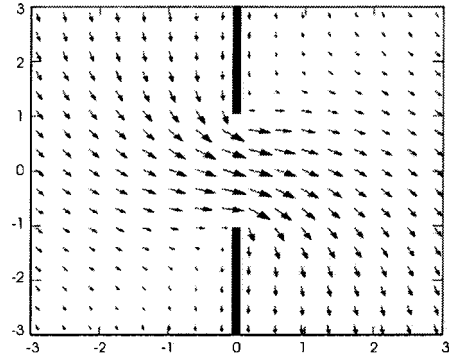


FIG. 5. Vector plot of the current density for $\phi = \pi/2$ and $q = 0.5$. Numbers mark x and y axes in the units of the contact size a .

the distances $\rho \sim \xi_0 \gg a$) the interference current is spread out and the current density is equal to its value in the banks.

A simple and transparent expression for the current-density distribution $\mathbf{j}(\rho)$ can be found for temperatures close to the critical temperature ($T_c - T \ll T_c$). At the distances from the contact, which are less than the coherence length ξ_0 and arbitrary in comparison to the size a , $\mathbf{j}(\rho)$ takes the form

$$\mathbf{j}(\rho, \phi, \mathbf{v}_s) = \mathbf{j}_J(\rho, \phi) + \mathbf{j}_T(\mathbf{v}_s) + \mathbf{j}_{JT}(\rho, \phi, \mathbf{v}_s), \quad (7)$$

$$\mathbf{j}_J(\rho, \phi) = 2j_c \sin \phi \langle \hat{\mathbf{v}} \text{sign}(v_x) \rangle_{\hat{\mathbf{v}} \in \alpha(\rho)},$$

$$\mathbf{j}_T(\mathbf{v}_s) = -j_c k \langle \hat{\mathbf{v}} \hat{v}_y \rangle_{\hat{\mathbf{v}}},$$

$$\mathbf{j}_{JT}(\rho, \phi, \mathbf{v}_s) = j_c k (1 - \cos \phi) \langle \hat{\mathbf{v}} \hat{v}_y \rangle_{\hat{\mathbf{v}} \in \alpha(\rho)},$$

where

$$j_c(T, v_s) = \frac{\pi|e|N(0)v_F}{8} \frac{\Delta^2(T, v_s)}{T_c} \quad (8)$$

is a critical current density of the contact at $T \approx T_c$ and $k = [14\zeta(3)/\pi^3](v_s p_F / T_c)$. We detach explicitly the Josephson current $\mathbf{j}_J(\rho, \phi)$, and the spatially homogeneous (transport) current density $\mathbf{j}_T(\mathbf{v}_s)$ produced by the superfluid velocity \mathbf{v}_s , and write the total current (7) as the sum of three components \mathbf{j}_J , \mathbf{j}_T , and the rest, i.e., the "interference" current \mathbf{j}_{JT} . The macroscopic quantum interference takes place

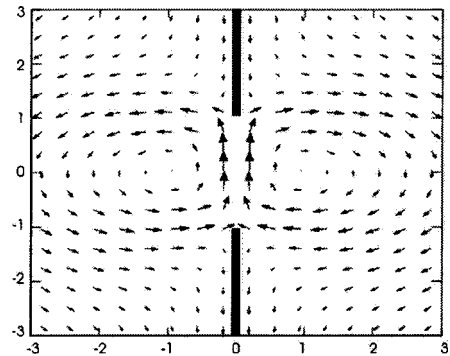


FIG. 6. Vector plot of the current density for $\phi = \pi$ and $q = 0.5$.

in the vicinity of the contact region where both coherent current densities $\mathbf{j}_J(\rho, \phi)$ and $\mathbf{j}_T(\mathbf{v}_s)$ exist. We emphasize that at ϕ near π at the slit the “interference” current \mathbf{j}_{JT} is antiparallel to \mathbf{j}_T . If the phase difference $\phi = \pi$, the current $\mathbf{j}_{JT} = -2\mathbf{j}_T$. When there is no phase difference (at $\phi = 0$), we have $\mathbf{j}_{JT} = 0$.

In conclusion, we have investigated the coherent current states in the Josephson ballistic point contact simultaneously subjected to the order parameter phase difference ϕ and to the tangential to the junction interface the superfluid velocity v_s in the banks. The current-phase relation $I_J(\phi)$ is shown to be controlled by the transport superconducting current $I_T(\mathbf{v}_s)$. Thus, by varying $I_T(\mathbf{v}_s)$, the characteristics of the weak link, such as the shape of the Josephson current-phase relation and the value of the critical current, can be changed. A similar effect can be produced by increasing the temperature T of the system. But, as compared to controlling by the

transport supercurrent, increasing the temperature leads to additional thermal noise.

Moreover, the current distribution pattern in the vicinity of the contact was obtained. The current pattern drastically depends on the external phase difference ϕ ; in particular, at $\phi = \pi$ the existence of two antisymmetric vortexlike current structures is predicted. Considering the current pattern, we have also demonstrated that the superposition of the supercurrents in the vicinity of the weak link is not just their vector sum. These results can be relevant in a wide range of problems, in which the current (and corresponding magnetic-field) distribution in the vicinity of the weak link is important.

We thank I. Dmitrenko, I. Yanson, A. Zagoskin, and A. Amin for stimulating discussions. We acknowledge support from D-Wave Systems, Inc. (Vancouver).

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