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Influence of a single defect on the conductance of a tunnel point contact between a normal metal and a superconductor

Ye. S. Avotina and Yu. A. Kolesnichenko

B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Lenin Ave., Kharkov 61103, Ukraine; Kamerlingh Onnes Laboratorium, Universiteit Leiden, Postbus 9504, 2300 Leiden, The Netherlands

J. M. van Ruitenbeek

Kamerlingh Onnes Laboratorium, Universiteit Leiden, Postbus 9504, 2300 Leiden, The Netherlands
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We have investigated theoretically the conductance of a normal-superconductor point contact in the tunnel limit and analyzed the quantum interference effects originating from the scattering of quasi-particles by point-like defects. Analytical expressions for the oscillatory dependence of the conductance on the position of the defect are obtained for a defect situated either in the normal metal or in the superconductor. It is found that the amplitude of oscillations increases significantly when the applied bias approaches the gap energy of the superconductor. The spatial distribution of the order parameter near the surface in the presence of a defect is also obtained.
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I. INTRODUCTION

Electron scattering by single surface[1] and subsurface[2] defects results in an oscillatory dependence of the scanning tunneling microscope (STM) conductance G on the distance r_0 between the contact and the defect. These oscillations originate from the interference of electron waves, which are scattered by the defect and reflected back by the contact. They have the same period ($G\sim\sin(2k_Fr_0+\delta)$, k_F is the Fermi wave vector) as the Friedel oscillations[3] of the local electron density of states in the vicinity of a scatterer. For subsurface point-like defects, the oscillatory dependence of the conductance in a STM-like geometry has been investigated theoretically in Refs. [4–8].

Although defects below a metal surface can be "visible" in STM data for up to ten interatomic distances,[9,10] the amplitude of the quantum oscillations in the conductance becomes very small with increasing defect depth. An effective way to enhance the STM sensitivity to such oscillation effects is to use a superconducting tip.[11] In Ref. 12, using a low-temperature STM with normal metal tungsten tips and superconducting niobium tips, the formation of electron standing waves near surface defects and step edges on the Au (111) surface has been observed. It was demonstrated that the amplitude of conductance oscillations is significantly enhanced when a superconducting tip is used and the applied bias $|eV|$ is close to the gap energy Δ_0 of the superconductor.

The STM investigation of various defects in superconductors is of interest in itself. For example, in Ref. 13 a bound state near a magnetic Mn adatom on the surface of superconducting Nb was observed by STM. The effect of single Zn defects on the superconductivity in high-T_c superconductors was investigated in Ref. 14, and the manifestation of d-wave symmetry of the order parameter was observed in a quasibound state near the defect.

For the reasons listed there is interest in theoretical investigations of the conductance of normal metal-superconductor (NS) tunnel contacts of small lateral size, in the vicinity of which a single defect is placed. The authors of Ref. 15 considered the conductance of an NS contact of finite size at low temperatures and for voltages $|eV|<\Delta_0$ using the tunneling Hamiltonian approximation. They found that when the radius a of the contact is smaller than the Fermi wavelength λ_F, the conductance of an NS point contact becomes $G_{ns}=(h/2e^2)G_{nn}^2\sim a^8$, where G_{nn} is the conductance of the contact in the normal state.[15] This dependence is fundamentally different from the result of a quasiclassical theory,[16] valid for $a\gg\lambda_F$.

The conductivity of large ($a\gg\lambda_F$) ballistic NS contacts in the presence of a "planar defect" has been investigated theoretically in several papers.[21–24] In those papers a planar NS structure and a δ-function potential barrier, playing the role of the defect, have been considered, from which "geometrical" resonances resulted due to combined Andreev and normal reflections.

In order to describe the effect of isolated point-like defects in a superconductor on the STM conductance one usually calculates the local density of states $n(\mathbf{r})$ (for a review, see Ref. 25), where it is assumed that the conductance of the small tunnel contact is proportional to the local density of electron states. While for subsurface defects this assumption remains qualitatively valid, it does not permit a correct description of the details of the conductance oscillations because the bulk electron density of states around the defect is modified by reflection from the interface, $\mathbf{r}\in\mathbf{\Sigma}$, and in the limit of zero tunneling probability we have $n(\mathbf{r}\in\mathbf{\Sigma})=0$. In this case, the problem of electron transmission through the small NS tunnel junction in the presence of the defect should be considered.

In this paper we present the results of a theoretical investigation of the conductance of an NS point contact (with

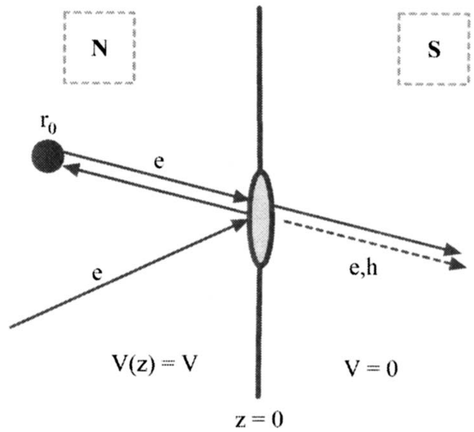

FIG. 1. Model of the contact. The point-like defect is situated in the normal half-space. The electron trajectories in the normal metal and the trajectories of "electron-like" and "hole-like" excitations in the superconductor are shown schematically.

$a \ll \lambda_F$) in the tunneling limit, and we analyze the quantum interference effects originating from the scattering of quasiparticles by a point-like defect. Analytical expressions are obtained for the dependence of the conductance on the position of the defect and on the applied voltage, for a defect situated in the normal metal or in the superconductor.

II. MODEL AND BASIC EQUATIONS

Our model is presented in Fig. 1. The normal and superconducting half-spaces are separated by an infinitely thin dielectric interface, which has an orifice of radius a. The potential barrier in the plane of interface $z=0$ is taken to be a δ function, $U(\mathbf{r})=U_0 f(\boldsymbol{\rho})\delta(z)$, where $\boldsymbol{\rho}$ is the value of the radius vector $\boldsymbol{\rho}$ in the plane $z=0$. The function $f(\boldsymbol{\rho})\to\infty$ at all points of the plane except in the contact ($\rho<a$), where $f(\boldsymbol{\rho})=1$. At the point $\mathbf{r}_0$ a nonmagnetic defect described by a spherically symmetric potential $D(|\mathbf{r}-\mathbf{r}_0|)$ is placed. A voltage V is applied between the two sides of the contact. We assume that the transmission probability $|t|$ of electrons through the barrier in the orifice is small ($|t|\approx\hbar^2 k_F/m^* U_0 \ll 1$, m^* is the effective electron mass). In that case the applied voltage drops entirely over the barrier, and the electric potential can be described by a step function, $V(z)=V\Theta(-z)$ with V a constant. Based on the same reasoning we use a step function for the superconducting order parameter $\Delta(\mathbf{r}) =\Delta(\mathbf{r})\Theta(z)$. We consider the case of low temperatures and in the calculations take $T=0$. At zero temperature a tunnel current flows through the contact for $|eV|>\Delta$. The applied bias is assumed to be small on the scale of the Debye frequency ω_D and the Fermi energy ε_F, $|eV|\ll\hbar\omega_D\ll\varepsilon_F$.

For definiteness we consider electron tunneling from the normal half-space ($z<0$) to the superconducting half-space ($z>0$), i.e., $eV>0$. In order to evaluate the total current through the contact, $I(V)$, and the differential conductance $G(V)=dI(V)/dV$, we should find the current density $\mathbf{j}_\mathbf{k}(\mathbf{r})$ of quasiparticles with momentum $\mathbf{k}$ at $z>0$, formed by electrons transmitted through the contact. The current density $\mathbf{j}_\mathbf{k}(\mathbf{r})$ can be expressed in terms of the coefficients $u_\mathbf{k}(\mathbf{r})$ and $v_\mathbf{k}(\mathbf{r})$ of the canonical Bogoliubov transformation[17,18]

$$\mathbf{j}_\mathbf{k}(\mathbf{r})=\frac{e\hbar}{m^*}\,\mathrm{Im}[u_\mathbf{k}(\mathbf{r})\,\nabla\,u^*_\mathbf{k}(\mathbf{r})f_F(E_\mathbf{k}) - v_\mathbf{k}(\mathbf{r})\,\nabla\,v^*_\mathbf{k}(\mathbf{r})f_F(-E_\mathbf{k})], \tag{1}$$

where $f_F(E)$ is the Fermi function, which at $T=0$ is simply the unit step function, $f_F(E)=\Theta(E)$. The functions $u_\mathbf{k}(\mathbf{r})$ and $v_\mathbf{k}(\mathbf{r})$ satisfy to the Bogoliubov-de Gennes (BdG) equations[19]

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2-\varepsilon_F+D(|\mathbf{r}-\mathbf{r}_0|)\right]u_\mathbf{k}(\mathbf{r})+\Delta(\mathbf{r})v_\mathbf{k}(\mathbf{r}) = E_\mathbf{k}u_\mathbf{k}(\mathbf{r}),$$

$$-\left[-\frac{\hbar^2}{2m^*}\nabla^2-\varepsilon_F+D(|\mathbf{r}-\mathbf{r}_0|)\right]v_\mathbf{k}(\mathbf{r})+\Delta^*(\mathbf{r})u_\mathbf{k}(\mathbf{r}) = E_\mathbf{k}v_\mathbf{k}(\mathbf{r}). \tag{2}$$

Equations (2) may be interpreted as wave equations for a two-component "wave function,"

$$\hat{\psi}_\mathbf{k}=\begin{pmatrix}u_\mathbf{k}\\ v_\mathbf{k}\end{pmatrix}, \tag{3}$$

of quasiparticles with energy $E_\mathbf{k}$. The conditions connecting the vector $\hat{\psi}_\mathbf{k}$ in the normal metal ($\hat{\psi}_{n\mathbf{k}}$) and in the superconductor ($\hat{\psi}_{s\mathbf{k}}$) at the interface $z=0$ are

$$\hat{\psi}_{n\mathbf{k}}(\rho,0)=\hat{\psi}_{s\mathbf{k}}(\rho,0)=\hat{\psi}_\mathbf{k}(\rho,0), \tag{4}$$

$$\frac{\partial}{\partial z}\hat{\psi}_{s\mathbf{k}}(\rho,0)-\frac{\partial}{\partial z}\hat{\psi}_{n\mathbf{k}}(\rho,0)=\frac{2m^*}{\hbar^2}U_0 f(\rho)\hat{\psi}_\mathbf{k}(\rho,0). \tag{5}$$

The order parameter in the superconductor should be determined from the self-consistency condition

$$\Delta(\mathbf{r})=\gamma\sum_{\mathbf{k},E_\mathbf{k}<\hbar\omega_D}u_\mathbf{k}(\mathbf{r})v^*_\mathbf{k}(\mathbf{r})[1-2f_F(E_\mathbf{k})], \tag{6}$$

$$\Delta(z\to+\infty)\to\Delta_0, \tag{7}$$

where the constant Δ_0 can be chosen real; γ is the pair potential constant. It can be easily shown[17] that Eq. (1) combined with the self-consistency condition (6) automatically satisfies the continuity equation

$$\mathrm{div}\sum_\mathbf{k}\mathbf{j}_\mathbf{k}(\mathbf{r})=0. \tag{8}$$

The current-voltage characteristic $I(V)$ of the contact in the presence of a defect can be found by means of integration of the current density $\mathbf{j}_\mathbf{k}(\mathbf{r})$ over the momentum $\mathbf{k}$ (within the energy interval $\Delta_0\leqslant E_\mathbf{k}\leqslant eV$) and over a surface overlapping the contact in the superconducting half-space. For this surface we choose a half-sphere of large radius $r\gg r_0,\xi_0$ (ξ_0 is the coherence length of the superconductor) centered at the contact $r=0$. On this half-sphere we assume $\Delta(\mathbf{r})=\Delta_0$ and hence $E_\mathbf{k}=\sqrt{\xi^2_\mathbf{k}+\Delta^2_0}$, where $\xi_\mathbf{k}=\hbar^2k^2/2m^*-\varepsilon_F$ is the kinetic energy measured from the Fermi level. The conductance $G(V)$ of the contact (at $T=0$) is given by

$$G(V) = 4\pi r e^2 N(0) \int \frac{d\Omega}{4\pi}\Theta(z)\int_{-\infty}^{\infty} d\xi_{\mathbf{k}} \int \frac{d\Omega_{\mathbf{k}}}{4\pi}\Theta(k_z) \times (\mathbf{r}\mathbf{j}_{\mathbf{k}}(\mathbf{r}))\delta(E_{\mathbf{k}} - eV), \tag{9}$$

where $d\Omega$ and $d\Omega_{\mathbf{k}}$ are elements of solid angle in the real and momentum spaces, respectively, and $N(0)$ is the density of states for one direction of spin.

III. SOLUTION OF THE BOGOLIUBOV-DE GENNES EQUATION

Generally, a self-consistent solution of Eqs. (2) can be found only numerically. Such solution must fulfill the condition of conservation of the total current I through any surface overlapping the contact, in spite of the spatial dependence of the order parameter. In order to simplify the task we will exploit the condition of a small barrier transparency and find an analytical solution of Eqs. (2) using the approximation of a constant order parameter $\Delta(\mathbf{r}) = \Delta_0\Theta(z)$. By means of this solution the coordinate dependence of $\Delta(\mathbf{r})$ can be found (see Appendix).

In this Section we generalize the method developed in the papers.[4,20] We search the solutions of Eqs. (2) as a series expansion in the small transmission probability $|t| \sim 1/U_0$,

$$\hat{\psi}_{\mathbf{k}}(\mathbf{r}) = \hat{\psi}_{\mathbf{k}0}(\mathbf{r}) + \hat{\psi}_{\mathbf{k}1}(\mathbf{r}) + \dots , \tag{10}$$

where $\hat{\psi}_{\mathbf{k}0}(\mathbf{r})$ satisfies the zero boundary condition at $z=0$, and $\hat{\psi}_{\mathbf{k}1}(\mathbf{r}) \sim 1/U_0$. For calculation of the current in the leading approximation in the transmission coefficient ($I \sim 1/U_0^2$) it is enough to find the first correction $\hat{\psi}_{\mathbf{k}1}(\mathbf{r})$. Substituting the expansion (10) into the boundary conditions (4) and (5), we find that the function $\hat{\psi}_{\mathbf{k}1}(\mathbf{r})$ satisfies the condition of continuity at $z=0$, and its value at $z=+0$ (in the superconducting half-space) is given by the relation

$$u_{s\mathbf{k}1}(\rho,0) = \frac{\hbar^2}{2m^* U_0 f(\rho)}\frac{\partial}{\partial z}u_{n\mathbf{k}0}(\rho,0); \quad v_{s\mathbf{k}1}(\rho,0) = 0. \tag{11}$$

The bondary condition does not contain Andreev reflections, which appear in the next approximation in $1/U_0$ (Ref. 30). Thus we shall not consider Andreev resonances, which were analyzed in Refs. 21–24 for a one-dimensional model.

The quasiparticle scattering by the defect will be taken into account by perturbation theory in the strength of the interaction with the defect. First, we find the solution of Eqs. (2) for the contact without the defect.

Let us consider an electron with energy $E_{\mathbf{k}} > \Delta_0$, which moves towards the interface from the normal metal. When $D(\mathbf{r})=0$ (the defect is absent) and $1/U_0=0$ (the interface is impenetrable for electrons), in the normal half-space we have

$$u_{u\mathbf{k}0}(\mathbf{r}) = e^{i\varkappa\rho}(e^{ik_z z} - e^{-ik_z z}), \quad v_{n\mathbf{k}0}(\mathbf{r}) = 0, \tag{12}$$

where $\mathbf{k}=(\varkappa,k_z)$, $k_z = k\cos(\vartheta)$, ϑ is the angle between the vector $\mathbf{k}$ and the z axis, and $\boldsymbol{\varkappa}$ is the component of the wave vector parallel to the interface.

Making use of the Fourier transform of the $\hat{\psi}_{\mathbf{k}}(\mathbf{r})$ components with respect to the coordinate $\boldsymbol{\rho}$ in the plane parallel to the interface,

$$\hat{\psi}_{\mathbf{k}1}(\boldsymbol{\rho},z) = \int_{-\infty}^{\infty} d\boldsymbol{\varkappa}' \hat{\Psi}_{\mathbf{k}1}(\boldsymbol{\varkappa}',z)e^{i\boldsymbol{\varkappa}'\cdot\boldsymbol{\rho}}. \tag{13}$$

and finding $\hat{\Psi}_{\mathbf{k}1}(\varkappa,0)$ from the simplified boundary condition (11), we find the solution of Eqs. (2) in the superconducting half-space:

$$u_{\mathbf{k}1}(\mathbf{r}) = t(k_z)\frac{1}{u_0^2 - v_0^2}[u_0^2\varphi_0^{(+)}(\mathbf{r}) + v_0^2\varphi_0^{(-)}(\mathbf{r})], \tag{14}$$

$$v_{\mathbf{k}1}(\mathbf{r}) = t(k_z)\frac{u_0 v_0}{u_0^2 - v_0^2}[\varphi_0^{(+)}(\mathbf{r}) + \varphi_0^{(-)}(\mathbf{r})], \tag{15}$$

where

$$\varphi_0^{(\pm)}(\mathbf{r}) = \pm\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} d\boldsymbol{\varkappa}' e^{i\boldsymbol{\varkappa}'\cdot\boldsymbol{\rho}}\int_{-\infty}^{\infty} d\boldsymbol{\rho}'\frac{e^{i(\boldsymbol{\varkappa}-\boldsymbol{\varkappa}')\cdot\boldsymbol{\rho}'}}{f(\boldsymbol{\rho})} \times e^{\pm ik_z^{(\pm)}z}, \tag{16}$$

$$k_z^{(\pm)} = \frac{\sqrt{2m^*}}{\hbar}\left[\varepsilon_F - \frac{\hbar^2\varkappa^2}{2m^*} \pm \sqrt{E_{\mathbf{k}}^2 - \Delta_0^2}\right]^{1/2}, \tag{17}$$

$$u_0^2 = 1 - v_0^2 = \frac{1}{2}\left(1 + \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right), \tag{18}$$

and $t(k_z) = \hbar^2 k_z / i m^* U_0$ is the amplitude of the electron wave after tunneling through a homogeneous barrier with a large U_0. The functions $u_{\mathbf{k}1}(\mathbf{r})$ and $v_{\mathbf{k}1}(\mathbf{r})$ contain the sum of two solutions $\varphi_0^{(\pm)}(\mathbf{r})$ of equations (2), which correspond to "electron-like" ($k_z^{(+)} > k_{zF} = 1/\hbar\sqrt{2m^*(\varepsilon_F - \hbar^2\varkappa^2/2m^*)}$ and "hole-like" ($k_z^{(-)} < k_{zF}$) quasiparticles having a positive z component of the group velocity $\mathbf{v}_g = dE_{\mathbf{k}}/\hbar d\mathbf{k}$.

For a small radius of the contact (in the limit $a \to 0$) the function (16) takes the form[8]

$$\varphi_0^{(\pm)}(\mathbf{r},k) = \frac{(k^{(\pm)}a)^2\cos\theta}{2}h_1^{(1)}(k^{(\pm)}r), \tag{19}$$

$$k^{\pm}(E_{\mathbf{k}}) = \frac{\sqrt{2m^*}}{\hbar}\left[\varepsilon_F \pm \sqrt{E_{\mathbf{k}}^2 - \Delta_0^2}\right]^{1/2}. \tag{20}$$

Here $h_1^{(1)}(x)$ is the spherical Hankel function of the first kind.

In the presence of the defect the functions $u_{\mathbf{k}1}(\mathbf{r})$ and $v_{\mathbf{k}1}(\mathbf{r})$ can be found in first approximation in the electron-impurity interaction potential $D(|\mathbf{r}-\mathbf{r}_0|)$ by means of Eqs. (2).

1. If the defect is situated in the normal half-space the functions $u_{\mathbf{k}1}(\mathbf{r})$ and $v_{\mathbf{k}1}(\mathbf{r})$ in the superconductor have the same form as Eqs. (14) and (15) in which the amplitude $t(k_z)$ must be replaced by the value

$$\tilde{t}(k_z) = t(k_z) + \frac{4\pi^2 m^* k}{\hbar^2}g t(k) u_{n\mathbf{k}0}(\mathbf{r}_0)h_1^{(1)}(kr_0), \tag{21}$$

where g is the electron-defect interaction constant:

$$g = \int d\mathbf{r} D(|\mathbf{r}-\mathbf{r}_0|), \tag{22}$$

In order to obtain Eq. (21) we assume that the characteristic radius of the scattering potential is much smaller than the Fermi wavelength λ_F (point defect). This condition permits taking the functions $u_{\mathbf{k}1}(\mathbf{r})$ and $h_1^{(1)}(kr)$ outside the integral at the point $\mathbf{r}=\mathbf{r}_0$. The variations in the amplitudes of the "wave functions" $u_{\mathbf{k}1}(\mathbf{r})$ and $v_{\mathbf{k}1}(\mathbf{r})$ result from the fact that the wave incident on the contact is a superposition of a plane wave and a spherical wave that comes from the scattering by the defect.

2. If the defect is situated inside the superconductor, the additions $\Delta u_{\mathbf{k}1}(\mathbf{r})$ and $\Delta v_{\mathbf{k}1}(\mathbf{r})$ to the functions (14) and (15) due to the defect scattering take the form

$$\Delta u_{\mathbf{k}1}(\mathbf{r}) = \frac{2\pi m^* g}{\hbar^2}\frac{1}{v_0^2-u_0^2}\int_{-\infty}^{\infty} d\boldsymbol{\varkappa}\, e^{i\boldsymbol{\varkappa}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_0)}\left\{\frac{1}{k_z^{(+)}}u_0 \sin(k_z^{(+)}z)e^{ik_z^{(+)}z}[u_0u_{\mathbf{k}1}(\mathbf{r}_0)-v_0v_{k1}(\mathbf{r}_0)] + \frac{1}{k_z^{(-)}}v_0\sin(k_z^{(-)}z)\,e^{-ik_z^{(-)}z}[u_0v_{\mathbf{k}1}(\mathbf{r}_0)-v_0u_{\mathbf{k}1}(\mathbf{r}_0)]\right\}; \tag{23}$$

$$\Delta v_{\mathbf{k}1}(\mathbf{r}) = \frac{2\pi m^* g}{\hbar^2}\frac{1}{v_0^2-u_0^2}\int_{-\infty}^{\infty} d\boldsymbol{\varkappa}\, e^{i\boldsymbol{\varkappa}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}_0)}\left\{\frac{1}{k_z^{(+)}}v_0 \sin(k_z^{(+)}z)\,e^{ik_z^{(+)}z}[u_0u_{\mathbf{k}1}(\mathbf{r}_0)-v_0v_{k1}(\mathbf{r}_0)] - \frac{1}{k_z^{(-)}}u_0\sin(k_z^{(-)}z)\,e^{-ik_z^{(-)}z}[u_0v_{\mathbf{k}1}(\mathbf{r}_0)-v_0u_{\mathbf{k}1}(\mathbf{r}_0)]\right\}. \tag{24}$$

It is known that the order parameter $\Delta(\mathbf{r})$ displays Friedel-like oscillations near a defect[26,27] or a surface.[28,29] The current I through the tunnel contact is defined by the average value of $\Delta(\mathbf{r})$, which coincides with Δ_0. In the Appendix we analyze the spatial dependence of $\Delta(\mathbf{r})$ near the surface of the superconductor, in the vicinity of which a nonmagnetic defect is placed (at a distance less than the coherence length ξ_0). Figure 2 illustrates the results of these calculations. An inhomogeneous spatial distribution of the order parameter is visible. We removed from the plot the region of radius $\lambdabar_F$ (black circle) near the defect where Eq. (A9) is not valid.

IV. CONDUCTANCE OF THE CONTACT

By means of the solutions of the BdG equations obtained in the previous Section, we calculated the conductance G of the NS tunnel point contact. In the linear approximation in the electron-defect interaction constant g the conductance G can be presented as the sum of two terms,

$$G(V,r_0) = G_{0ns}(V) + \Delta G_{\rm osc}(V,r_0), \quad eV > \Delta_0. \tag{25}$$

The first term $G_{0ns}(V)$ in Eq. (25) is the conductance of the NS tunnel point contact in the absence of the defect,

$$G_{0ns}(V) = G_{0nn}\frac{eV}{\sqrt{(eV)^2-\Delta_0^2}}; \quad G_{0nn} = \frac{2e^2a^4m^*\varepsilon_F^3}{9\pi\hbar^3U_0^2}. \tag{26}$$

where G_{0nn} is the conductance of a contact between normal metals, which is multiplied by the normalized density of states of the superconductor at $E=eV$ in Eq. (26). The second term describes the oscillatory dependence of the conductance on the distance between the contact and the defect.

If the defect is situated in the normal metal half-space, $\Delta G_{\rm osc}(V,r_0)$ is given by

$$\Delta G_{\rm osc}(V,r_0) = -G_{0ns}(V)\frac{12}{\pi}\tilde{g}\left(\frac{\lambda_F}{r_0}\right)^2 \times (k_Fz_0)^2 j_1(k_Fr_0)y_1(k_Fr_0), \tag{27}$$

where

$$\tilde{g} = \frac{2\pi m^* k_F}{\hbar^2}g \tag{28}$$

is the dimensionless electron-defect interaction constant, $j_l(x)$ and $y_l(x)$ are the spherical Bessel functions of the first and second kinds,[31] and $\lambdabar_F=\hbar/\sqrt{2m^*\varepsilon_F}$. In Fig. 3 the dependence of $\Delta G_{\rm osc}(V,r_0)$ on the distance ρ_0 is shown for two values of the bias eV, one of which is very close to the gap energy $(eV/\Delta_0=1.1)$ and the other is $eV=2\Delta_0$. The figure

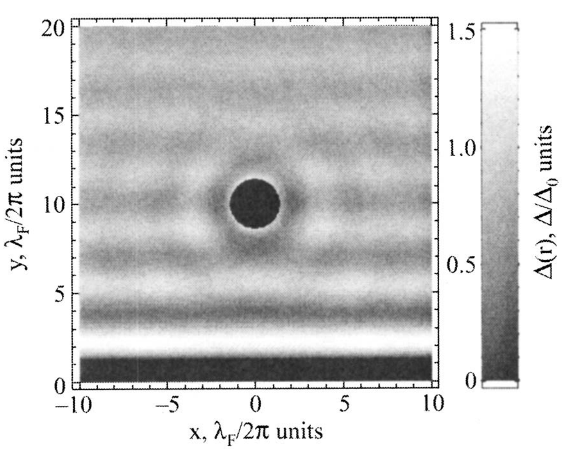

FIG. 2. Real-space image of $\Delta(\mathbf{r})/\Delta_0$ near the surface of the superconductor in the plane passing through the defect; the image was obtained by using Eq. (A9) with the parameters $z_0=10\lambdabar_F$, $\xi_0=10^4\lambdabar_F$, and $\tilde{g}=4\pi$.

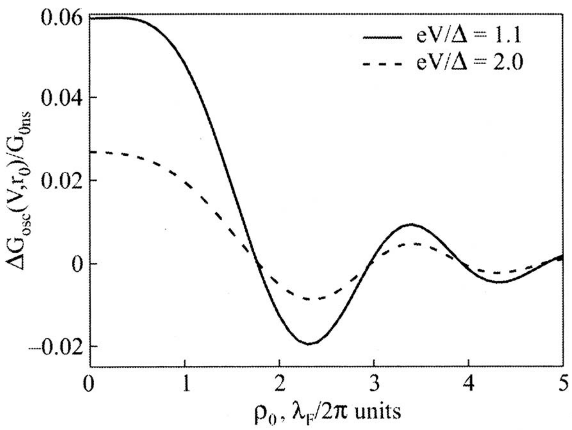

FIG. 3. Dependence of the normalized oscillatory part of the conductance $\Delta G_{\rm osc}/G_{0ns}$, Eq. (27), on the distance ρ_0 between the defect and the contact axis for two values of the applied voltage. The defect is situated in the normal metal at a depth $z_0=5\lambda\!\!\!^{-}_F$. The dimensionless interaction constant is taken as $\tilde{g}=0.01$.

illustrates the increasing amplitude of the conductance oscillations near $eV\simeq\Delta_0$.

For the defect in the superconducting half-space the oscillatory part of the conductance consists of two terms:

$$\Delta G_{\rm osc}(V,r_0) = -G_{0ns}(V)\frac{12}{\pi}\tilde{g}\left(\frac{\lambda}{r_0}\right)^2 (k_F z_0)^2 \times \sum_{\alpha=\pm} \psi_\alpha(eV) j_1(k_\alpha r_0) y_1(k_\alpha r_0), \tag{29}$$

where

$$\psi_\pm = \begin{cases} u_0 \\ v_0 \end{cases}, \quad k_\pm = \frac{\sqrt{2m^*}}{\hbar}\left[\varepsilon_F \pm \sqrt{(eV)^2-\Delta_0^2}\right]^{1/2}. \tag{30}$$

In Eqs. (26)–(29) we have neglected all small terms of order Δ_0/ε_F and eV/ε_F. Nevertheless we have kept the second term in square brackets in the formula for k_+ [see Eq. (30)] because for a relatively large r_0, $(\sqrt{(eV)^2-\Delta_0^2}/\varepsilon_F)$ $(r_0/\lambda\!\!\!^{-}_F)\simeq 1$, the phase shift of the oscillations may be important. In Fig. 4 we show the difference between the dependences of the normalized oscillatory parts of the conductance $\Delta G_{\rm osc}/G_{0ns}$ on the distance ρ_0 for a contact between normal metals ($\Delta_0=0$) and for an NS contact. An observable shift of the conductance oscillations results from the voltage dependence of the wave vectors $k_\pm$ (30).

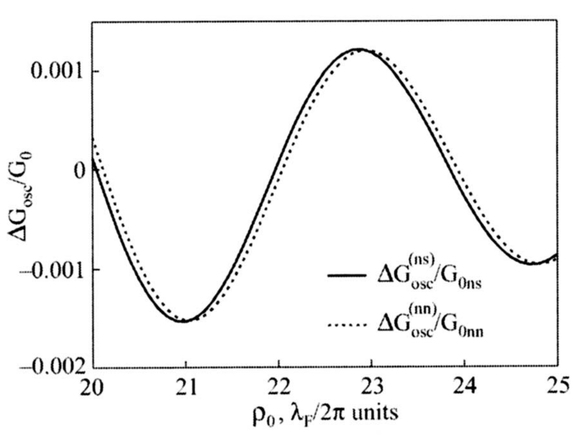

FIG. 4. The dependence of the oscillatory parts of the conductance $\Delta G_{\rm osc}/G_0$ (29) on the distance ρ_0 between the defect and contact axis for contact between normal metals $(\Delta G_{\rm osc}^{(nm)}/G_{0mn})$ and an NS contact $(\Delta G_{\rm osc}^{(ns)}/G_{0mn})$. The defect is situated in the right metal (the superconductor) at a depth of $10\lambda\!\!\!^{-}_F$; $eV/\Delta_0=5$; $\tilde{g}=0.01$.

V. CONCLUSION

Thus we have analyzed the conductance G of an NS tunnel point contact with radius a smaller han the Fermi wavelength $\lambda\!\!\!^{-}_F$, at low temperatures $(T=0)$ and for applied bias eV larger than the gap energy Δ_0 of the superconductor. The effect of quantum interference of quasiparticles scattered by a single defect situated in the vicinity of the contact has been taken into account. We have shown that in leading approximation in the parameters $eV/\varepsilon_F \ll 1$, $\Delta_0/\varepsilon_F \ll 1$ the conductance of a small NS contact is $G_{0ns}=G_{0nn}N_s(eV)$, Eq. (26), i.e., the product of the conductance of the same contact between normal metals, $G_{0nn}\sim a^4$, and the normalized density of states of the superconductor $N_s(eV)$, similar to the case of a planar tunnel contact. Although such result is not unexpected and has been confirmed by experiment,[11] for a contact of radius $a<\lambda\!\!\!^{-}_F$ it was not obvious, and it is obtained in this paper for the first time.

If the defect is situated in the normal metal, the conductance displays oscillations with a period defined by the Fermi wave vector, $\Delta G_{\rm osc}(V,r_0)\sim \sin 2k_F r_0$ at $k_F r_0 \gg 1$ [Eq. (27), Fig. 3], as in the case of a contact between normal metals.[4] In this case the defect plays the role of an additional "barrier" between the normal and superconducting metals and results in oscillations of the transmission coefficient. The underlying principle here is similar to resonance transmission through a two-barrier system.

In the superconductor the electron wave incident on the contact from the normal metal is transformed into a superposition of "electron-like" and "hole-like" quasiparticles. In the case when the defect is located in the superconducting half-space, quantum interference takes place between the partial waves transmitted and scattered by the defect, for both types of quasiparticles independently [Eq. (29)]. Although the difference between the wave vectors $k^{(\pm)}(eV)$ of "electrons" and "holes" is small, the shift $(k^{(+)}-k^{(-)})r_0$ between the two oscillations should be observable (Fig. 4).

APPENDIX: OSCILLATIONS OF THE ORDER PARAMETER NEAR THE SURFACE IN THE PRESENCE OF A DEFECT

When calculating the conductance to first order in the transmission probability we should know the order parameter $\Delta(\mathbf{r})$ in the limit of a nontransparent interface (surface), $U_0\to\infty$. According to Ref. 32,

$$\Delta^*(\mathbf{r}) = \gamma T \sum_{n=-\infty}^{\infty} F_\omega^+(\mathbf{r},\mathbf{r})\Theta(\omega_D-\omega), \tag{A1}$$

where $\omega=\pi T(2n+1)$ are the Matsubara frequencies. The Fourier components $G_\omega(\mathbf{r},\mathbf{r}')$ and $F_\omega^+(\mathbf{r},\mathbf{r}')$ of the Green functions satisfy the Gor'kov equations, which in the absence of a defect potential have the form

$$\left(i\omega - \frac{\hbar^2\nabla^2}{2m^*} - \varepsilon_F\right)G_\omega(\mathbf{r},\mathbf{r}') + \Delta(\mathbf{r})F_\omega^+(\mathbf{r},\mathbf{r}') = \delta(\mathbf{r}-\mathbf{r}'),$$

$$\left(i\omega+\frac{\hbar^2\nabla^2}{2m^*}+\varepsilon_F\right)F^+_\omega(\mathbf{r},\mathbf{r}')+\Delta^*(\mathbf{r})G_\omega(\mathbf{r},\mathbf{r}')=0. \quad (A2)$$

For a homogeneous superconductor $\Delta(\mathbf{r})=\Delta_0=$const, and the solutions $G_\omega(\mathbf{r},\mathbf{r}')=G^{(0)}_\omega(\mathbf{r}-\mathbf{r}')$ and $F^+_\omega(\mathbf{r},\mathbf{r}')=F^{+(0)}_\omega(\mathbf{r}-\mathbf{r}')$ of Eqs. (A2) can be found to be

$$G^{(0)}_\omega(\mathbf{r}-\mathbf{r}')=-\frac{\pi N(0)}{k_F r}\left[\cos k_F r+\frac{i\omega}{\sqrt{\Delta_0^2+\omega^2}}\sin k_F r\right]\times\exp\left(-\frac{r}{v_F\hbar}\sqrt{\Delta_0^2+\omega^2}\right). \quad (A3)$$

$$F^{+(0)}_\omega(\mathbf{r}-\mathbf{r}')=\frac{\pi N(0)\Delta_0^*}{\sqrt{\Delta_0^2+\omega^2}}\frac{\sin k_F r}{k_F r}\exp\left(-\frac{r}{v_F\hbar}\sqrt{\Delta_0^2+\omega^2}\right), \quad (A4)$$

where $r=|\mathbf{r}-\mathbf{r}'|$, v_F is the Fermi velocity, and $\omega\ll\varepsilon_F$. For the semi-infinite superconducting half-space any component of the matrix Green function

$$\hat{G}^{(s)}_\omega(\mathbf{r},\mathbf{r}')=\begin{pmatrix} G^{(s)}_\omega(\mathbf{r},\mathbf{r}') & F^{(s)}_\omega(\mathbf{r},\mathbf{r}') \\ F^{+(s)}_\omega(\mathbf{r},\mathbf{r}') & -G^{(s)}_\omega(\mathbf{r}',\mathbf{r}) \end{pmatrix} \quad (A5)$$

can be written as

$$\hat{G}^{(s)}_\omega(\mathbf{r},\mathbf{r}')=\hat{G}^{(0)}_\omega(\mathbf{r}-\mathbf{r}')-\hat{G}^{(0)}_\omega(\mathbf{r}-\tilde{\mathbf{r}}'), \quad (A6)$$

where $\tilde{\mathbf{r}}'=(x',y',-z')$. Equation (A6) is exact and it provides the zero value of $\Delta(\mathbf{r})$ at the surface $z=0$. The fact that the order parameter vanishes at the nontransparent interface can by seen from Eq. (6).

The Green function for the superconducting half-space in the presence of the point defect can be found from the Dyson equation

$$\hat{G}_\omega(\mathbf{r},\mathbf{r}')=\hat{G}^{(s)}_\omega(\mathbf{r},\mathbf{r}')+\int d\mathbf{r}''\hat{G}^{(s)}_\omega(\mathbf{r},\mathbf{r}'')D(|\mathbf{r}''-\mathbf{r}_0|)\tau_3\hat{G}_\omega(\mathbf{r}'',\mathbf{r}'), \quad (A7)$$

where τ_3 is a Pauli matrix. Making use of the small radius of the defect potential in the first-order approximation in the interaction constant g (22), we obtain

$$F^+_\omega(\mathbf{r},\mathbf{r})=F^{+(s)}_\omega(\mathbf{r},\mathbf{r}')+g[F^{+(s)}_\omega(\mathbf{r},\mathbf{r}_0)G^{(s)}_\omega(\mathbf{r}_0,\mathbf{r}')+G^{(s)}_{-\omega}\times(\mathbf{r}_0,\mathbf{r})F^{+(s)}_\omega(\mathbf{r}_0,\mathbf{r}')]. \quad (A8)$$

As a first step for the self-consistent solution, the functions $G^{(0)}_\omega(\mathbf{r}-\mathbf{r}')$ (A3) and $F^{+(0)}_\omega(\mathbf{r}-\mathbf{r}')$ (A4) may be used. At $T\to 0$ the summation over Matsubara frequencies in Eq. (A1) can be replaced by an integration. Substituting Eqs. (A3) and (A4) into Eq. (A6) and using Eq. (A8), we find the spatial distribution of the order parameter (A1) in the next (after $\Delta=\Delta_0=$const) approximation:

$$\Delta(\mathbf{r})=\Delta_0\left\{1-\frac{\sin 2k_F z}{2k_F z}\ln^{-1}\left(\frac{2\omega_D}{\Delta_0}\right)\mathcal{K}\left(\frac{2\pi z}{\xi_0};\frac{\omega_D}{\Delta_0}\right)+\frac{1}{4\pi}\tilde{g}\ln^{-1}\left(\frac{2\omega_D}{\Delta_0}\right)\left[\frac{\sin 2k_F s_0}{2(k_F s_0)^2}\mathcal{K}\left(\frac{2\pi s_0}{\xi_0};\frac{\omega_D}{\Delta_0}\right)+\frac{\sin 2k_F\tilde{s}_0}{2(k_F\tilde{s}_0)^2}\mathcal{K}\left(\frac{2\pi\tilde{s}_0}{\xi_0};\frac{\omega_D}{\Delta_0}\right)-\frac{\sin k_F(s_0+\tilde{s}_0)}{k_F^2 s_0\tilde{s}_0}\mathcal{K}\left(\frac{\pi(s_0+\tilde{s}_0)}{\xi_0};\frac{\omega_D}{\Delta_0}\right)\right]\right\}. \quad (A9)$$

Here

$$\mathcal{K}(a;b)=\int_0^{\operatorname{arcsinh} b}dt e^{-a\cosh t}, \quad (A10)$$

$s_0=|\mathbf{r}-\mathbf{r}_0|$; $\tilde{s}_0=|\mathbf{r}-\tilde{\mathbf{r}}_0|$, and $\xi_0=\hbar v_F/\pi\Delta_0$ is the coherence length. At $ab\gg 1$, $\mathcal{K}(a;b)\simeq K_0(a)$, the modified Bessel function.[31] Equation (A9) is valid at distances from the defect larger than the characteristic radius of the potential $D(|\mathbf{r}-\mathbf{r}_0|)$. The correction to the constant value of the order parameter Δ_0 decreases at small distances $r\ll\xi_0$ from the surface or the defect according to a power law, and vanishes exponentially ($\sim e^{-2\pi r/\xi_0}$) at larger distances $r\gg\xi_0$. A gray-scale plot of $\Delta(\mathbf{r})$ obtained by means of Eq. (A9) is presented in Fig. 2. In the plot we used an unrealistically large value of the constant $\tilde{g}$ in order to show the influence on the order parameter of the defect and the surface in the same plot. For realistic values $\tilde{g}\sim 0.01$ the spatial oscillations of $\Delta(\mathbf{r})$ resulting from the scattering by the defect have a much smaller amplitude than the second term in the braces of Eq. (A9). The matching procedure can be continued when we put $\Delta(\mathbf{r})$ of Eq. (A9) into Gor'kov's equations [Eqs. (A2)] or the BdG equations (2). Unfortunately, starting with this step, the solutions may be obtained only numerically.

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