# Magnetoelastic generation of electromagnetic fields by sound waves in weak ferromagnets 

Yu. A. Kolesnichenko ${ }^{\text {a) }}$ and D. I. Stepanenko
B. Verkin Institute for Low Temperature Physics and Engineering, National Academy of Sciences of Ukraine, pr. Lenina 47, Kharkov 61103, Ukraine
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A study is made of the generation of electromagnetic oscillations by a sound wave in a weak ferromagnet with a higher-order symmetry axis. The electric fields are calculated for different states with different orientations of the antiferromagnetic vector. It is shown that the magnetoelastic interaction leads to an additional phase difference between the electromagnetic and sound waves. © 2006 American Institute of Physics. [DOI: 10.1063/1.2178472]

The interconversion of electromagnetic and sound waves in substances possessing magnetic ordering is an independent research topic in solid state physics. Magnetic materials are characterized by specific mechanisms of excitation, interconversion, and interaction of sound, spin, and electromagnetic waves. Many papers have been written on the subject (see, e.g., Refs. $1-5$ and the references cited therein). The main focus of attention has been on the study of the features of the spectrum of magnetoacoustic and spin waves and also the question of electromagnetic generation of sound, while the mechanisms of excitation of electromagnetic waves by a sound wave have been the subject of a limited number of papers. ${ }^{6,7}$ Recently experiments have been done to study the kinetic phenomena in rare-earth ( R ) nickel borocarbides $\left(\mathrm{RNi}_{2} \mathrm{~B}_{2} \mathrm{C}\right)$. These substances are a new and extremely interesting class of objects. ${ }^{8-11}$ These objects are attracting attention because of a number of peculiar properties. While having the same tetragonal body-centered crystal structure and electrical conductivity comparable to that of ordinary metals, borocarbides can exhibit a transition to the superconducting state $(\mathrm{R}=\mathrm{Y}, \mathrm{Lu})$, possess heavy-fermion properties $(\mathrm{R}=\mathrm{Yb})$, and demonstrate coexistence of superconductivity and magnetism ( $\mathrm{R}=\mathrm{Tm}, \mathrm{Er}, \mathrm{Ho}, \mathrm{Dy}$ ) or only magnetic ordering $(\mathrm{R}=\mathrm{Tb}, \mathrm{Gd})$. The magnetic properties are due to localization of the $4 f$ electrons of the rare-earth elements. Compounds containing Tb and Er exhibit weak ferromagnetism. The small magnetic moment arises because of a deviation of the antiferromagnetic ordering of the magnetic moments from a strictly antiparallel orientation. In this paper we calculate the electromagnetic field amplitude and the phase difference of the electromagnetic and sound waves in an antiferromagnet with a weak ferromagnetic moment in various orientational states as functions of the saturation magnetization, anisotropic energy, magnetostriction constants, and other parameters containing magnetic materials.

The energy density of an antiferromagnet, which is a sum of magnetic $w_{m}$, magnetoelastic $w_{m e}$, and elastic $w_{e}$ energies, is conveniently written in terms of the relative antiferromagnetic and magnetization vectors:

$$
\mathbf{l}=\frac{\mathbf{M}_{1}-\mathbf{M}_{2}}{2 M_{0}}, \quad \mathbf{m}=\frac{\mathbf{M}_{1}+\mathbf{M}_{2}}{2 M_{0}}
$$

where $m \ll l$, and $\mathbf{M}_{1}$ and $\mathbf{M}_{2}$ are the sublattice magnetizations, and the electrons responsible for the magnetic properties are assumed to be localized on atoms of the lattice. Assuming that the usual condition for low temperatures, $M_{1}^{2}$ $=M_{2}^{2}=M_{0}^{2}$, it is easy to show that the vectors $\mathbf{l}$ and $\mathbf{m}$ satisfy the following relations: $\mathbf{l}^{2}+\mathbf{m}^{2}=\mathbf{1}, \mathbf{l} \cdot \mathbf{m}=0$.

For tetragonal crystals with a high-order symmetry axis $O Z$ the magnetic energy density divided by $4 M_{0}^{2}$ can be written in the form

$$
\begin{align*}
w_{m}= & \frac{1}{2} A m^{2}+\frac{1}{2} a m_{z}^{2}+\frac{1}{2} b l_{z}^{2}+d_{f}\left(l_{x} m_{y}-l_{y} m_{x}\right) \\
& +\frac{1}{2} \alpha \frac{\partial \mathbf{l}}{\partial x_{i}} \frac{\partial \mathbf{l}}{\partial x_{i}}+\frac{1}{2} \alpha_{1} \frac{\partial \mathbf{m}}{\partial x_{i}} \cdot \frac{\partial \mathbf{m}}{\partial x_{i}}-\mathbf{h} \cdot \mathbf{m} \tag{1}
\end{align*}
$$

Here the first term is the exchange energy, the second and third terms are the uniaxial anisotropy energy, and the fourth term is the Dzyaloshinskii interaction, which is responsible for the weak ferromagnetism. The last three terms determine the exchange energy due to the nonuniformity of the magnetic moment, and the Zeeman energy of the magnetic moment in a magnetic field $\mathbf{H}$ is, accordingly, $\mathbf{h}$ $\equiv \mathbf{H} / 2 M_{0}$. Expressions for $w_{m e}$ and $w_{e}$ in the approximation of isotropy of the magnetostrictive and elastic properties in the basal plane can be written in the form ${ }^{3}$

$$
\begin{align*}
w_{m e}= & b_{11}\left(u_{x x} l_{x}^{2}+u_{y y} l_{y}^{2}\right)+b_{12}\left(u_{x x} l_{y}^{2}+u_{y y} l_{x}^{2}\right)++b_{33} u_{z z} l_{z}^{2} \\
& +2 b_{44}\left(u_{y z} l_{y}+u_{x z} l_{x}\right) l_{z}+2 b_{66} u_{x y} l_{x} l_{y}  \tag{2}\\
w_{e}= & c_{11}\left(u_{x x}^{2}+u_{y y}^{2}\right)+c_{12} u_{x x} u_{y y}+c_{13}\left(u_{x x}+u_{y y}\right) u_{z z} \\
& +c_{33} u_{z z}^{2}+2 c_{44}\left(u_{y z}^{2}+u_{x z}^{2}\right)+2 c_{66} u_{x y}^{2} . \tag{3}
\end{align*}
$$

Here $u_{i j}$ is the strain tensor, $b_{i j}$ and $c_{i j}$ are the nonzero components of the fourth-rank tensor of magnetoelastic and elastic constants, respectively, divided by $4 M_{0}^{2}$. These constants are related by the expressions $b_{66}=b_{11}-b_{12}$ and $c_{66}=c_{11}$ $-c_{12}$. Formula (3) describes the change of the elastic energy due to a change of the direction of the vector $\mathbf{l}$.

Assuming that the sublattice magnetizations satisfy the Landau-Lifshitz equations with a relaxation term of the

Gilbert form, we easily find the following equations of motion for the magnetization and antiferromagnetic vectors:

$$
\begin{align*}
\frac{d \mathbf{m}(\mathbf{r}, \mathbf{t})}{d t} & =-\omega_{0}\left[\mathbf{m}(\mathbf{r}, t) \times \mathbf{H}_{\mathbf{m}}+\mathbf{l}(\mathbf{r}, t) \times \mathbf{H}_{\mathbf{l}}\right]+\mathbf{R}_{\mathbf{m}}  \tag{4}\\
\frac{d \mathbf{l}(\mathbf{r}, \mathbf{t})}{d t} & =-\omega_{0}\left[\mathbf{m}(\mathbf{r}, t) \times \mathbf{H}_{\mathbf{l}}+\mathbf{l}(\mathbf{r}, t) \times \mathbf{H}_{\mathbf{m}}\right]+\mathbf{R}_{\mathbf{l}} \tag{5}
\end{align*}
$$

where $\omega_{0}=g|e| M_{0} / m_{e} c, g$ is the gyromagnetic ratio, $e$ is the charge of the electron, $m_{e}$ is the mass of the electron, $c$ is the speed of light,

$$
\begin{aligned}
& \mathbf{R}_{\mathbf{m}}=\lambda\left[\mathbf{m}(\mathbf{r}, t) \times \frac{d}{d t} \mathbf{m}(\mathbf{r}, t)+\mathbf{l}(\mathbf{r}, t) \times \frac{d}{d t} \mathbf{l}(\mathbf{r}, t)\right], \\
& \mathbf{R}_{\mathbf{l}}=\lambda\left[\mathbf{m}(\mathbf{r}, t) \times \frac{d}{d t} \mathbf{l}(\mathbf{r}, t)+\mathbf{l}(\mathbf{r}, t) \times \frac{d}{d t} \mathbf{m}(\mathbf{r}, t)\right]
\end{aligned}
$$

are the relaxation terms, $\lambda=1 / \omega_{0} \tau, \tau$ is the effective relaxation time of the sublattice magnetizations with respect to direction, $\mathbf{H}_{\mathbf{m}}(\mathbf{r}, t), \mathbf{H}_{1}(\mathbf{r}, t)$ are the effective magnetic fields, and

$$
\begin{align*}
& \mathbf{H}_{\mathbf{m}}(\mathbf{r}, t)=-\frac{\partial w}{\partial \mathbf{m}}+\frac{\partial}{\partial x_{i}} \frac{\partial w}{\partial\left(\partial \mathbf{m} / \partial x_{i}\right)} \\
& \mathbf{H}_{\mathbf{l}}(\mathbf{r}, t)=-\frac{\partial w}{\partial \mathbf{l}}+\frac{\partial}{\partial x_{i}} \frac{\partial w}{\partial\left(\partial \mathbf{l} / \partial x_{i}\right)} \tag{6}
\end{align*}
$$

For finding the electric fields $\mathbf{E}$ excited by an external sound field, Eqs. (4) and (5) must be supplemented by Maxwell's equations

$$
\begin{equation*}
\operatorname{rot} \mathbf{H}=\frac{4 \pi}{c} \mathbf{j}, \operatorname{rot} \mathbf{E}=-\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \operatorname{div} \mathbf{B}=0, \operatorname{div} \mathbf{j}=0 \tag{7}
\end{equation*}
$$

In the case when the mean free path of the charge carriers is much less than the skin depth, the current density can be written in the local limit:

$$
\begin{equation*}
\mathbf{j}=\sigma\left(\mathbf{E}+\frac{1}{c}[\dot{\mathbf{u}} \times \mathbf{B}]-\frac{m_{e}}{e} \ddot{\mathbf{u}}\right) \equiv \sigma \widetilde{\mathbf{E}} \tag{8}
\end{equation*}
$$

where $\sigma$ is the electrical conductivity, $\mathbf{B}=\mathbf{H}+8 \pi M_{0} \mathbf{m}$ is the magnetic induction, and $\mathbf{u}$ is the lattice displacement vector. The force acting on the charges in the field of the sound wave is determined by the effective electric field $\widetilde{\mathbf{E}}$, which includes the a Lorentzian term and an inertial term. The elastic and acoustic properties of the antiferromagnet are determined by the constants $c_{i j}$, and the coupling of the spin subsystem with the lattice is determined by the constants $b_{i j}$. The constants $c_{i j}$ are usually several orders of magnitude greater than the constants $b_{i j}$, since coupled magnetoacoustic waves exist only when certain resonance conditions are met. In the remaining cases the spin waves and elastic waves can be treated separately. We shall not investigate further the features of the spectrum of acoustic and spin waves, assuming that the sound field is external and the frequency is specified. We note that under the condition $\max \left|b_{i j}\right| B^{2} / 8 \pi \rho s^{2} \ll 1$, which holds for magnetic fields up to the order of $10^{5} \mathrm{Oe}$, the frequency of the sound wave can be assumed equal to $\omega=s k$ (here $s$ is the sound velocity, $\rho$ is the density of the magnet, and $\mathbf{k}$ is the wave vector).

Let us first consider the case when $b+d_{f}^{2} / A<0$ and the external magnetic field $\mathbf{H}_{0}$ is directed along the symmetry axis $O Z$. We restrict consideration to the field region $H_{0}$ $<A M_{0}$, in which $m<l l$. Investigation of the minimum of expression (1) together with the equations of motion (4), (5) in the equilibrium state shows that the following states of the antiferromagnet, differentiating in the orientation of the vector $\mathbf{I}$, are possible:

1) $h_{0} \equiv H_{0} / 2 M_{0}<h_{1}=\sqrt{(A+a)\left|b+d_{f}^{2} / A\right|}$ - the antiferromagnet is found in a state with a compensated magnetic moment $\mathbf{m}=0$, and the vector $\mathbf{l}$ is directed along the $O Z$ axis;
2) $h_{0}>h_{1}$ - the vector $\mathbf{m}$ is perpendicular to the $O Z$ axis, and the longitudinal and transverse magnetization are equal to

$$
m_{z}=\frac{h_{0}}{A+a}, \quad m_{\perp}=-\frac{d_{f}\left(\mathbf{e}_{z} \times \mathbf{l}\right)}{A}
$$

where $\mathbf{e}_{z}$ is the unit vector along the $O Z$ axis.
Taking into account the smallness of the anisotropy constants and the relaxation constant in comparison with the exchange interaction constant $A$ and neglecting terms of order $m^{2}$, we can write Eqs. (4), (5) approximately as

$$
\begin{align*}
\dot{\mathbf{l}}= & \omega_{0} \mathbf{l} \times\left(A \mathbf{m}-\mathbf{h}+d_{f}\left(\mathbf{e}_{z} \times \mathbf{l}\right)\right),  \tag{9}\\
\dot{\mathbf{m}}= & \omega_{0}\left[\mathbf{l} \times\left(b l_{z} \mathbf{e}_{z}+d_{f}\left(\mathbf{m} \times \mathbf{e}_{z}\right)-\alpha \frac{\partial^{2} \mathbf{l}}{\partial x_{i}}+\mathbf{F}_{l}\right)\right. \\
& \left.+\mathbf{m} \times\left(d_{f}\left(\mathbf{e}_{z} \times \mathbf{l}\right)-\mathbf{h}\right)\right]+\lambda(\mathbf{l} \times \mathbf{i}) . \tag{10}
\end{align*}
$$

Here an overdot indicates a derivative with respect to time, and $\mathbf{F}_{1} \equiv \partial w_{m e} / \partial \mathbf{l}$. In the case when the transverse sound wave propagates along the symmetry axis

$$
\begin{equation*}
\mathbf{u}=\mathbf{u}_{0} \exp (-i \omega t+i k z), \quad u_{0}=\left(u_{0 x}, u_{0 y}, 0\right) \tag{11}
\end{equation*}
$$

it follows from expression (2) that

$$
\begin{equation*}
\mathbf{F}_{1}=i k b_{44} l_{z} \mathbf{u}+i \mathbf{k} b_{44}\left(u_{y} l_{y}+u_{x} l_{x}\right) \tag{12}
\end{equation*}
$$

We set $\mathbf{m}=\mathbf{m}_{0}+\mathbf{m}^{\sim}, \mathbf{l}=\mathbf{l}_{0}+\mathbf{l}^{\sim}$, and $\mathbf{h}=\mathbf{h}_{0}+\mathbf{h}^{\sim}$, where $\mathbf{m}_{0}$ and $\mathbf{l}_{0}$ are the equilibrium values of the magnetization and antiferromagnetic vectors, and $\mathbf{m}^{\sim}, \mathbf{I}^{\sim}$, and $\mathbf{h}^{\sim}$ are the variable corrections proportional to $\exp (-i \omega t+i k z)$, determined from the linearized system of Eqs. (9) and (10).

In the oriented state 1) the longitudinal components of the vectors $\mathbf{I}^{\sim}$ and $\mathbf{m}^{\sim}$ are equal to zero, while the circularly polarized components $l_{+}^{\sim}=l_{x}^{\sim}+i l_{y}^{\sim}, m_{+}^{\sim}=m_{x}^{\sim}+i m_{y}^{\sim}$ have the form

$$
\begin{align*}
l_{+}^{\sim} & =\frac{\left(\omega+\omega_{H}-i d_{f} \omega_{0}\right) \omega_{0} h_{+}^{\sim}+i k b_{44} A \omega_{0}^{2} u_{+}}{\left(\omega+\omega_{H}\right)^{2}-\left(\omega_{1}^{2}+\omega_{l}^{2}+i \omega \gamma\right)},  \tag{13}\\
m_{+}^{\sim} & =\frac{1}{A}\left(-\omega_{0}^{-1}\left(\omega+\omega_{H}+i d_{f} \omega_{0}\right) l_{+}^{\sim}+h_{+}^{\sim}\right) \\
& =\chi_{+} h_{+}^{\sim}-i \beta_{+} k u_{+} \tag{14}
\end{align*}
$$

where

$$
\begin{aligned}
& \chi_{+}=\frac{1}{A}\left[1-\frac{\left(\omega+\omega_{H}\right)^{2}+\omega_{0}^{2} d_{f}^{2}}{\left(\omega+\omega_{H}\right)^{2}-\left(\omega_{k}^{2}+\omega_{1}^{2}-i \omega \gamma\right)}\right], \\
& \beta_{+}=\frac{\left(\omega+\omega_{H}+i d_{f} \omega_{0}\right) \omega_{0} b_{44}}{\left(\omega+\omega_{H}\right)^{2}-\left(\omega_{k}^{2}+\omega_{1}^{2}-i \omega \gamma\right)}, \\
& \omega_{H}=\omega_{0} h_{0}=g \frac{|e| H_{0}}{2 m_{e} c}, \quad \omega_{k}^{2}=A \alpha k^{2} \omega_{0}^{2}, \\
& \omega_{1}^{2}=\omega_{0}^{2} A\left|b+d_{f}^{2} / A\right|=\omega_{0} h_{1}^{2}, \quad \gamma=\omega_{0} \lambda A .
\end{aligned}
$$

The components $l_{-}^{\sim}=l_{x}^{\sim}-i l_{y}^{\sim}, m_{-}^{\sim}=m_{x}^{\sim}-i m_{y}^{\sim}$ are found from formulas (13), (14) by taking the complex conjugate and then making the substitution $\omega \rightarrow-\omega$.

Substituting (14) into the Maxwell's equation

$$
\begin{equation*}
k^{2} h_{+}^{\sim}=i \frac{4 \pi \sigma \omega}{c^{2}}\left(h_{+}^{\sim}+4 \pi m_{-}^{\sim}\right)+\frac{4 \pi \sigma \omega}{c^{2}} h_{0}\left(1+\frac{\omega}{\omega_{H}}\right) k u_{+}, \tag{15}
\end{equation*}
$$

we find the magnetic and electric fields induced by a sound wave:

$$
\begin{align*}
h_{+}^{\sim}= & i \frac{4 \pi \beta_{-}+h_{0}\left(1+\omega / \omega_{H}\right)}{1+4 \pi \chi_{+}+i k^{2} \delta^{2}} k u_{+},  \tag{16}\\
E_{+}= & u_{+} \frac{\omega}{c}\left[-2 i M_{0} 2^{2} \delta^{2} \frac{4 \pi \beta_{+}+h_{0}\left(1+\omega / \omega_{H}\right)}{1+4 \pi \chi_{+}+i k^{2} \delta^{2}}\right. \\
& \left.+H_{0}\left(1+\frac{\omega}{\omega_{H}}\right)\right], \tag{17}
\end{align*}
$$

where $\delta=c / \sqrt{4 \pi \sigma \omega}$ is the skin depth.
The poles of the magnetic susceptibility $\chi_{ \pm}$determine the eigenfrequencies of spin waves of left and right circular polarization; in the limit $\lambda \rightarrow 0$ they are equal to $\omega_{ \pm}(k)$ $=\sqrt{\omega_{k}^{2}+\omega_{1}^{2}} \mp \omega_{H}$. If the frequency of the sound wave does not coincide with either of these frequencies or if the relaxation constant is not small, then the quantity $4 \pi \chi_{ \pm} \sim A^{-1} \ll 1$ in Eq. (17) can be neglected. In the case when the sound wavelength is large compared to the skin depth, $k \delta \ll 1$, the effective electric field $\widetilde{E}_{-}=E_{+}-(\omega / c) u_{-} H_{0}\left(1+\omega / \omega_{H}\right)$ can be written in the form

$$
\begin{align*}
\widetilde{E}_{+}= & -2 i M_{0} k^{2} \delta^{2} u_{+} \frac{\omega}{c} \\
& \times\left[\frac{4 \pi\left(\omega+\omega_{H}-i d_{f} \omega_{0}\right) \omega_{0} b_{44}}{\left(\omega+\omega_{H}\right)^{2}-\left(\omega_{k}^{2}+\omega_{l}^{2}-i \omega \gamma\right)}+h_{0}\left(1+\omega / \omega_{H}\right)\right] \tag{18}
\end{align*}
$$

The phase shift $\Delta \varphi$ between the sound wave and the effective electric field is determined by the constants $\lambda$ and $d_{f}$. In the limiting case when the relaxation constant is negligibly small $(\lambda \rightarrow 0)$ we obtain from formula (18)
$\Delta \varphi=-\frac{\pi}{2}+\arctan$

$$
\begin{equation*}
\times \frac{4 \pi d_{f} \omega_{H} \omega_{0}^{2} b_{44}}{\left(\omega+\omega_{H}\right)\left\{4 \pi \omega_{H} \omega_{0} b_{44}+h_{0}\left[\left(\omega+\omega_{H}\right)^{2}-\left(\omega^{2}+\omega_{1}^{2}\right)\right]\right\}} \tag{19}
\end{equation*}
$$

In the orientational state 2) for $h_{1}>h_{0}$ the magnetoelastic interaction, according to Eq. (12), is due to the component of the displacement vector directed along $\mathbf{l}_{0}$. We choose a coordinate system in which the $O X$ axis is parallel to the vector $\mathbf{I}_{0}$, and then from Eqs. (9), (10), and (12) we find the following expressions for the components of the variable magnetization:

$$
\begin{align*}
& m_{x}^{\sim}=\chi_{x x} h_{x}^{\sim}+\chi_{x y} h_{y}^{\sim}+\chi_{x z} h_{z}^{\sim}+\beta_{1} k u_{x}, \\
& m_{y}^{\sim}=\chi_{y x} h_{x}^{\sim}+\chi_{y y} h_{y}^{\sim}+\beta_{2} k u_{x}, \\
& m_{z}^{\sim}=\chi_{z x} h_{x}^{\sim}+\chi_{z z} h_{z}^{\sim}, \tag{20}
\end{align*}
$$

where

$$
\begin{align*}
& \chi_{x x}=-\frac{1}{A}\left(\frac{\omega_{0}^{2} d_{f}^{2}}{\omega^{2}-\omega_{k}^{2}+i \omega \gamma}+\frac{\omega_{H}^{2}}{\omega^{2}-\Omega^{2}+i \omega \gamma}\right),  \tag{21}\\
& \chi_{x y}=-\chi_{y x}=\frac{i \omega \omega_{H} A^{-1}}{\omega^{2}-\Omega^{2}+i \omega \gamma}, \\
& \chi_{x z}=-\chi_{z x}=\frac{i \omega \omega_{0} d_{f} A^{-1}}{\omega^{2}-\omega_{k}^{2}+i \omega \gamma}, \\
& \chi_{y y}=\frac{1}{A}\left(1-\frac{\omega^{2}}{\omega^{2}-\Omega^{2}+i \omega \gamma}\right), \\
& \chi_{z z}=\frac{1}{A}\left(1-\frac{\omega^{2}}{\omega^{2}-\omega_{k}^{2}+i \omega \gamma}\right), \quad \beta_{1}=-\frac{i \omega_{0} \omega_{H} b_{44}}{\omega^{2}-\Omega^{2}-i \omega \gamma}, \\
& \beta_{2}=\frac{\omega \omega_{0} b_{44}}{\omega^{2}-\Omega^{2}+i \omega \gamma}, \quad \Omega^{2}=\omega_{k}^{2}+\omega_{H}^{2}-\omega_{1}^{2} .
\end{align*}
$$

The frequency of the sound wave is much lower than the cyclotron frequency in a magnetic field $H_{0}>2 M_{0} h_{1}$ $\sim 10^{4}-10^{5} \mathrm{Oe}$, corresponding to the orientational state 2 ), and therefore the inertial term in the expression (8) for the current density can be neglected, and Maxwell's equations for the Fourier components of the variable fields take the form

$$
\begin{align*}
& k^{2} \mathbf{h}_{\perp}^{\sim}=i \frac{4 \pi \sigma \omega}{c^{2}}\left(\mathbf{h}_{\perp}^{\sim}+4 \pi \mathbf{m}_{\perp}^{\sim}\right)+\frac{4 \pi \sigma \omega}{c^{2}} b_{0} k \mathbf{u}, \\
& j_{z}=\sigma\left(E_{z}+4 \pi \frac{i \omega}{c} \frac{d_{f}}{A} u_{x}\right)=0, \quad h_{z}^{\sim}+4 \pi m_{z}^{\sim}=0 . \tag{22}
\end{align*}
$$

Here $\mathbf{h}_{\perp}^{\sim}=\left(h_{x}^{\sim}, h_{y}^{\sim}, 0\right), \mathbf{m}_{\perp}^{\sim}=\left(m_{x}^{\sim}, m_{y}^{\sim}, 0\right)$ are the transverse components of the magnetization and magnetic field, $b_{0}$ $=h_{0}+4 \pi / A \approx h_{0}$ is the $z$ component of the constant magnetic induction. After determining $\mathbf{h}^{\sim}$ from Eqs. (22), we find the effective electric field generated by the sound wave:

$$
\begin{align*}
\widetilde{E}_{x}= & -2 i M_{0} k^{2} \delta^{2} \frac{\omega}{c} D^{-1}\left[( 1 + 4 \pi \tilde { \chi } _ { x x } + i k ^ { 2 } \delta ^ { 2 } ) \left(-4 \pi \beta_{2} u_{x}\right.\right. \\
& \left.\left.+i b_{0} u_{y}\right)-4 \pi \chi_{y x}\left(-4 \pi \beta_{1} u_{x}+i b_{0} u_{x}\right)\right], \\
\widetilde{E}_{y}= & -2 i M_{0} k^{2} \delta^{2} \frac{\omega}{c} D^{-1}\left[\left(1+4 \pi \chi_{y y}+i k^{2} \delta^{2}\right)\right. \\
& \left.\times\left(-4 \pi \beta_{1} u_{x}+i b_{0} u_{x}\right)-4 \pi \chi_{x y}\left(-4 \pi \beta_{2} u_{x}+i b_{0} u_{y}\right)\right] \\
\widetilde{E}_{z}= & 0 \tag{23}
\end{align*}
$$

where

$$
\begin{aligned}
& D=\left(1+4 \pi \widetilde{\chi}_{x x}+i k^{2} \delta^{2}\right)\left(1+4 \pi \chi_{y y}+i k^{2} \delta^{2}\right) \\
& -16 \pi^{2} \chi_{x y} \chi_{y x}, \quad \tilde{\chi}_{x x}=\chi_{x x}-4 \pi \chi_{z x} /\left(1+4 \pi \chi_{z z}\right)
\end{aligned}
$$

If the sound wave frequency does not coincide with one of the poles of the components of the magnetic susceptibility tensor, and the sound wavelength is large in comparison with the skin depth, then formulas (23) become

$$
\begin{align*}
& \widetilde{E}_{x}=2 M_{0} k^{2} \delta^{2} \frac{\omega}{c}\left(\frac{4 \pi i \omega \omega_{0} b_{44}}{\omega^{2}-\Omega^{2}+i \omega \gamma} u_{x}+b_{0} u_{y}\right),  \tag{24}\\
& \widetilde{E}_{y}=-2 M_{0} k^{2} \delta^{2} u_{x} \frac{\omega}{c}\left(\frac{4 \pi \omega_{0} \omega_{H} b_{44}}{\omega^{2}-\Omega^{2}+i \omega \gamma}+b_{0}\right) . \tag{25}
\end{align*}
$$

Between the component of the effective electric field $\widetilde{E}_{x}$ and the elastic wave there is a phase shift $\Delta \varphi_{x}$ due to the interaction of the spin subsystem with the lattice. In the limit $\lambda \rightarrow 0$ we obtain from Eq. (24)

$$
\begin{equation*}
\Delta \varphi_{x}=-\arctan \frac{4 \pi \omega \omega_{0} b_{44} \tan \vartheta}{\left(\Omega^{2}-\omega^{2}\right) b_{0}} \tag{26}
\end{equation*}
$$

where $\tan \vartheta=u_{0 x} / u_{0 y}$.
In the case $b+d_{f}^{2} / A>0$ the vector $\mathbf{1}_{0}$ lies in the basal plane perpendicular to the $O Z$ axis, and the antiferromagnet has a weak magnetic moment $\mathbf{m}_{\perp}=-d_{f}\left(\mathbf{e}_{z} \times \mathbf{1}_{0}\right) / A$ in the absence of external magnetic field. The induced magnetic moment is directed along the external magnetic field, and its value in the leading approximation with respect to $A^{-1}$ is equal to $m \approx h_{0} / A$. The electric field generated by an acoustic wave (11) in an easy-plane antiferromagnet in a magnetic
field directed along the $O Z$ axis is given by Eqs. (23)-(25) in which the substitution $\omega_{1}^{2} \rightarrow-\omega_{1}^{2}$ or $\Omega^{2} \rightarrow \omega_{k}^{2}+\omega_{H}^{2}+\omega_{1}^{2}$ $\left(\mathbf{l}_{0} \| O X\right)$ has been made.

Formulas (18), (24), and (25) for the electric fields radiated by a sound wave consist of two terms. The first term in the brackets describes the contribution of the magnetoelastic interaction, which is inherent only to magnetically ordered substances, and the second term is due to the Lorentzian mechanism taking place in ordinary metals. It follows from expressions (17)-(19) and (23)-(26) that the magnetoelastic interaction has a substantial influence on the polarization and phase of the variable electric field.

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${ }^{\text {a) }}$ Electronic mail: kolesnichenko@ilt.kharkov.ua
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