On the theory of electromagnetic fields radiated by an elastic wave in a ferromagnet

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(Submitted September 13, 2004)
Fiz. Nizk. Temp. **31**, 536–541 (May 2005)

The generation of electromagnetic oscillations by a sound wave in a substance having magnetic order is investigated under conditions of the normal skin effect. The amplitude of the electric field and the phase difference of the electromagnetic and sound waves are calculated as functions of the saturation magnetization, anisotropy energy, magnetostriction constants, and other parameters characterizing the magnetic material.
[DOI: 10.1063/1.1925367]

Interest in the theoretical study of the interconversion of electromagnetic and sound waves in ferromagnetic metals has heightened in the last decade in connection with the discovery of a family of rare-earth (R) nickel borocarbides (RNi_2B_2C),[1–4] a significant number of which have magnetic order. With the same crystal structure, the borocarbides can exhibit a transition to the superconducting state (R=Y, Lu), possess heavy-fermion properties (R=Yb), or demonstrate coexistence of superconductivity and magnetism (R=Tm, Er, Ho, Dy) or only magnetic ordering (R=Tb, Gd). The superconducting borocarbides are among the so-called unconventional superconductors, the order parameter in which corresponds to $s+g$ symmetry,[5–7] in contrast to the isotropic s-wave pairing in ordinary metals. Experimental and theoretical study of the magnetoacoustic processes in magnetic borocarbides in the normal state can elucidate the influence of the magnetic order on their kinetic and thermodynamic characteristics. Besides the mechanisms of interconversion of the boson branches of the spectrum which are inherent to normal metals,[8,9] magnetic materials have specific mechanisms of excitation and interaction of sound, spin, and electromagnetic waves. Because of the magnetoelastic interaction, the propagation of elastic waves in ferromagnets and antiferromagnets is accompanied by oscillations of the magnetization. Although magnetoacoustic oscillations have been widely studied theoretically (see, e.g., Refs. 10 and 11) those studies were limited to the approximation of magnetostatics, and the electric fields were not considered in them. In the present paper we study the generation of electromagnetic oscillations by a sound wave in substances having magnetic order. We consider the case realized in practice wherein the electrons responsible for the magnetic properties are localized in lattice atoms, and the mean free time of the conduction electrons τ_e is short in relation to the frequencies of the alternating fields and the electron cyclotron frequency in the external magnetic field.

We start from the following expression for the energy density of a ferromagnet:

$$w_\Sigma = w_e(\mathbf{M}^2) + w_a(\mathbf{M}) + \frac{1}{2}\alpha_{ik}\frac{\partial \mathbf{M}}{\partial x_i}\cdot\frac{\partial \mathbf{M}}{\partial x_k} + \frac{\mathbf{H}^{(m)2} + \mathbf{E}\cdot\mathbf{D}}{8\pi} - \mathbf{M}\cdot\mathbf{H}^{(\mathrm{ext})} + \frac{1}{2}\rho\dot{\mathbf{u}}^2 + \frac{1}{2}\eta_{iklm}u_{ik}u_{lm} + f_{ik}(\mathbf{M})u_{ik} + \mathcal{E}(\mathbf{r},t). \quad (1)$$

Here $w_e(\mathbf{M}^2)$ is the exchange energy, $w_a(\mathbf{M})$ is the anisotropy energy, the third term is the exchange energy due to the nonuniformity of the magnetic moment density $\mathbf{M}$. The fourth and fifth terms are the energy of the electromagnetic field and the energy of the magnetic moment in an external uniform magnetic field $\mathbf{H}^{(\mathrm{ext})}$; $\mathbf{H}^{(m)}$ is the magnetic field produced by the magnetization, $\mathbf{E}$ and $\mathbf{D}$ are the electric field and electric displacement, respectively. The next three terms determine the elastic and magnetostriction energies; ρ is the density of the ferromagnet, $\mathbf{u}$ is the displacement vector of the lattice points with coordinates $\mathbf{r}$ at time t, $\dot{\mathbf{u}}$ is the time derivative, η_{iklm} is the tensor of elastic constants, u_{ik} is the strain tensor, and f_{ik} is the tensor characterizing the magnetostriction.

The energy of the system of conduction electrons with a dispersion relation $\varepsilon_0(\mathbf{p})$ can be written in the form[12]

$$\mathcal{E}(\mathbf{r},t) = \int \frac{2d^3p}{(2\pi\hbar)^3}[\varepsilon_0(\mathbf{p}) + \delta\varepsilon(\mathbf{r},\mathbf{p},t)]f(\mathbf{r},\mathbf{p},t), \quad (2)$$

where $\delta\varepsilon(\mathbf{r},\mathbf{p},t) = \lambda_{ik}(\mathbf{p})u_{ik} - (\mathbf{p} - m\partial\varepsilon_0/\partial\mathbf{p})\dot{\mathbf{u}}$ is the additional energy of an electron in the field of a sound wave, $\lambda_{ik}(\mathbf{p})$ is the deformation potential,[13] and m is the free electron mass. The distribution function $f(\mathbf{r},\mathbf{p},t)$ of the electronic system perturbed by alternating fields is conveniently represented as the sum of an instantaneous equilibrium distribution function and a nonequilibrium admixture:

$$f(\mathbf{r},\mathbf{p},t) = f_0(\varepsilon_0(\mathbf{p}) + \delta\varepsilon(\mathbf{r},\mathbf{p},t) - \delta\mu) - \psi(\mathbf{r},\mathbf{p},t)\frac{\partial f_0(\varepsilon_0)}{\partial\varepsilon_0}, \quad (3)$$

where $f_0(\varepsilon_0(\mathbf{p}))$ is the Fermi function, $\delta\mu = u_{ik}\langle\lambda_{ik}\rangle/\langle 1\rangle$ is the variation of the chemical potential determined from the condition of conservation of electron density, and the angle brackets

$$\langle\ldots\rangle = \int \frac{2d^3p}{(2\pi\hbar)^3}\left(-\frac{\partial f_0}{\partial\varepsilon_0}\right)\ldots$$

denotes averaging over the Fermi surface.

The equation of motion of the magnetic moment density has the form[10,14]

$$\frac{d\mathbf{M}(\mathbf{r},t)}{dt} = g[\mathbf{M}(\mathbf{r},t)\times\mathbf{H}^{\mathrm{eff}}] + \mathbf{R}, \tag{4}$$

where $g = -\gamma 2\mu_B/\hbar$, $\mu_B = |e|\hbar/2mc$ is the Bohr magneton, and γ is the gyromagnetic ratio of the ferromagnet. The effective magnetic field $\mathbf{H}^{\mathrm{eff}}(\mathbf{r},t)$ is written as a functional derivative of the energy of the ferromagnet with respect to $\mathbf{M}(\mathbf{r},t)$:

$$\mathbf{H}^{\mathrm{eff}}(\mathbf{r},t) = -\frac{\delta W}{\delta\mathbf{M}(\mathbf{r},t)}, \quad W = \int w_\Sigma d^3r. \tag{5}$$

The relaxation term $\mathbf{R}$, according to Ref. 10, can be written in the form

$$\mathbf{R} = \frac{1}{\tau_2}\left(\mathbf{H}^{\mathrm{eff}} + \frac{1}{2g}\,\mathrm{curl}\;\dot{\mathbf{u}}\right) - \frac{1}{\tau_1}\left\{\mathbf{m}\times\left[\mathbf{m}\times\left(\mathbf{H}^{\mathrm{eff}} + \frac{1}{2g}\,\mathrm{curl}\;\dot{\mathbf{u}}\right)\right]\right\}, \tag{6}$$

where $\mathbf{m} = \mathbf{M}/M$, and τ_1 and τ_2 are the temperature-dependent relaxation time of the direction and magnitude of the magnetic moment.

It follows from expressions (1) and (5) that

$$H_i^{\mathrm{eff}} = H_i^{(\mathrm{in})} - \frac{\partial w_a(\mathbf{M})}{\partial M_i} - 2M_i w_e'(\mathbf{M}^2) + \alpha_{lk}\frac{\partial^2 M_i}{\partial x_l \partial x_k} - u_{lk}\frac{\partial f_{lk}(\mathbf{M})}{\partial M_i}, \tag{7}$$

$\mathbf{H}^{(\mathrm{in})} = \mathbf{H}^{(\mathrm{ext})} + \mathbf{H}^{(m)}$ is the field inside the ferromagnet.

The equilibrium state corresponds to a minimum of the energy of the ferromagnet, and therefore the equation $\mathbf{H}^{\mathrm{eff}}(\mathbf{r}) = -\delta W/\delta\mathbf{M}(\mathbf{r}) = 0$ together with the equations of magnetostatics determines the equilibrium values of the magnetization $\mathbf{M}(\mathbf{r})$ and field $\mathbf{H}^{(m)}(\mathbf{r})$. We shall ignore effects deriving from the existence of domain structure, assuming that the equilibrium magnetization is uniform and is found from the equation

$$H_i^{(\mathrm{in})} - \frac{\partial w_a(\mathbf{M})}{\partial M_i} - 2M_i w_e'(\mathbf{M}^2) = 0. \tag{8}$$

Equation (4) must be supplemented by the equation of motion of the elastic medium[10,12]

$$\rho\ddot{u}_i = \eta_{iklm}\frac{\partial u_{lm}}{\partial x_k} + \frac{\partial f_{ik}(\mathbf{M})}{\partial M_l}\frac{\partial M_l}{\partial x_k} + \mathbf{M}\cdot\frac{\partial\mathbf{H}^{\mathrm{eff}}}{\partial x_i} + \frac{1}{2g}(\mathrm{curl}\;\mathbf{R})_i + \frac{1}{c}[\mathbf{j}\times\mathbf{B}]_i - \frac{m}{e}\frac{\partial j_i}{\partial t} + \frac{\partial}{\partial x_k}\langle\Lambda_{ik}\psi\rangle \tag{9}$$

and Maxwell's equations

$$\mathrm{curl}\;\mathbf{H}^{(\mathrm{in})} = \frac{4\pi}{c}\mathbf{j}, \quad \mathrm{curl}\;\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$
$$\mathrm{div}\,\mathbf{B} = 0, \quad \mathrm{div}\,\mathbf{j} = 0. \tag{10}$$

Here

$$\mathbf{j} = e\langle\mathbf{v}\psi\rangle \tag{11}$$

is the current density, $\mathbf{B} = \mathbf{H}^{(\mathrm{in})} + 4\pi\mathbf{M}$ is the magnetic induction, $\Lambda_{ik} = \lambda_{ik} - \langle\lambda_{ik}\rangle/\langle 1\rangle$, and $\mathbf{v} = \partial\varepsilon_0/\partial\mathbf{p}$.

The system of equations of the problem is closed by the kinetic equation in the τ approximation for the equilibrium component of the distribution function:

$$\frac{\partial\psi}{\partial t} + \mathbf{v}\frac{\partial\psi}{\partial\mathbf{r}} + \frac{e}{c}[\mathbf{v}\times\mathbf{B}]\frac{\partial\psi}{\partial\mathbf{p}} + \frac{\psi}{\tau_e} = e\mathbf{v}\tilde{\mathbf{E}} - \Lambda_{ik}\dot{u}_k, \tag{12}$$

where $\tilde{\mathbf{E}} = \mathbf{E} + 1/c\,[\dot{\mathbf{u}}\times\mathbf{B}] - (m/e)\ddot{\mathbf{u}} - \nabla\delta\mu/e$ is the effective electric field.

In the local limit we can easily obtain from Eqs. (11) and (12) an expression for the current density:

$$j_l = \tau_e e^2\langle v_l v_k\rangle\tilde{E}_k - \tau e\langle v_l\Lambda_{ik}\rangle\dot{u}_{ik} \equiv \sigma_{lk}\tilde{E}_k + \gamma_{lik}\dot{u}_{ik}, \tag{13}$$

which is a sum of the electronic current and the deformation current due to the lattice displacement.

The system of equations (4), (7), (9)–(12) together with the boundary conditions determines the electromagnetic and acoustic fields arising in an elastically strained ferromagnet.

Let us consider a ferromagnet with a cubic crystal lattice in a magnetic field $\mathbf{H}^{(\mathrm{ext})} = (0,0,H^{(\mathrm{ext})})$ occupying the half space $z>0$. In the case when the x, y, and z axes are directed along the edges of a cube, the anisotropy energy can be written in the form

$$w_a(\mathbf{M}) = -\frac{1}{2}\beta(M_x^4 + M_y^4 + M_z^4) \tag{14}$$

(we have neglected the influence of the magnetostrictive strains on the crystal structure). For $\beta>0$ the crystal has three equivalent directions of easy magnetization along the x, y, and z axes, and the equilibrium magnetization $\mathbf{M}_0$ will be parallel to the vector $\mathbf{H}^{(\mathrm{ext})}$. We set

$$\mathbf{M}(\mathbf{r},t) = \mathbf{M}_0 + \mathbf{M}^{\sim}(\mathbf{r},t), \quad \mathbf{H}^{(\mathrm{in})}(\mathbf{r},t) = \mathbf{H}_0 + \mathbf{h}(\mathbf{r},t), \tag{15}$$

where $\mathbf{M}^{\sim}(\mathbf{r},t)$ and $\mathbf{h}(\mathbf{r},t)$ are small deviations from the equilibrium values. Noting that for a cubic crystal $\alpha_{ik} = \alpha\delta_{ik}$, one can easily obtain from formulas (7) and (8) a linearized expression for the effective magnetic field:

$$H_i^{\mathrm{eff}} = h_i + \alpha\frac{\partial^2 M_i^{\sim}}{\partial x_k^2} - \left(\frac{H_0}{M_0} + 2\beta\mathbf{M}_0^2\right)M_i^{\sim} - 4M_{0_i}w_e''(\mathbf{M}_0^2) \times(\mathbf{M}^{\sim}\mathbf{M}_0) - u_{lk}\frac{\partial f_{lk}(\mathbf{M}_0)}{\partial M_{0_i}}. \tag{16}$$

Under experimental conditions the elastic wave usually propagates along the normal $\mathbf{n}$ to the surface, and the displacement at the boundary is assumed to be specified:

$$\mathbf{u}(0,t)=\mathbf{u}_0 e^{-i\psi t}, \quad \mathbf{u}_0 \perp \mathbf{n}. \tag{17}$$

The character of the processes occurring over a sufficiently extended time interval is determined by the boundary regime, since the influence of the initial conditions is weakened because of the dissipation inherent to all real systems. For steady-state oscillations the time dependence of all the alternating quantities has the form $e^{-i\omega t}$.

Assuming that the elastic and magnetostriction properties of the ferromagnet are isotropic, we use the following expressions[10,15] for the tensors η_{iklm} and f_{ik}:

$$\eta_{iklm}=\rho(s_l^2-2s_t^2)\delta_{ik}\delta_{lm}+\rho s_t^2(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}),$$

$$f_{ik}=f_1 M_i M_k + f_2 M^2 \delta_{ik}, \tag{18}$$

where s_t and s_l are the velocities of transverse and longitudinal sound, and f_1 and f_2 are the magnetostriction constants. Neglecting effects due to anisotropy of the charge carrier dispersion relation, we can write the conductivity tensor σ_{ik} and the renormalized deformation potential Λ_{ik} in the form

$$\sigma_{ik}=\tau e^2\langle v_l v_k\rangle=\sigma\delta_{ik},$$

$$\Lambda_{ik}=\Lambda(\varepsilon_0)\left(v_i v_k-\frac{1}{3}v^2\delta_{ik}\right), \tag{19}$$

where $\Lambda(\varepsilon_0)$ depends only on the electron energy. Under these conditions $\gamma_{lik}=0$, and the strain contribution to the current is equal to zero, while the circular components $u_+=u_x+iu_y$, $h_+=h_x+ih_y$ of the vectors **u**, **h**, **E**, and $\mathbf{M}^\sim$ satisfy the equations

$$\left[\overline{\psi}_0\left(\frac{H_0}{M_0}+2\beta\mathbf{M}_0^2\right)-\omega\right]M_+^\sim-\overline{\omega}_0\alpha\frac{\partial^2 M_+^\sim}{\partial z^2}$$

$$=\overline{\omega}_0 h_+-\left(f_1\overline{\omega}_0 M_0+\frac{i\omega}{2g\tau}\right)\frac{\partial u_+}{\partial z}, \tag{20}$$

$$-\omega^2 u_+-s_t^2\frac{\partial^2 u_+}{\partial z^2}=\frac{f_1 M_0}{\rho}\frac{\partial M_+^\sim}{\partial z}+\frac{B_0}{4\pi\rho}\left(1+\frac{\omega}{\omega_B}\right)\frac{\partial h_+}{\partial z}$$

$$+\frac{i}{2\rho g}\frac{\partial R_+}{\partial z}, \tag{21}$$

$$-\frac{\partial^2 h_+}{\partial z^2}=i\frac{4\pi\sigma\omega}{c^2}(h_++4\pi M_+^\sim)$$

$$-i\frac{4\pi\sigma\omega}{c^2}B_0\left(1+\frac{\omega}{\omega_B}\right)\frac{\partial u_+}{\partial z}, \tag{22}$$

$$E_+=-i\frac{\omega}{ck}B_+^\sim, \quad B_+^\sim=h_++4\pi M_+^\sim. \tag{23}$$

Here $\overline{\omega}_0=\omega_0-i\tau^{-1}$, $\omega_0=gM_0$, $\tau^{-1}=\tau_2^{-1}+\tau_1^{-1}$, $B_0=H_0+4\pi M_0$, and $\omega_B=|e|B_0/mc$.

The boundary conditions for the electric and magnetic fields satisfying system (20)–(23) reduce to continuity of the components h_+ and E_+ at the boundary of the ferromagnet with free space. Continuity of the normal component of the energy flux density at the surface of the ferromagnet and formula (1) imply the following boundary condition for the magnetization:

$$\frac{\partial M_+^\sim(0)}{\partial z}=0. \tag{24}$$

In the case when the inequality $\max(B_0^2/8\pi\rho s_t^2, f_1 M_0^2/\rho s_t^2)\ll 1$ holds, the nondissipative terms on the right-hand side of equation (21) constitute a small correction. Taking them into account leads to renormalization of the sound velocity and rotation of the plane of polarization of the vector **u**. If over the time of passage of the wave through the sample the angle of rotation of the amplitude of the displacement vector is small, then in neglect of dissipative effects the acoustic field can be assumed equal to the external field

$$u_+(z,t)=u_{0_+}e^{-i\omega t+iqz}, \quad q=\frac{\omega}{s_t}, \tag{25}$$

and Eqs. (20), (22) and (23) can be regarded as independent of Eq. (21).

The solution of the inhomogeneous system of differential equations (20), (22), (23) should be sought in the form of a sum of the solution of the corresponding homogeneous system and an induced solution describing the field induced by the sound wave. The induced solution of equations (20) and (22) is a plane wave, $M_+^\sim(z,q)=M_+^\sim q e^{iqz}$, $h_+(z,q)=h_+(q)e^{iqz}$ with wave number q and amplitudes

$$h_+(q)=\frac{qu_{0_+}}{D}\left[\overline{B}_0+\frac{4\pi\left(f_1\overline{\omega}_0 M_0+\frac{i\omega}{2g\tau}\right)}{d_1}\right], \tag{26}$$

$$M_+^\sim(q)=\frac{qu_{0_+}}{d_1}\left[\frac{\overline{\omega}_0}{D}\overline{B}_0-i\left(f_1\overline{\omega}_0 M_0+\frac{i\omega}{2g\tau}\right)\right.$$

$$\left.\times\left(1+i\frac{4\pi\overline{\omega}_0}{d_1 D}\right)\right], \tag{27}$$

where

$$d_1\equiv d_1(\omega,q)=\overline{\omega}_0\left(\frac{H_0}{M_0}+2\beta\mathbf{M}_0^2+\alpha q^2\right)-\omega,$$

$$D=(q\delta)^2-i\left(1+\frac{4\pi\overline{\omega}_0}{d_1}\right),$$

$\delta=c/\sqrt{4\pi\sigma\omega}$ is the depth of the skin layer, $\overline{B}_0=B_0(1+\omega/\omega_B)$.

From Eqs. (23) it is easy to find the ac electric field

$$E_+(z,q)=E_+(q)e^{iqz},$$

$$E_+(q)=-i\frac{\omega}{c}u_{0_+}\left\{\frac{\overline{B}_0}{D}\left(1+\frac{4\pi\overline{\omega}_0}{d_1}\right)\right.$$

$$-i\frac{4\pi\left(f_1\overline{\omega}_0 M_0+\frac{i\omega}{2g\tau}\right)}{d_1}$$

$$\left.\times\left[1+\frac{i}{D}\left(1+\frac{4\pi\overline{\omega}_0}{d_1}\right)\right]\right\}, \tag{28}$$

induced by the sound wave. In the limit $q\delta\to 0$ we obtain $E_+^{(0)}(q)=u_+\overline{B}_0\omega/c$, and the effective field

$$\tilde{E}_+ = E_+ - \frac{\omega}{c} u_+ \overline{B}_0 \tag{30}$$

goes to zero. The conduction current and the field $\tilde{\mathbf{E}}$ is produced by the following term in the expansion of (28) in powers of $(q\delta)^2$:

$$\tilde{E}_+(q) = -i(q\delta)^2 \frac{\omega}{c} u_{0_+} \frac{\overline{B}_0 d_1 + 4\pi\left(f_1 \overline{\omega}_0 M_0 + \frac{i\omega}{2g\tau}\right)}{d_1 + 4\pi\overline{\omega}_0}. \tag{31}$$

Separating Eq. (31) into real and imaginary parts,

$$\tilde{E}_+(q) = -\frac{i(q\delta)^2 \omega u_{0_+}}{c[\Omega^2 + (\zeta+4\pi)^2\tau^{-2}]} \times \Bigg\{ \Omega[\overline{B}_0(\omega_0\zeta - \omega) + 4\pi f_1 \omega_0 M_0] + (\zeta+4\pi)\tau^{-2}\left(\overline{B}_0\zeta + 4\pi f_1 M_0 - \frac{2\pi\omega}{g}\right) - i4\pi\omega\tau^{-1}\left[\overline{B}_0 - \left(f_1 + \frac{1}{2}(\zeta+4\pi)\right)M_0 + \frac{\omega}{2g}\right]\Bigg\}, \tag{32}$$

we find the phase difference of the electromagnetic and sound waves:

$$\varphi = -\frac{\pi}{2} - \arctan \frac{4\pi\omega\tau^{-1}\left[\overline{B}_0 - \left(f_1 + \frac{1}{2}(\zeta+4\pi)\right)M_0 + \frac{\omega}{2g}\right]}{\Omega[\overline{B}_0(\omega_0\zeta-\omega) + 4\pi f_1\omega_0 M_0] + (\zeta+4\pi)\tau^{-2}\left(\overline{B}_0\zeta + 4\pi f_1 M_0 - \frac{2\pi\omega}{g}\right)}. \tag{33}$$

Here

$$\Omega = \omega_0(\zeta+4\pi) - \omega = \mathrm{Re}(d_1 + 4\pi),$$

$$\zeta = \frac{H_0}{M_0} + 2\beta\mathbf{M}_0^2 + \alpha q^2.$$

For the asymptotic representation of the magnetic field and magnetization for $(q\delta)^2 \ll 1$ it is sufficient to consider only the zeroth approximation in the small parameter $(q\delta)^2$:

$$h_+^{(0)}(q) = iqu_{0_+} \frac{\overline{B}_0 d_1 + 4\pi\left(f_1\overline{\omega}_0 M_0 + \frac{i\omega}{2g\tau}\right)}{d_1 + 4\pi\overline{\omega}_0}, \tag{34}$$

$$M_+^{(0)\sim}(q) = iqu_{0+} \frac{\overline{B}_0\overline{\omega}_0 - \left(f_1\overline{\omega}_0 M_0 + \frac{i\omega}{2g\tau}\right)}{d_1 + 4\pi\overline{\omega}_0}. \tag{35}$$

The solutions of the homogeneous system of differential equations corresponding to (20), (22), and (23) describe modes which are damped at distances of the order of the skin depth. Assuming that the spatial dependence of all the variable quantities is of the form e^{ikz}, we obtain from (20), (22), and (23) the dispersion relation giving the wave number as a function of frequency:

$$D(\omega,k) = (\delta k)^2 - i\left(1 + \frac{4\pi\overline{\omega}_0}{d_1(\omega,k)}\right) = 0. \tag{36}$$

In the case $\alpha\delta^2 \ll H_0/M_0 + 2\beta\mathbf{M}_0^2$ equation (36) takes the form

$$k^2(\omega) = \frac{i}{\delta^2}\left(1 + 4\pi\frac{(\omega_0^2+\tau^{-2})\zeta - \omega_0\omega + i\omega\tau^{-1}}{(\omega_0\xi-\omega)^2 + \xi^2\tau^{-2}}\right), \tag{37}$$

where $\xi = H_0/M_0 + 2\beta\mathbf{M}_0^2$. From the two roots it is necessary to choose the solution for which the mode is damped at $z\to\infty$.

Applying the boundary condition (24) and using Maxwell's equations, we can easily express the amplitudes of the skin solutions,

$$M_+^s(k,q) = -\frac{q}{k} M_+^{\sim}(q),$$

$$h_+^s(k,q) = -\frac{q}{k}\frac{d_1(\omega,k)}{\overline{\omega}_0} M_+^{\sim}(q), \tag{38}$$

$$E_+^s(k,q) = i\frac{\omega q}{k^2 c}\frac{d_1(\omega,k) + 4\pi\overline{\omega}_0}{\overline{\omega}_0} M_+^{\sim}(q)$$

in terms of the amplitude of the displacement at the boundary of the ferromagnet. Here $M_+^{\sim}(q)$ is determined by formula (27). It follows from expressions (38) for $q = \omega/s_t \ll |k| \sim \delta^{-1}$ that the amplitudes of the skin-effect modes are small compared to the amplitudes of the forced oscillations.

The electromagnetic waves radiated by the sound wave (25) in a ferromagnetic insulator are determined by the equation of motion of the magnetization (20) and the wave equation, which for a ferromagnetic insulator with dielectric constant ε becomes

$$-\frac{\partial^2 h_+}{\partial z^2} = \varepsilon\frac{\omega^2}{c^2}(h_+ + 4\pi M_+^{\sim}). \tag{39}$$

It follows from this equation that the magnetic field excited by a sound wave with wave number $q = \omega/s_t$ is significantly less than the ac magnetization:

$$h_+ \approx 4\pi\varepsilon\frac{s_t^2}{c^2} M_+^{\sim} \ll M_+^{\sim} = -i\frac{q}{d_1}\left(f_1\overline{\omega}_0 M_0 + \frac{i\omega}{2g\tau}\right)u_+. \tag{40}$$

The electric field induced by the sound wave in a ferromagnetic insulator and the phase difference of the electromagnetic and sound waves are determined by the expressions

$$E_+^{\sim}=4\pi\frac{\omega}{kc}M_+^{\sim}=-i\frac{4\pi\omega}{d_1c}\left(f_1\overline{\omega_0}M_0+\frac{i\omega}{2g\tau}\right)u_+, \qquad (41)$$

$$\varphi=-\frac{\pi}{2}-\arctan\frac{\tau^{-1}\omega\left[-f_1M_0+\frac{1}{2g}(\omega-\zeta\omega_0)\right]}{f_1M_0[\zeta(\tau^{-2}+\omega_0^2)-\omega\omega_0]-\frac{1}{2g}\zeta\tau^{-2}\omega}. \qquad (42)$$

From the homogeneous system of equations of the problem it is easy to obtain the dispersion relation that determines the spectrum of free oscillations of the electromagnetic field:

$$k^2=\varepsilon\frac{\omega^2}{c^2}\left(1+\frac{4\pi\overline{\omega_0}}{d_1(\omega,k)}\right). \qquad (43)$$

It follows from the boundary condition (24) that the amplitudes of the free oscillations of the electromagnetic field and magnetization M_+^f in a ferromagnetic insulator substantially exceed the amplitude of the induced oscillations: $M_+^f \simeq (c/\varepsilon s_t)M_+^{\sim}$.

The electric fields produced by a sound wave in a substance having magnetic order depends substantially on the magnetostriction constants and magnetization relaxation times. In the case when the relaxation times of the magnitude and direction of the magnetization are large, $\tau^{-1}\equiv\tau_1^{-1}+\tau_2^{-1}<\omega|\omega_0|$, the phase difference $\Delta\varphi$ of the electromagnetic and sound waves tends toward $-\pi/2$. The deviation $\Delta\varphi$ from $-\pi/2$ is maximum for frequencies of the order of $\omega\sim|\omega_0|\sim\tau^{-1}$.

The authors thank V. D. Fil' for calling their attention to the importance of the problem. This study was supported by the CRDF program (Grant No. UPI-2566-KH-03).

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Translated by Steve Torstveit