

## Josephson effect in point contacts between “*f*-wave” superconductors

R. Mahmoodi

*Institute for Advanced Studies in Basic Sciences, 45195-159, Gava Zang, Zanjan, Iran*

S. N. Shevchenko

*B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Lenin Ave., 61103 Kharkov, Ukraine*

Yu. A. Kolesnichenko\*

*Institute for Advanced Studies in Basic Sciences, 45195-159, Gava Zang, Zanjan, Iran and B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 47 Lenin Ave., 61103 Kharkov, Ukraine*

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A stationary Josephson effect in point contacts between triplet superconductors is analyzed theoretically for the most-probable models of the order parameter in  $\text{UPt}_3$  and  $\text{Sr}_2\text{RuO}_4$ . The consequence of misorientation of the crystals in the superconducting banks on this effect is considered. We show that different models for the order parameter lead to quite different current-phase relations. For certain angles of misorientation a boundary between superconductors can generate a spontaneous current parallel to the surface. In a number of cases the state with a zero Josephson current and minimum of the free energy corresponds to a spontaneous phase difference. This phase difference depends on the misorientation angle and may possess any value. We conclude that experimental investigations of the current-phase relations of small junctions can be used for determination of the order parameter symmetry in the superconductors mentioned above. © 2002 American Institute of Physics. [DOI: 10.1063/1.1468521]

### 1. INTRODUCTION

Triplet superconductivity, which is an analog of superfluidity in  $^3\text{He}$ , was first discovered in the heavy-fermion compound  $\text{UPt}_3$  more than ten years ago.<sup>1,2</sup> Recently, a novel triplet superconductor  $\text{Sr}_2\text{RuO}_4$  was found.<sup>3,4</sup> In these compounds, the triplet pairing can be reliably determined, for example, by Knight shift experiments,<sup>5,6</sup> but the identification of the symmetry of the order parameter is a much more difficult task. A large number of experimental and theoretical investigations done on  $\text{UPt}_3$  and  $\text{Sr}_2\text{RuO}_4$  are concerned with different thermodynamic and transport properties, but the precise order-parameter symmetry still remains to be worked out (see, for example, Refs. 7, 10–12, and original references therein).

Calculations of the order parameter  $\hat{\Delta}(\hat{\mathbf{k}})$  in  $\text{UPt}_3$  and  $\text{Sr}_2\text{RuO}_4$  as a function of the momentum direction  $\hat{\mathbf{k}}$  on the Fermi surface is a very complex problem. Some general information about  $\hat{\Delta}(\hat{\mathbf{k}})$  can be obtained from the symmetry of the normal state:  $G_{\text{spin-orbit}} \times \tau \times U(1)$ , where  $G_{\text{spin-orbit}}$  represents the point group with inversion,  $\tau$  is the time-inversion operator, and  $U(1)$  is a gauge transformation group. A superconducting state breaks one or more symmetries. In particular, a transition to the superconducting state implies the appearance of a phase coherence corresponding to breaking of the gauge symmetry. According to the Landau theory<sup>13</sup> of second-order phase transitions, the order parameter transforms only according to irreducible representations of the symmetry group of the normal state. Conventional superconducting states have the total point symmetry of the crystal

and belong to the even unitary representation  $A_{1g}$ . In conventional superconductors this symmetry is broken. The parity of a superconductor with inversion symmetry can be specified using the Pauli principle. Because for triplet pairing the spin part of  $\hat{\Delta}$  is a symmetric second-rank spinor, the orbital part has to belong to an odd representation. In the general case the triplet pairing is described by an order parameter of the form  $\hat{\Delta}(\hat{\mathbf{k}}) = i\mathbf{d}(\hat{\mathbf{k}})\hat{\sigma}\hat{\sigma}_2$ , where the vector  $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ , and  $\hat{\sigma}_i$  are Pauli matrices in the spin space. A vector  $\mathbf{d}(\hat{\mathbf{k}}) = -\mathbf{d}(-\hat{\mathbf{k}})$  in spin space is frequently referred to as an order parameter or a gap vector of the triplet superconductor. This vector defines the axis along which the Cooper pairs have zero spin projection. If  $\mathbf{d}$  is complex, the spin components of the order parameter spontaneously break time-reversal symmetry.

Symmetry considerations reserve for the order parameter considerable freedom in the selection of irreducible representation and its basis functions. Therefore in many papers (see, for example, Refs. 7, 10–12, 14–16) authors consider different models (so-called scenarios) of superconductivity in  $\text{UPt}_3$  and  $\text{Sr}_2\text{RuO}_4$ , which are based on possible representations of crystallographic point groups. The subsequent comparison of theoretical results with experimental data makes it possible to draw conclusions about the symmetry of the order parameter.

In real crystalline superconductors there is no classification of Cooper pairing by angular momentum (*s*-wave, *p*-wave, *d*-wave, *f*-wave pairing, etc.). However, these terms are often used for unconventional superconductors in the

meaning that the point symmetry of the order parameter is the same as that for the corresponding representation of the  $SO_3$  symmetry group of an isotropic conductor. In this terminology conventional superconductors can be referred to as  $s$ -wave. For example, “ $p$ -wave” pairing corresponds to the odd two-dimensional representation  $E_{1u}$  of the point group  $D_{6h}$  or the  $E_u$  representation of the point group  $D_{4h}$ . The order parameter for these representations has the same symmetry as for the superconducting state with angular momentum  $l=1$  of Cooper pairs in an isotropic conductor. If the symmetry of  $\hat{\Delta}$  cannot be formally related to any irreducible representation of the  $SO_3$  group, these states are usually referred to as hybrid states.

Apparently, in crystalline triplet superconductors the order parameter has a more complex dependence on  $\hat{\mathbf{k}}$  in comparison with the well-known  $p$ -wave order parameter for superfluid phases of  $^3\text{He}$ . The heavy-fermion superconductor  $\text{UPt}_3$  belongs to the hexagonal crystallographic point group ( $D_{6h}$ ), and it is most likely that the pairing state belongs to the  $E_{2u}$  (“ $f$ -wave” state) representation. The layered perovskite material  $\text{Sr}_2\text{RuO}_4$  belongs to the tetragonal crystallographic point group ( $D_{4h}$ ). Initially the simplest “ $p$ -wave” model based on the  $E_u$  representation was proposed for the superconducting state in this compound.<sup>8,9</sup> However, this model was inconsistent with available experimental data, and later<sup>10,11</sup> other “ $f$ -wave” models of the pairing state were proposed.

Theoretical studies of the specific heat, thermal conductivity, and ultrasound absorption for different models of triplet superconductivity show considerable quantitative differences between calculated dependences.<sup>7,10,11,16</sup> The Josephson effect is much more sensitive to dependence of  $\hat{\Delta}$  on the momentum direction on the Fermi surface. One of the possibilities for forming a Josephson junction is to create a point contact between two massive superconductors. A microscopic theory of the stationary Josephson effect in ballistic point contacts between conventional superconductors was developed in Ref. 17. Later this theory was generalized for a pinhole model in  $^3\text{He}$  (Refs. 18 and 19) and for point contacts between “ $d$ -wave” high- $T_c$  superconductors.<sup>20,21</sup> It was shown that current-phase relations for the Josephson current in such systems are quite different from those of conventional superconductors, and states with a spontaneous phase difference become possible. Theoretical and experimental investigations of this effect in novel triplet superconductors seem to be interesting and enable one to distinguish among different candidates for the superconducting state.

In Ref. 22 the authors study the interfacial Andreev bound states and their influence on the Josephson current between clean “ $f$ -wave” superconductors both self-consistently (numerically) and non-self-consistently (analytically). The temperature dependence of the critical current is presented. However, in that paper there is no detailed analysis of the current-phase relations for different orientations of the crystals in the superconducting banks.

In this paper we theoretically investigate the stationary Josephson effect in a small ballistic junction between two bulk triplet superconductors with different orientations of the crystallographic axes with respect to the junction normal. In Sec. 2 we describe our model of the junction and present the

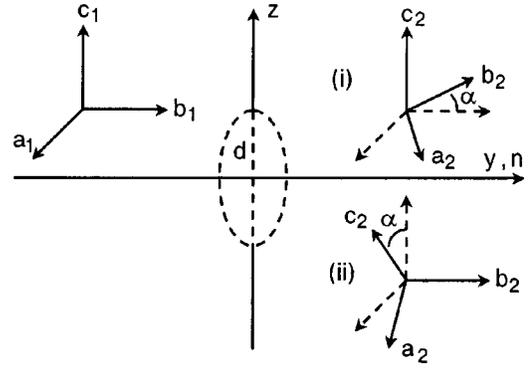


FIG. 1. Scheme of a contact in the form of an orifice between two superconducting banks, which are misoriented by an angle  $\alpha$ .

full set of equations. In Sec. 3 the current density in the junction plane is calculated analytically for a non-self-consistent model of the order parameter. In Sec. 4 the current-phase relations for the most-likely models of “ $f$ -wave” superconductivity in  $\text{UPt}_3$  and  $\text{Sr}_2\text{RuO}_4$  are analyzed for different mutual orientations of the banks. We end in Sec. 5 with some conclusions.

## 2. MODEL OF THE CONTACT AND FORMULATION OF THE PROBLEM

We consider a model of a ballistic point contact as an orifice of diameter  $d$  in a partition impenetrable to electrons, between two superconducting half spaces (Fig. 1). We assume that the contact diameter  $d$  is much larger than the Fermi wavelength and use the quasiclassical approach. In order to calculate the stationary Josephson current in point contact we use “transport-like” equations for  $\xi$ -integrated Green functions  $\check{g}(\hat{\mathbf{k}}, \mathbf{r}, \varepsilon_m)$  (Ref. 23)

$$[i\varepsilon_m \check{\tau}_3 - \check{\Delta}, \check{g}] + i v_F \hat{\mathbf{k}} \nabla \check{g} = 0, \quad (1)$$

and the normalization condition

$$\check{g} \check{g} = -1. \quad (2)$$

Here  $\varepsilon_m = \pi T(2m+1)$  are discrete Matsubara energies,  $v_F$  is the Fermi velocity,  $\hat{\mathbf{k}}$  is a unit vector along the electron velocity, and  $\check{\tau}_3 = \hat{\tau}_3 \otimes \hat{I}$ ;  $\hat{\tau}_i$  ( $i=1, 2, 3$ ) are Pauli matrices in a particle-hole space.

The Matsubara propagator  $\check{g}$  can be written in the form:<sup>24</sup>

$$\check{g} = \begin{pmatrix} g_1 + \mathbf{g}_1 \hat{\sigma} & (g_2 + \mathbf{g}_2 \hat{\sigma}_2) i \hat{\sigma}_2 \\ i \hat{\sigma}_2 (g_3 + \mathbf{g}_3 \hat{\sigma}) & g_4 - \hat{\sigma}_2 \mathbf{g}_4 \hat{\sigma} \hat{\sigma}_2 \end{pmatrix}; \quad (3)$$

as can be done for an arbitrary Nambu matrix. Matrix structure of the off-diagonal self-energy  $\check{\Delta}$  in Nambu space is

$$\check{\Delta} = \begin{pmatrix} 0 & i \mathbf{d} \hat{\sigma} \hat{\sigma}_2 \\ i \hat{\sigma}_2 \mathbf{d}^* \hat{\sigma} & 0 \end{pmatrix}. \quad (4)$$

Below we consider so-called unitary states, for which  $\mathbf{d} \times \mathbf{d}^* = 0$ .

The gap vector  $\mathbf{d}$  has to be determined from the self-consistency equation:

$$\mathbf{d}(\hat{\mathbf{k}}, \mathbf{r}) = \pi TN(0) \sum_m \langle V(\hat{\mathbf{k}}, \hat{\mathbf{k}}') \mathbf{g}_2(\hat{\mathbf{k}}', \mathbf{r}, \varepsilon_m) \rangle, \quad (5)$$

where  $V(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$  is a pairing interaction potential;  $\langle \dots \rangle$  stands for averaging over directions of an electron momentum on the Fermi surface;  $N(0)$  is the electron density of states.

Solutions of Eqs. (1), (5) must satisfy the conditions for the Green functions and vector  $\mathbf{d}$  in the banks of superconductors far from the orifice:

$$g(\mp\infty) = \frac{i\varepsilon_m \check{\tau}_3 - \check{\Delta}_{1,2}}{\sqrt{\varepsilon_m^2 + |\mathbf{d}_{1,2}|^2}}, \quad (6)$$

$$\mathbf{d}(\mp\infty) = \mathbf{d}_{1,2}(\hat{\mathbf{k}}) \exp\left(\mp \frac{i\phi}{2}\right), \quad (7)$$

where  $\phi$  is the external phase difference. Equations (1) and (5) have to be supplemented by the boundary continuity conditions at the contact plane and conditions of reflection at the interface between superconductors. Below we assume that this interface is smooth and that electron scattering is negligible.

### 3. CALCULATION OF THE CURRENT DENSITY

The solution of Eqs. (1) and (5) allows us to calculate the current density:

$$\mathbf{j}(\mathbf{r}) = 2\pi eTvFN(0) \sum_m \langle \hat{\mathbf{k}} g_1(\hat{\mathbf{k}}, \mathbf{r}, \varepsilon_m) \rangle. \quad (8)$$

We consider the simple model of a constant order parameter up to the surface. The pair breaking and the scattering on the partition and in the junction are ignored. This model can be rigorously found for calculations of the current density (8) in ballistic point contacts between conventional superconductors in the zero approximation in the small parameter  $d/\xi_0$  ( $\xi_0$  is the coherence length).<sup>17</sup> In anisotropically paired superconductors the order parameter changes at distances of the order of  $\xi_0$  even near a specular surface.<sup>25,26</sup> Thus for calculations of the current (8) in the leading approximation in the parameter  $d/\xi_0$  it is necessary to solve Eq. (5) near the surface of a semi-infinite superconductor. It can be done only numerically and will be the subject of our future investigations. Below we assume that the order parameter does not depend on coordinates and in each half space is equal to its value (7) far from the point contact. For this non-self-consistent model the current-phase relation of a Josephson junction can be calculated analytically. This makes it possible to analyze the main features of the current-phase relations for different scenarios of “*f*-wave” superconductivity. We believe that under this strong assumption our results describe the real situation qualitatively, as has been justified for point contacts between “*d*-wave” superconductors<sup>20</sup> and pinholes in <sup>3</sup>He.<sup>27</sup> It was also shown in Ref. 22 that for a contact between “*f*-wave” superconductors there is also good qualitative agreement between the self-consistent and non-self-consistent solutions (although, of course, quantitative distinctions are present).

In a ballistic case the system of 16 equations for functions  $g_i$  and  $\mathbf{g}_i$  can be decomposed into independent blocks of equations. The set of equations which enables us to find the Green function  $g_1$  is

$$iv_F \hat{\mathbf{k}} \nabla g_1 + (\mathbf{g}_3 \mathbf{d} - \mathbf{g}_2 \mathbf{d}^*) = 0; \quad (9)$$

$$iv_F \hat{\mathbf{k}} \nabla \mathbf{g}_- + 2i(\mathbf{d} \times \mathbf{g}_3 + \mathbf{d}^* \times \mathbf{g}_2) = 0; \quad (10)$$

$$iv_F \hat{\mathbf{k}} \nabla \mathbf{g}_3 - 2i\varepsilon_m \mathbf{g}_3 - 2g_1 \mathbf{d}^* - i\mathbf{d}^* \times \mathbf{g}_- = 0; \quad (11)$$

$$iv_F \hat{\mathbf{k}} \nabla \mathbf{g}_2 + 2i\varepsilon_m \mathbf{g}_2 + 2g_1 \mathbf{d} - i\mathbf{d} \times \mathbf{g}_- = 0; \quad (12)$$

where  $\mathbf{g}_- = \mathbf{g}_1 - \mathbf{g}_4$ . Equations (9)–(12) can be solved by integrating over ballistic trajectories of electrons in the right and left half spaces. The general solution satisfying the boundary conditions (6) at infinity is

$$g_1^{(n)} = \frac{i\varepsilon_m}{\Omega_n} + iC_n \exp(-2s\Omega_n t); \quad (13)$$

$$\mathbf{g}_-^{(n)} = \mathbf{C}_n \exp(-2s\Omega_n t); \quad (14)$$

$$\mathbf{g}_2^{(n)} = -\frac{2C_n \mathbf{d}_n - \mathbf{d}_n \times \mathbf{C}_n}{-2s\eta\Omega_n + 2\varepsilon_m} \exp(-2s\Omega_n t) - \frac{\mathbf{d}_n}{\Omega_n}; \quad (15)$$

$$\mathbf{g}_3^{(n)} = -\frac{2C_n \mathbf{d}_n^* + \mathbf{d}_n^* \times \mathbf{C}_n}{2s\eta\Omega_n + 2\varepsilon_m} \exp(-2s\Omega_n t) - \frac{\mathbf{d}_n^*}{\Omega_n}; \quad (16)$$

where  $t$  is the time of flight along the trajectory,  $\text{sgn}(t) = \text{sgn}(z) = s$ ;  $\eta = \text{sgn}(v_z)$ ;  $\Omega_n = \sqrt{\varepsilon_m^2 + |\mathbf{d}_n|^2}$ . By matching the solutions (13)–(16) at the orifice plane ( $t=0$ ), we find the constants  $C_n$  and  $\mathbf{C}_n$ . Index  $n$  numbers the left ( $n=1$ ) and right ( $n=2$ ) half spaces. The function  $g_1(0) = g_1^{(1)}(-0) = g_1^{(2)}(+0)$ , which determines the current density in the contact, is

$$g_1(0) = \frac{i\varepsilon_m(\Omega_1 + \Omega_2) \cos \zeta + \eta(\varepsilon_m^2 + \Omega_1 \Omega_2) \sin \zeta}{\vec{\Delta}_1 \vec{\Delta}_2 + (\varepsilon_m^2 + \Omega_1 \Omega_2) \cos \zeta - i\varepsilon_m \eta(\Omega_1 + \Omega_2) \sin \zeta}. \quad (17)$$

In formula (17) we have taken into account that for unitary states the vectors  $\mathbf{d}_{1,2}$  can be written as

$$\mathbf{d}_n = \vec{\Delta}_n \exp i\psi_n, \quad (18)$$

where  $\vec{\Delta}_{1,2}$  are real vectors.

Knowing the function  $g_1(0)$ , one can calculate the current density at the orifice plane  $\mathbf{j}(0)$ :

$$\mathbf{j}(0) = 4\pi eN(0)v_FT \sum_{m=0}^{\infty} \int d\hat{\mathbf{k}} \hat{\mathbf{k}} \text{Re } g_1(0), \quad (19)$$

where

$$\text{Re } g_1(0) = \frac{[\Delta_1^2 \Delta_2^2 \cos \zeta + (\varepsilon_m^2 + \Omega_1 \Omega_2) \vec{\Delta}_1 \vec{\Delta}_2] \sin \zeta}{[\vec{\Delta}_1 \vec{\Delta}_2 + (\varepsilon_m^2 + \Omega_1 \Omega_2) \cos \zeta]^2 + \varepsilon_m^2 (\Omega_1 + \Omega_2)^2 \sin^2 \zeta} \quad (20)$$

or, alternatively,

$$\text{Re } g_1(0) = \frac{\Delta_1 \Delta_2}{2} \sum_{\pm} \frac{\sin(\zeta \pm \theta)}{\varepsilon_m^2 + \Omega_1 \Omega_2 + \Delta_1 \Delta_2 \cos(\zeta \pm \theta)}, \quad (21)$$

where  $\theta$  is defined by  $\vec{\Delta}_1(\hat{\mathbf{k}}) \vec{\Delta}_2(\hat{\mathbf{k}}) = \Delta_1(\hat{\mathbf{k}}) \Delta_2(\hat{\mathbf{k}}) \cos \theta$ , and  $\zeta(\hat{\mathbf{k}}) = \psi_2(\hat{\mathbf{k}}) - \psi_1(\hat{\mathbf{k}}) + \phi$ .

Misorientation of the crystals would generally result in the appearance of current along the interface,<sup>20,22</sup> as can be calculated by projecting the vector  $\mathbf{j}$  on the corresponding direction.

We consider a rotation  $R$  only in the right-hand superconductor (see Fig. 1), (i.e.,  $\mathbf{d}_2(\hat{\mathbf{k}}) = R \mathbf{d}_1(R^{-1} \hat{\mathbf{k}})$ ). The  $c$  axis in the left half space is chosen along the partition between superconductors (along the  $z$  axis in Fig. 1). To illustrate results obtained by computing Eq. (19), we plot the current-phase relation for different below-mentioned scenarios of “ $f$ -wave” superconductivity for two different geometries corresponding to different orientations of the crystals to the right and to the left at the interface (see Fig. 1):

(i) The basal  $ab$  plane to the right is rotated about the  $c$  axis by an angle  $\alpha$ ;  $\hat{\mathbf{c}}_1 \parallel \hat{\mathbf{c}}_2$ .

(ii) The  $c$  axis to the right is rotated about the contact axis ( $y$  axis in Fig. 1) by an angle  $\alpha$ ;  $\hat{\mathbf{b}}_1 \parallel \hat{\mathbf{b}}_2$ .

Further calculations require a certain model of the vector order parameter  $\mathbf{d}$ .

#### 4. CURRENT-PHASE RELATION FOR DIFFERENT SCENARIOS OF “ $F$ -WAVE” SUPERCONDUCTIVITY

The model which has been successful to explain properties of the superconducting phases in  $\text{UPt}_3$  is based on the odd-parity  $E_{2u}$  representation of the hexagonal point group  $D_{6h}$  for strong spin-orbital coupling with vector  $\mathbf{d}$  locked along the  $\mathbf{c}$  axis of the lattice:<sup>10</sup>  $\mathbf{d} = \Delta_0 \hat{z} [\eta_1 Y_1 + \eta_2 Y_2]$ , where  $Y_1 = k_z(k_x^2 - k_y^2)$  and  $Y_2 = 2k_x k_y k_z$  are the basis function of the representation.<sup>1</sup> The coordinate axes  $x, y, z$  here and below are chosen along the crystallographic axes  $\hat{\mathbf{a}}, \hat{\mathbf{b}}, \hat{\mathbf{c}}$  as at the left in Fig. 1. This model describes the hexagonal analog of spin-triplet “ $f$ -wave” pairing. For the high-temperature  $A$ -phase ( $\eta_2 = 0$ ) the order parameter has an equatorial line node and two longitudinal line nodes. In the low-temperature  $B$  phase ( $\eta_2 = i$ ) or the axial state

$$\mathbf{d} = \Delta_0 \hat{z} k_z (k_x + i k_y)^2 \quad (22)$$

the longitudinal line nodes are closed and there is a pair of point nodes. The  $B$  phase (22) breaks the time-reversal symmetry. The function  $\Delta_0 = \Delta_0(T)$  in Eq. (22) and below describes the dependence of the order parameter  $\mathbf{d}$  on temperature  $T$  (in carrying out numerical calculations we assume  $T = 0$ ).

Other candidates for describing the orbital states, which imply that the effective spin-orbital coupling in  $\text{UPt}_3$  is weak, are the unitary planar state

$$\mathbf{d} = \Delta_0 k_z [\hat{x}(k_x^2 - k_y^2) + \hat{y} 2k_x k_y] \quad (23)$$

[or  $\mathbf{d} = \Delta_0(Y_1, Y_2, 0)$ ] and the non-unitary bipolar state  $\mathbf{d} = \Delta_0(Y_1, iY_2, 0)$ .<sup>7</sup> In Fig. 2 we plot the Josephson current-phase relation  $j_J(\phi) = j_J(y=0)$  calculated from Eq. (19) for both the axial [with the order parameter given by Eq. (22)] and the planar [Eq. (23)] states for a particular value of  $\alpha$

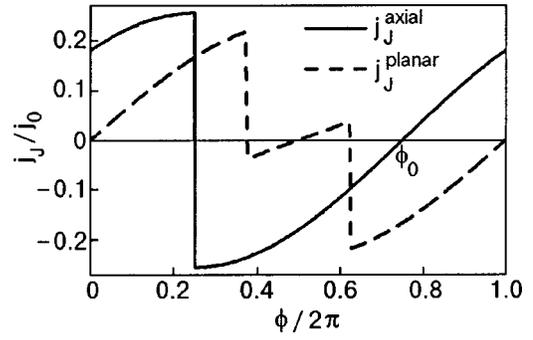


FIG. 2. Josephson current densities versus phase  $\phi$  for axial (22) and planar (23) states in the geometry (i); misorientation angle  $\alpha = \pi/4$ ; the current is given in units of  $j_0 = (\pi/2)eN(0)v_F \Delta_0(0)$ .

under the rotation of the basal  $ab$  plane to the right [the geometry (i)]. For simplicity we use a spherical model of the Fermi surface. For the axial state the current-phase relation is just a slanted sinusoid and for the planar state it shows a “ $\pi$  state.” The appearance of the  $\pi$  state at low temperatures is due to the fact that different quasiparticle trajectories contribute to the current with different effective phase differences  $\zeta(\hat{\mathbf{k}})$  [see Eqs. (19) and (21)].<sup>19</sup> Such a different behavior can be a criterion for distinguishing between the axial and the planar states, taking advantage of the phase-sensitive Josephson effect. Note that for the axial model the Josephson current formally does not equal zero at  $\phi = 0$ . This state is unstable (does not correspond to a minimum of the Josephson energy), and a state with a spontaneous phase difference (value  $\phi_0$  in Fig. 2), which depends on the misorientation angle  $\alpha$ , is realized.

The remarkable influence of the misorientation angle  $\alpha$  on the current-phase relation is shown in Fig. 3 for the axial state in the geometry (ii). For some values of  $\alpha$  (in Fig. 3 it is  $\alpha = \pi/3$ ) there are more than one state, which correspond to minima of the Josephson energy ( $j_J = 0$  and  $dj_J/d\phi > 0$ ).

The calculated  $x$  and  $z$  components (which are parallel to the surface) of the current  $\mathbf{j}_s(\phi)$  are shown in Fig. 4 for the same axial state in the geometry (ii). Note that the current tangential to the surface as a function of  $\phi$  is not zero when the Josephson current (Fig. 3) is zero. This spontaneous tangential current (see also Ref. 22) is due to a specific “proximity effect” similar to the spontaneous current in contacts between “ $d$ -wave” superconductors.<sup>20,28</sup> The total current is

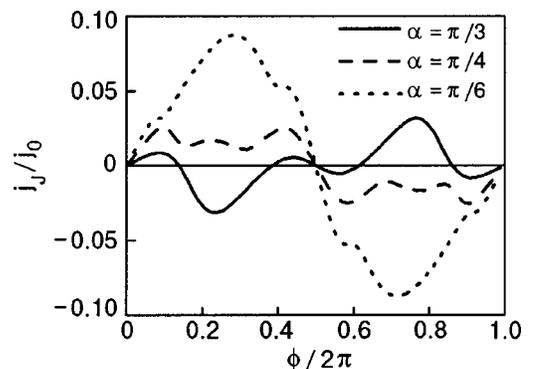


FIG. 3. Josephson current versus phase  $\phi$  for the axial (22) state in the geometry (ii) for different  $\alpha$ .

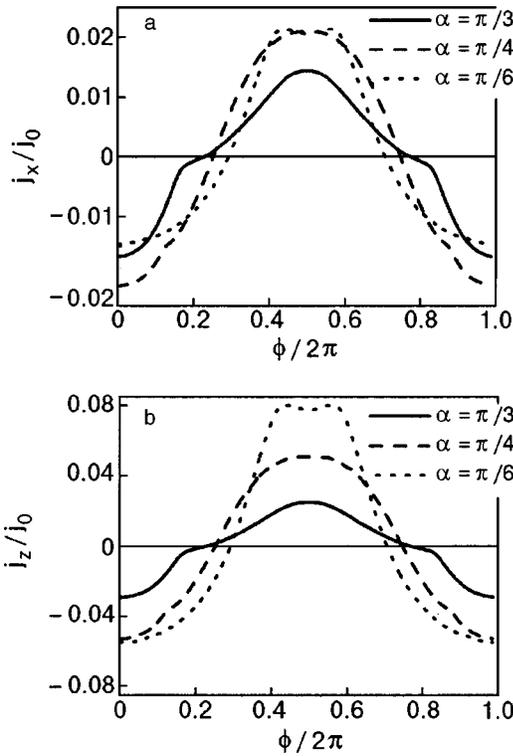


FIG. 4. The  $x$  (a) and  $z$  (b) components of the tangential current versus phase  $\phi$  for the axial state (22) in the geometry (ii) for different  $\alpha$ .

determined by the Green function, which depends on the order parameters in both superconductors. As a result of this, for nonzero misorientation angles a current parallel to the surface can be generated. In the geometry (i) the tangential current for both the axial and planar states at  $T=0$  is absent.

The first candidate for the superconducting state in  $\text{Sr}_2\text{RuO}_4$  was the “ $p$ -wave” model<sup>8</sup>

$$\mathbf{d} = \Delta_0 \hat{z} (\hat{k}_x + i\hat{k}_y). \quad (24)$$

Recently<sup>11,12</sup> it was shown that the pairing state in  $\text{Sr}_2\text{RuO}_4$  most likely has lines of nodes. It was suggested that this can occur if the spin-triplet state belongs to a nontrivial realization of the  $E_u$  representation of the group  $D_{4h}$ , with either  $B_{2g} \otimes E_u$  (Ref. 12) or  $B_{1g} \otimes E_u$  (Ref. 11) symmetry:

$$\mathbf{d} = \Delta_0 \hat{z} \hat{k}_x \hat{k}_y (\hat{k}_x + i\hat{k}_y), \quad \text{for } B_{2g} \otimes E_u \text{ symmetry}; \quad (25)$$

$$\mathbf{d} = \Delta_0 \hat{z} (\hat{k}_x^2 - \hat{k}_y^2) (\hat{k}_x + i\hat{k}_y), \quad \text{for } B_{1g} \otimes E_u \text{ symmetry}. \quad (26)$$

Note that models (24)–(26) of the order parameter spontaneously break time-reversal symmetry.

Taking into account a quasi-two-dimensional electron energy spectrum in  $\text{Sr}_2\text{RuO}_4$ , we calculate the current (19) numerically using the model of a cylindrical Fermi surface. The Josephson current for the hybrid “ $f$ -wave” model of the order parameter Eq. (26) is compared to the “ $p$ -wave” model (Eq. (24)) in Fig. 5 (for  $\alpha = \pi/4$ ). Note that the critical current for the “ $f$ -wave” model is several times smaller (for the same value of  $\Delta_0$ ) than for the “ $p$ -wave” model. This different character of the current-phase relations enables us to distinguish between the two states.

In Figs. 6 and 7 we present the Josephson current and the tangential current for the hybrid “ $f$ -wave” model for differ-

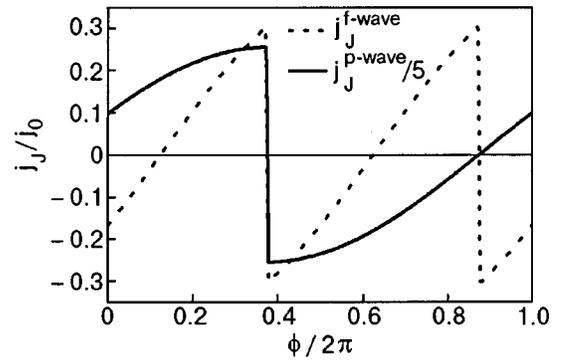


FIG. 5. Josephson current versus phase  $\phi$  for hybrid “ $f$ -wave” and “ $p$ -wave” states in the geometry (i);  $\alpha = \pi/4$ .

ent misorientation angles  $\alpha$  (for the “ $p$ -wave” model it equals zero). Just as in Fig. 2 for the “ $f$ -wave” order parameter (22), in Fig. 6 for the hybrid “ $f$ -wave” model (25) the steady state of the junction with zero Josephson current corresponds to a nonzero spontaneous phase difference if the misorientation angle  $\alpha \neq 0$ .

## CONCLUSION

We have considered the stationary Josephson effect in point contacts between triplet superconductors. Our analysis is based on models with “ $f$ -wave” symmetry of the order parameter belonging to the two-dimensional representations of the crystallographic symmetry groups. It is shown that the current-phase relations are quite different for different models of the order parameter. Because the order parameter phase depends on the momentum direction on the Fermi surface, misorientation of the superconductors leads to a spontaneous phase difference that corresponds to zero Josephson current and to the minimum of the weak-link energy. This phase difference depends on the misorientation angle and can possess any values. We have found that in contrast to the “ $p$ -wave” model, in the “ $f$ -wave” models the spontaneous current may be generated in a direction which is tangential to the orifice plane. Generally speaking this current is not equal to zero in the absence of the Josephson current. We demonstrate that the study of the current-phase relation of a small Josephson junction for different crystallographic orientations of the banks enables one to assess the applicability of different models to the triplet superconductors  $\text{UPt}_3$  and  $\text{Sr}_2\text{RuO}_4$ .

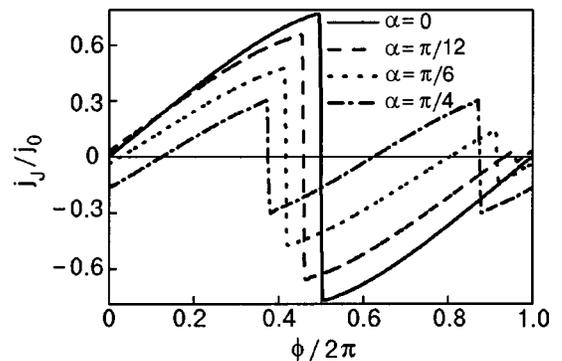


FIG. 6. Josephson current versus phase  $\phi$  for the hybrid “ $f$ -wave” state in the geometry (i) for different  $\alpha$ .

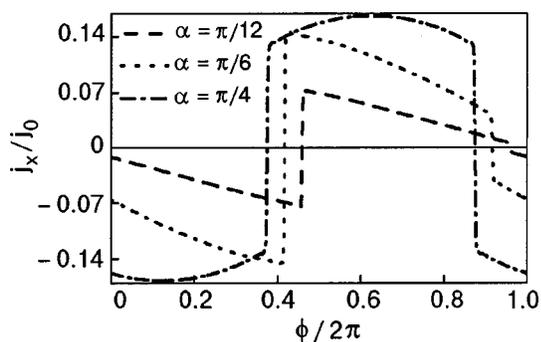


FIG. 7. Tangential current density versus phase  $\phi$  for the hybrid “*f*-wave” state in the geometry (i) for different  $\alpha$ .

It is clear that such experiments require very clean superconductors and perfect structures of the junction because of pair-breaking effects of nonmagnetic impurities and defects in triplet superconductors. The influence of single impurities and interfacial roughness in the plane of the contact, which may essentially decrease the critical current of the junction, will be analyzed in our next paper.

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\*E-mail: kolesnichenko@ilt.kharkov.ua

<sup>1)</sup>Strictly speaking, in crystals with a strong spin-orbit coupling the spin is a “bad” quantum number, but the electronic states are twofold degenerate and can be characterized by pseudospins.

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