LOW-DIMENSIONAL AND DISORDERED SYSTEMS

Overheating effect and hole-phonon interaction in SiGe heterostructures

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(Submitted June 26, 2008)
Fiz. Nizk. Temp. **34**, 1192–1196 (November 2008)

The effect of charge-carrier overheating in a two-dimensional (2D) hole gas is realized in a $Si_{1-x}Ge_x$ quantum well, where $x=0.13$, 0.36, 0.8, or 0.95. The Shubnikov–de Haas (SdH) oscillation amplitude is used as a "thermometer" to measure the temperature of overheated holes. The temperature dependence of the hole-phonon relaxation time is found from an analysis of the change of the dependence of the amplitude of the SdH oscillations on temperature and applied electrical field. Analysis of the temperature dependence of the hole-phonon relaxation time reveals a transition of the 2D system from the regime of "partial inelasticity" to conditions of small-angle scattering. © *2008 American Institute of Physics*. [DOI: 10.1063/1.3009592]

INTRODUCTION

The surface of a crystal experiences surface acoustic waves:[1] Rayleigh and Lamb waves and so on. The interaction of electrons with them differs essentially from the electron-phonon interaction in the bulk of the crystal. Specific features of the electron-phonon interaction are present in thin films of metals and semiconductors because the surface processes more strongly affect the kinetic properties of these objects. In free thin films with $d<\lambda$ (d is the film thickness, and λ is the phonon wavelength) there are bending waves obeying the quadratic dispersion law $\omega \propto q^2$ (ω and q are the phonon frequency and momentum, respectively).[2] Films on a substrate can experience Love waves with shear horizontal polarization. These waves obey an unusual dispersion law $\omega \propto q^{1/2}$ (Ref. 3). The specifics of the phonon spectrum of films affect the character of the electron-phonon interaction. Besides, the space quantization of the electron spectrum can be an additional factor of influence in semimetallic and semiconducting films. The situation is quite a bit simpler at the interface in semiconducting heterostructures. Here the electrons occupy the quantum states in the quantum well (QW) and form a two-dimensional (2D) electron gas, whereas the phonons can be thought of as three-dimensional (3D), since the elastic properties of the crystal are identical on both sides of the interface. Heterostructures in semiconductors offer the possibility of investigating the electron-phonon interaction between a 2D electron gas and 3D phonons.

The aim of this study is to investigate the hole-phonon interaction in a 2D hole gas in SiGe heterostructures and, in particular, to obtain information about the time and the temperature dependence of the hole-phonon interaction. The use of the quantum corrections to conductivity caused by the effects of weak localization on interaction cannot provide information about the electron-phonon relaxation time at very low temperatures, because in this condition the electron-electron scattering predominates over other inelastic processes. This relaxation time can, however, be found from the electron overheating effect, where the electron temperature T_e increases above the phonon temperature T_{ph} due to a strong electric field (current) or other heating factors. It should be noted that the electron overheating effect in a 2D electron gas is induced by the heat transport through the interface. Estimation of the electron-phonon relaxation time is possible because the transfer of the excess energy from the electron system to the phonon system is controlled by this time, even under the condition of strong elastic scattering. Experimentally, this problem amounts to estimating the electron temperature T_e, which changes under the influence a strong current. To achieve the electron overheating effect, the phonon should be free to leave the conducting layer and enter the surrounding crystal (i.e., good acoustic coupling is required between the conducting layer and the crystal). This requirement is met in heterostructures.

In the present study the overheating effect of charge carriers was realized in p-type heterostructures with a $Si_{1-x}Ge_x$ quantum well. The Shubnikov–de Haas oscillation (SdHO) amplitude was used as a "thermometer" to measure the temperature of overheated holes.

TABLE I. Characteristic parameters of the samples.

Sample	Quantum well	ρ_{xx}, kΩ	p_{Hall}, 10^{11} cm^{-2}	p_{SdH}, 10^{11} cm^{-2}	μ_{Hall}, 10^4 cm^2·V^{-1}·s^{-1}	$m^*(m_0)$	D, cm^2·s^{-1}
A	$Si_{0.87}Ge_{0.13}$	3.04	1.89	2.0	1.16	0.24	20.7
B	$Si_{0.64}Ge_{0.36}$	4.78	6.42	6.7	0.22	0.24	13.9
C	$Si_{0.2}Ge_{0.8}$	3.17	15.8	14.6	0.11	0.16	29.3
D	$Si_{0.05}Ge_{0.95}$	0.54	17.5	16.2	0.68	0.156	179

SAMPLE DESCRIPTION

Four of the samples studied in this work (labeled A-D, see Table I) were prepared by the molecular beam epitaxy technique.[4] In samples A and B the layers are arranged as follows: a single-crystal n-type Si (100) substrate, a pure, undoped Si epitaxial layer, a $Si_{1-x}Ge_x$ quantum well (~10 nm thick), an undoped Si spacer (~20 nm), a Si supply layer boron-doped at about 2.5×10^{18} cm^{-3}, and a pure Si cap (10 nm). In samples C and D the composition of the crystal beneath the QW, the spacer, and the boron-doped layers was $Si_{0.7}Ge_{0.3}$ or $Si_{0.37}Ge_{0.63}$, respectively, instead of pure Si.

For the conductivity measurements Hall bars were prepared, shaped as a "double cross," i.e., a narrow (~0.5 mm) strip with two pairs of narrow (0.05 mm) potential bars about 2 mm apart. The diagonal R_{xx} and off-diagonal R_{xy} components of the resistance tensor were measured as functions of magnetic field up to 11 T on samples A and C and up to 6 T on samples B and D. The sample parameters: diagonal resistivity ρ_{xx}, hole concentration p_{Hall} and p_{SdH} from the Hall and SdHO measurements, mobility μ_{Hall}, effective mass m^*, and hole diffusion coefficient D measured at the lowest temperature are shown in Table I.

The variations in magnetoresistances $\rho_{xx}(B)$ and $\rho_{xy}(B)$ with field, at the lowest temperatures, are illustrated in Fig. 1 (ρ stands for the resistance per square of a 2D electron system). The curves exhibit pronounced Shubnikov–de Haas oscillations at $B\geqslant1$ T and clear steps of the quantum Hall effect in sample A.

EXPERIMENTAL RESULTS AND DISCUSSION

Quantum interference effects were used in Ref. 5 to estimate the electron temperature during electron overheating. Electron overheating becomes more obvious when the SdHO are observed.[6–8] In the cited papers the falloff of the amplitude of the oscillations with increasing applied electric field was used to find a relation between the electron temperature and the rate of loss of excess energy by the electrons. In Ref. 8 the dependence of the energy loss time on the overheating temperature was found, and it was concluded that the main

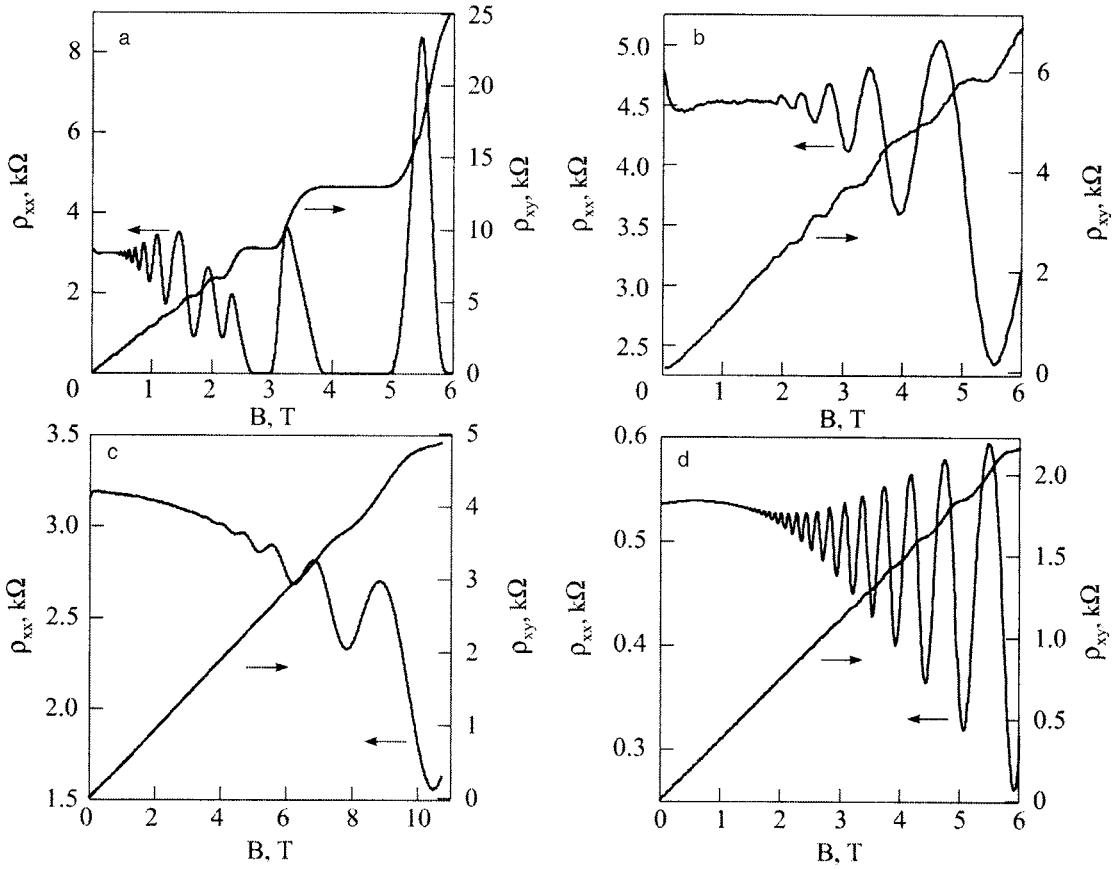

FIG. 1. Magnetoresistances ρ_{xx} and ρ_{xy} of samples A (a), B (b), C (c), D (d). T=33 mK for sample A, and $T\sim0.3$ K for the other samples.

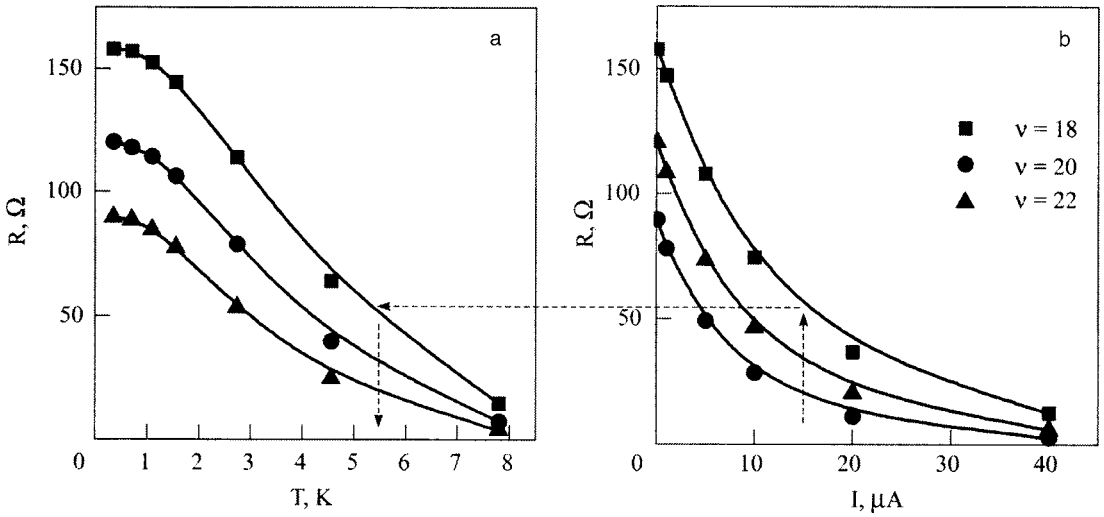

FIG. 2. Dependence of Shubnikov-de Haas oscillation amplitudes for sample D (a) with temperature and (b) with driving current. The solid lines are a guide to the eye.

channel of electron energy loss is the emission of acoustic phonons. In the present study, the overheating effect of the charge carriers is used to calculate, straightforwardly, the temperature dependence of the hole-phonon relaxation time in p-type $Si_{1-x}Ge_x$ quantum wells.

In our experiments, the hole temperature T_h was found by comparing the SdHO amplitude change with current and with temperature. As an example, the SdHO amplitude observed in sample D at low current and varying temperature (a) and at constant temperature and varying current (b) are shown in Fig. 2.

The amplitude variation in these two cases were analyzed for three extrema (with filling factors ν=18, 20, and 22) in the magnetic field region 0.8–3.5 T. The quantum numbers ν are found from the off-diagonal component of the resistivity $\rho_{xy}(B)$ using the equation for resistance quantization under the condition of the quantum Hall effect ($h/e^2\nu$).

The electron overheating effect has been considered in a number of theoretical studies (their results are applicable to hole overheating as well). In Ref. 9 an expression

$$T_e - T_{\text{ph}} = \frac{E^2\sigma}{\gamma T_{e\text{-ph}}}\tau_{e\text{-ph}}(T_{e\text{-ph}}), \tag{1}$$

was obtained from the heat balance equation, which contains the characteristic of our interest, i.e., the electron-phonon relaxation time $\tau_{e\text{-ph}}$ at a certain temperature $T_{e\text{-ph}}$, characterizing the electron-phonon interaction under the electron overheating condition. The prefactor γ describes the temperature dependence of the electronic specific heat $C_e(T) = \gamma T$. Since this parameter is unknown, it is reasonable to pass on to the equation of Ref. 10:

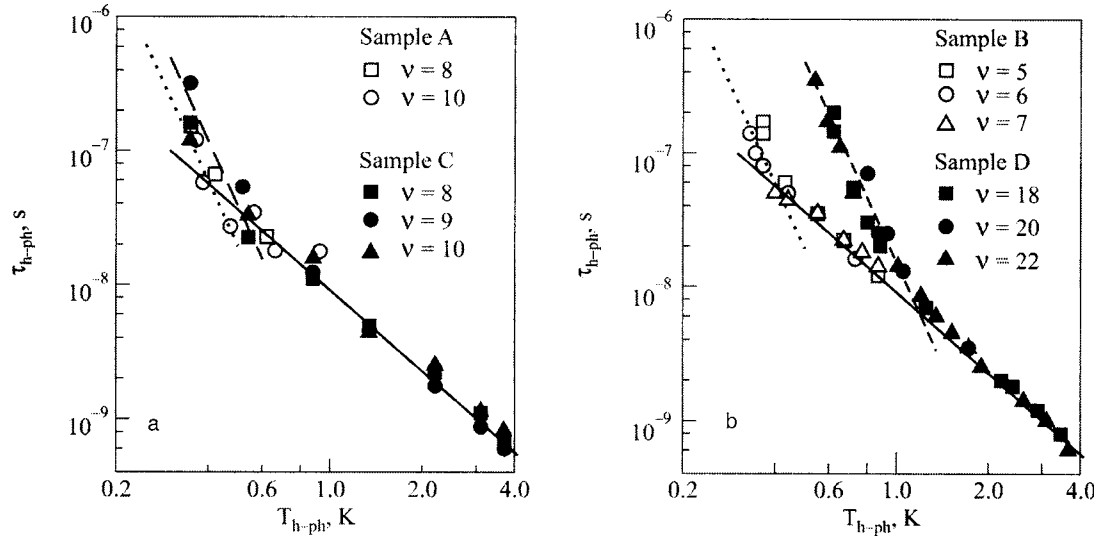

FIG. 3. Temperature dependence of hole-phonon relaxation times $\tau_{h\text{-ph}}$ found from the overheating effect. The solid line is the dependence $\tau_{e\text{-ph}}^{-1}=9\times10^{-9}T^2$, the dotted line is the dependence $\tau_{e\text{-ph}}^{-1}=0.6\times10^{-9}T^5$, the dashed line is the dependence $\tau_{e\text{-ph}}^{-1}=1.2\times10^{-9}T^5$, and the short-dash line is the dependence $\tau_{e\text{-ph}}^{-1}=1.5\times10^{-8}T^5$.

$$(kT_e)^2 = (kT_{\text{ph}})^2 + \left(\frac{6}{\pi^2}\right)(eE)^2 D\tau_{e\text{-ph}}. \quad (2)$$

This follows from Eq. (1) if we write down the heat capacity and conductivity in terms of the density of states ν_{ds}: $C_e = (\pi^2/3)k^2\nu_{ds}T$ and $\sigma = e^2\nu_{ds}D$, and besides it is necessary to use expressions for a 2D electron gas: $\nu_{ds} = m^*/(\pi\hbar^2)$, $D = (1/2)v_F^2\tau$ ($v_F = (\hbar/m^*)/(2\pi n)^{1/2}$ is the Fermi velocity, and n is the concentration of 2D electrons). Equation (2) is quite convenient because it includes only one characteristic of the samples, namely the electron diffusion coefficient D. The calculations according to Eq. (2) give the possibility of obtaining the dependence $\tau_{e\text{-ph}}(T_{e\text{-ph}})$. The temperature $T_{e\text{-ph}}$ is taken to be the mean $T_{e\text{-ph}} = (T_{\text{ph}} + T_e)/2$;[11,12] here T_{ph} corresponds to the temperature of the crystal.

The temperature dependence of the hole-phonon relaxation time $\tau_{h\text{-ph}}$ of all the samples is shown in Fig. 3. The dependences $\tau_{h\text{-ph}}(T_{h\text{-ph}})$ above 1 K for sample D and above $\sim 0.5-0.6$ K for the other samples can be approximated by the power law $\tau_{h\text{-ph}}^{-1} = 9\times 10^{-9}T^2$ (Fig. 3, solid lines). Importantly, this power law is common to all the samples despite their different characteristics. Moreover, the points corresponding to the filling factors ν for each different sample fall on the same curve, which suggests that the $\tau_{h\text{-ph}}$ value is independent of the magnetic field. A stronger dependence $\tau_{h\text{-ph}}(T_{h\text{-ph}})$ appears below the temperatures specified above.

The results obtained must be interpreted in the contexts of the theoretical studies considering the temperature variations of electron-phonon relaxation for 2D electrons interacting with 3D phonons.[13,14] In Ref. 14 these variations are analyzed in a wide range of temperatures. The energy is quantized in a QW:

$$E_1 = \frac{p_x^2}{2m_x^*} + \frac{p_y^2}{2m_y^*} + \frac{\pi^2\hbar^2}{2m_z^*L^2}n^2,$$

where L is the width of the well and n is the quantum number. Electrons occupy the ground state in the QW with the energy $E_1 = \pi^2\hbar^2/2m_z^*L^2$. The electron-phonon interaction of 2D carriers is limited on variation of the transverse momentum of the electrons in direction of quantization (in the z direction). The transverse component of the momenta of the emitted (absorbed) phonons is determined by the width of the QW, $q_\perp \sim 2\pi/L$. At high temperatures the thermal phonon momentum $q_T > 2\pi/L$, and the electron-phonon scattering is accompanied by emission (absorption) of phonons with wave vectors mainly perpendicular to the QW. The process is characterized by a dependence $\tau_{e\text{-ph}}^{-1} \propto T$. At lower temperatures the momentum of the thermal phonon $q_T = k_BT/\hbar s$ (s is the sound velocity) is smaller than $2\pi/L$. But if the phonon is capable of changing the wave vector of the electron by the maximum value $2k_F$, the energy exchange between the electron and the lattice can be about k_BT. We expect a dependence $\tau_{e\text{-ph}}^{-1} \propto T^2$ in this region of "partial inelasticity." At even lower temperatures ($q_T < 2k_F$) the wave vector of the emitted (absorbed) phonon is limited by temperature, and the scattering is similar to the small-angle scattering in a 3D metal, following a dependence $\tau_{e\text{-ph}}^{-1} \propto T^5$.

The obtained dependences $\tau_{h\text{-ph}}^{-1} \propto T^2$ (Fig. 3) correspond to the region of "partial inelasticity."[14] With lowering temperatures the dependence becomes stronger. We attribute this to the small-angle mechanism predicted in theory,[14] which is characterized by a dependence $\tau_{e\text{-ph}}^{-1} \propto T^5$. The temperature of the transition to this dependence obeys qualitatively (to within a numerical coefficient $\ll 1$) the condition $q_T \sim 2k_F$, if we take $k_F = (2\pi n)^{1/2}$ for a 2D electron gas, i.e., the temperature grows as the carrier concentration increases. The cause of the quantitative discrepancy is not clear yet and may be connected with the conditional character of the formulas for a 2D electron gas in the real situation.

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