ELECTRONIC PROPERTIES OF METALS AND ALLOYS

Manifestation of the spin-orbit interaction in bismuth films in a parallel magnetic field

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The magnetic field dependence of the resistance of bismuth thin films (100–700 Å thick) at low temperatures (1.5–77 K) are analyzed in the conceptual framework of quantum corrections to the conductivity due to weak localization and electron interaction effects. It is shown that the diversity and variability of the magnetoresistance curves in a parallel field upon variations of the thickness and temperature are due to the fact that the spin-orbit interaction time τ_{so} increases with increasing field, altering the relationship between τ_{so} and the phase relaxation time τ_{φ}. This result supports the hypothesis that the strong spin-orbit interaction manifested in the surface scattering of electrons is due to the existence of a potential gradient near the metal surface, and a parallel magnetic field alters the orientation of the spins, accompanied by a decrease of the rate of spin-orbit processes. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2409638]

INTRODUCTION

The experimentally obtained spin-orbit interaction time τ_{so} in thin films indicates that in the elastic scattering of electrons in them, spin-flip processes occur at a higher rate than theoretical estimates predict.[1–4] A possible explanation for this is the hypothesis that because the surface scattering of electrons in thin films is dominant, it is accompanied by strong spin-orbit interaction.

Merservey and Tedrow[5] have analyzed the experimental data for the spin-orbit relaxation time determined from the Knight shift, the critical magnetic field of superconducting films, experiments on the tunneling of spin-polarized electrons into a superconductor, and spin resonance in metals. Those authors verified that τ_{so}^{-1} obeys the dependence on the atomic number Z of the metal predicted in the theory of Abrikosov and Gor'kov,[1] which considers the spin-orbit interaction in the scattering of electrons on impurities. That theory predicts the relation

$$\frac{\tau_{so}^{-1}}{\tau^{-1}} \approx (aZ)^4, \tag{1}$$

where τ is the elastic scattering time, and $a=e^2/\hbar c=1/137$ is the fine-structure constant. The ratio $\tau_{so}^{-1}/\tau^{-1}=\varepsilon$ is a phenomenological parameter introduced by Dyson[6] and has the meaning of the probability of surface scattering with a spin flip. The authors of Ref. 5 constructed the dependence of τ_{so}^{-1}/τ^{-1} on Z for 10 metals and were convinced that, despite the large error in the determination of τ_{so} for the methods indicated above, relation (1) is functionally satisfied to a first approximation. Since the data for τ_{so} analyzed in Ref. 5 pertained to thin films, the authors proceeded from the assumption that surface scattering is dominant in them and took the time τ to be the transit time of an electron between the two surfaces, $\tau^{sf}=L/v_F$, where L is the film thickness and v_F is the Fermi velocity. Meanwhile, it turned out that the numerical values of $\tau_{so}^{-1}/(\tau^{sf})^{-1}$ were severalfold greater than the values expected according to Eq. (1) and reached ~0.1–0.5 for heavy metals.

The appearance of a theory of weak localization (WL)[7–9] and electron interaction (EEI)[9–12] effects and the experimental observations of these effects has made it possible to obtain information about the spin-orbit scattering time from analysis of the magnetic field dependence of the resistance. It follows from the theory of the weak localization effect that in the case of a weak spin-orbit interaction ($\tau_{so} \gg \tau_{\varphi}$, where τ_{φ} is the relaxation time of the phase of the electron wave function) the quantum corrections give a negative magnetoresistance. For a strong spin-orbit interaction ($\tau_{so} \ll \tau_{\varphi}$) the magnetoresistance (MR) is positive, with a logarithmic saturation at high fields. For $\tau_{so} \leqslant \tau_{\varphi}$ the MR curve passes through a maximum in the positive region and subsequently becomes negative (see below for details). Although the behavior of the magnetoresistance is determined by the relationship between τ_{so} and the time τ_{φ}, which at helium temperatures is, as a rule, greater than τ, the shape of the MR curves is indicative for qualitative assessment.

The first experimental studies of the weak localization effect in thin films often found manifestations of strong spin-orbit interaction. For example, for metals with large and intermediate values of Z (for films of Bi,[13–16] Au,[17,18] Pt,[19] W, Ta, Mo, Zr,[20] Pd,[21] and Sb[22]) positive magnetoresistance ($\tau_{so} \ll \tau_{\varphi}$) was found; for metals with small or intermediate values of Z (for films of Ag[23,18] and Cu[24–26] and even the light Mg[27]) magnetoresistance curves with a maximum ($\tau_{so} \leqslant \tau_{\varphi}$) or negative magnetoresistance (for Cu[28] and Mg[29] films) were found.

The description of the experimental curves with the aid of the theoretical formulas makes it possible to obtain the values of τ_{φ} and τ_{so} to rather high accuracy (in cases where the diffusion coefficient is reliably known). This fitting pro-

cedure works best for MR curves with a maximum. The values of τ_{so} obtained have turned out to be relatively small: e.g., in Cu films the following values of τ_{so} have been obtained: 8.4×10^{-12} s,[24] 6.8×10^{-12} s,[30] and, according to Ref. 28, $10^{-12}\,\text{s}\leqslant\tau_{so}\leqslant10^{-11}$ s; in Ag films $\tau_{so}=2\times10^{-11}$ s;[24,18] and, in Mg films $\tau_{so}=1\times10^{-10}$ s.[27] The values of τ_{φ} and τ_{so} have also been found successfully in the case of positive MR: in Au films $\tau_{so}=8.2\times10^{-13}$ s;[18] in Sb films $\tau_{so}=5\times10^{-13}$ s;[22,31] in Bi films $\tau_{so}=3\times10^{-13}$ s,[14] $(1-5)\times10^{-13}$ s,[16] $(4-8)\times10^{-13}$ s;[30] and, in films of the refractory metals W, Ta, Mo, and Zr $\tau_{so}\simeq(3-8)\times10^{-13}$ s.[20] From papers in which the values of the elastic relaxation time τ are given, one can estimate the parameter ε. For example, according to Ref. 18, for Ag and Au films $\varepsilon\simeq0.015$ and $\varepsilon\simeq0.25$, respectively, which exceed the estimates from formula (1) by a factor of 3-4; in Sb films $\varepsilon\simeq0.02$,[31] which is 3 times greater than the estimate from formula (1), etc.

Lindelof and Wang[32] confirmed the previous assumption that a high probability of spin-orbit interaction is inherent to surface scattering. Assuming that the value of τ_{so} observed in Mg films is a sum of bulk and surface scattering contributions,

$$\tau_{so}^{-1}=(\tau_{so}^{b})^{-1}+(\tau_{so}^{sf})^{-1}=(\varepsilon^{b}/\tau^{b})+(\varepsilon^{sf}/\tau^{sf}),$$

where ε^{b} and ε^{sf} are the probabilities of spin flip during bulk and surface scattering, and τ^{b} and τ^{sf} are the corresponding elastic times, those authors showed that ε^{sf} is greater than ε^{b} by one or two orders of magnitude (depending on the type of substrate). Similarly, in Ref. 33 it was found for Au films that $\varepsilon^{sf}\simeq2\times10^{-2}$ and $\varepsilon^{b}\simeq4\times10^{-4}$, i.e., $\varepsilon^{sf}\gg\varepsilon^{b}$.

In Ref. 34 the behavior of the time τ_{so} with changing thickness L was investigated for bismuth films and it was found that τ_{so} tends to increase with increasing L. This attests to the fact that strong spin-orbit interaction in bismuth films is due to the surface scattering contribution, the role of which decreases with increasing film thickness.

In Ref. 34 it was conjectured that the enhancement of the spin-orbit interaction for surface scattering of electrons is due to a gradient of the internal crystalline potential near the metal surface, the existence of which follows from the boundary conditions for the electron wave function. Since in an electron's motion towards the surface and reflection from it there is no invariance associated with spin symmetry upon time inversion, the description of the process of reflection of an electron from the surface should include a term due to spin-orbit interaction. We note that this situation is similar to the problem of the asymmetry of the potential well formed in a semiconductor heterojunction, which was considered by Rashba and co-workers.[35,36]

It follows from the Hamiltonian of the spin-orbit interaction[37]

$$H_{so}=\frac{h}{(2m_0c)^2}[\nabla V(r)\times\mathbf{p}]\hat{\sigma} \tag{2}$$

(where $V(r)$ is the potential, $\mathbf{p}$ is the electron quasimomentum, and $\hat{\sigma}$ is a Pauli matrix) that the spin of an electron near the surface (or the axis of precession of the spin) turns parallel to the surface of the crystal. A magnetic field directed parallel to the surface of the film is taken into account by means of a term describing the Zeeman interaction,

$$H_z=g^{*}\mu_B B. \tag{3}$$

The combined influence of the spin-orbit and Zeeman interactions can alter the intensity of the spin-orbit scattering in the surface reflection of electrons.

The goal of the experiments reported here was to check the hypothesis of a possible influence of a parallel magnetic field on the characteristics of the spin-orbit scattering in bismuth thin films.

I. EXPERIMENT

A. Shape of the magnetoresistance curves

The experimental curves of the resistance of bismuth thin films as functions of the strength of a parallel magnetic field were obtained on the same four series of bismuth thin-film samples as in Ref. 34, where the behavior of the magnetoresistance in a perpendicular magnetic field was investigated. In each series of films there were three (or four) different thicknesses, in the interval 100–700 Å. The experimental technique is described in Ref. 34.

In a *perpendicular* magnetic field the presence of strong spin-orbit interaction in the films was manifested by a positive sign of the MR and the logarithmic saturation of the MR curves at high magnetic fields that is characteristic of the weak localization effect. With increasing film thickness and increasing temperature, this anomaly decreases in amplitude (see Fig. 1 in Ref. 34).

In a *parallel* magnetic field one observes an extraordinarily diverse and mutable picture in recording the MR of bismuth films. The shape of the curves and the sign of the MR change with the film thickness, temperature, and, so some degree, under the influence of changes of the structural characteristics.

For films of thickness 150–250 Å at temperatures of 4.2 K and below one observes positive MR, for which the shape of the MR curves is characteristic of the weak localization effect. When the temperature is raised to ~15–20 K in the region of fields above 0.5 T a downward bend appears on the MR curves, and in the interval 20–77 K the MR becomes negative and decreases in amplitude with increasing temperature.

For films of thickness 250–400 Å, one observes MR curves with a maximum even at helium temperatures, the amplitude of the maximum decreasing noticeably in the temperature interval 1.6–4.2 K (Fig. 1). After the maximum the MR curves pass into the negative region. With increasing temperature a completely negative MR is formed (Fig. 2). The temperature of the transition to a completely negative MR tends to decrease with increasing film thickness. When the temperature is raised to 77 K the value of the negative MR decreases noticeably.

In samples with more perfect structure the formation of the maximum on the magnetic field scale is delayed, and the transition of the MR to the negative region is not observed in the whole magnetic field interval investigated.

Films greater than 400 Å thick at temperatures of 4.2 K and below are also characterized by positive MR with a maximum or saturation (Fig. 3). With increasing tempera-

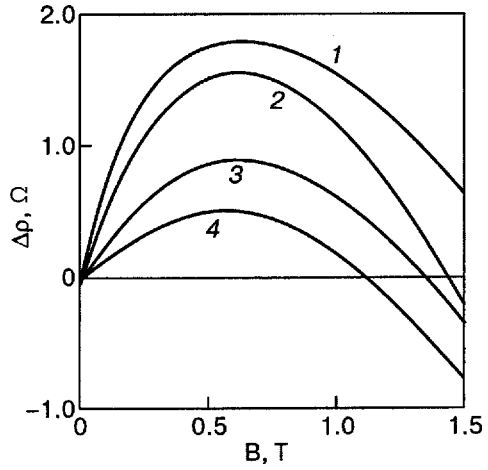

FIG. 1. Magnetic field dependence of the change of the resistance $\Delta\rho$ of a sample 320 Å thick at T [K]: 1.6 (*1*), 2.7 (*2*), 3.2 (*3*), 4.2 (*4*).

ture, however, negative MR (or a tendency toward its formation) is manifested only at temperatures of 10–20 K, while at higher temperatures the MR is positive.

At room temperatures the MR of the bismuth films of all thicknesses is of very low amplitude and approximately quadratic in magnetic field.

B. Qualitative explanation

The MR curves are shaped by the influence of a number of factors: the contribution of the classical (Drude) variation of the resistance in magnetic field, of the type μ^2B^2 (μ is the mobility), a possible contribution of the classical size effect, and the contributions of the weak localization and electron interaction effects. The contribution of the classical MR increases somewhat in magnitude with increasing film thickness owing to growth of μ and decreases with increasing temperature because of a decline in μ, but these changes of μ are slight, and, moreover, the classical change of the resistance gives only a positive MR. The classical size effect in thin films in a parallel magnetic field, predicted in Ref. 38, can be realized only under the condition $l \gg L$, where l is the electron mean free path. The hallmarks of the classical size effect, in the form a weak decrease of the resistance in the high magnetic field region, were observed in Ref. 39 in condensed Ag and Au films of micron thickness in magnetic fields of tens of kilooersteds. In our case the classical size effect is unlikely to be reflected in the change of resistance in magnetic field.

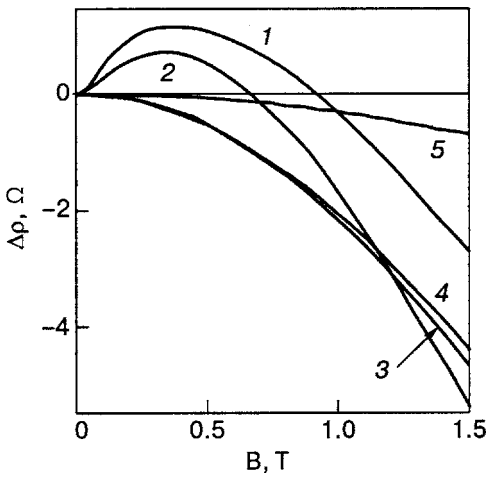

FIG. 2. Magnetic field dependence of the change of the resistance $\Delta\rho$ of a sample 380 Å thick at T [K]: 2 (*1*), 4.2 (*2*), 14 (*3*), 20 (*4*), 77 (*5*).

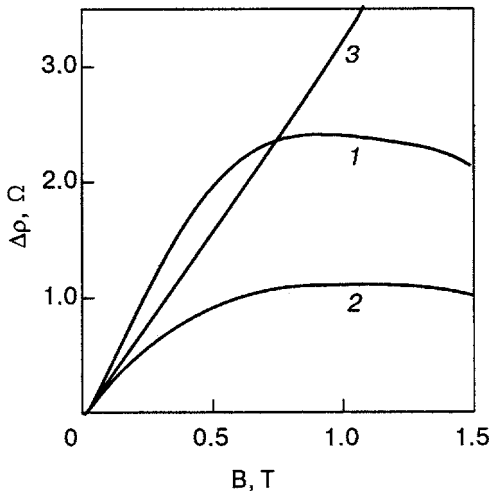

FIG. 3. Magnetic field dependence of the change of the resistance $\Delta\rho$ of a sample of thickness [Å]: 400 (*1*), 480 (*2*), 520 (*3*) at a temperature of 4.2 K.

It should be supposed that complex changes of the shape of the MR curves for bismuth films in a parallel magnetic field with increasing film thickness and temperature are due mainly to the contribution of quantum interference effects—weak localization and electron interaction. The interaction effect does not lead to a change of sign of the MR with increasing temperature. The governing role in the transformation of the MR curves of bismuth films is played by the weak localization effect. The change in sign of the MR in the weak localization effect occurs upon transition from strong to weak spin-orbit interaction.

The corrections to the conductivity of a thin film due to the weak localization of electrons vary according to the following relations:

— in a perpendicular magnetic field[9]

$$\Delta\sigma_{B_\perp}^{WL} = \frac{e^2}{2\pi^2\hbar}\left[\frac{3}{2}f_2\left(\frac{4eBD}{h}\tau_\varphi^*\right) - \frac{1}{2}f_2\left(\frac{4eBD}{h}\tau_\varphi\right)\right], \quad (4)$$

— in a parallel magnetic field[40]

$$\Delta\sigma_{B_\parallel}^{WL} = \frac{e^2}{2\pi^2\hbar}\left[\frac{3}{2}\ln\left(\frac{L^2e^2B^2D}{3\hbar^2}\tau_\varphi^* + 1\right) - \frac{1}{2}\ln\left(\frac{L^2e^2B^2D}{3\hbar^2}\tau_{\varphi+1}\right)\right], \quad (5)$$

where D is the diffusion coefficient, $f_2(x) = \ln(x) + \psi(1/x + 1/2)$, ψ is the logarithmic derivative of the Γ function, τ_φ is the phase relaxation time, and τ_φ^* is the time modified by the spin-orbit interaction: $(\tau_\varphi^*)^{-1} = \tau_\varphi^{-1} + 4/3\tau_{so}^{-1}$, where τ_{so} is the relaxation time of the spin due to the spin-orbit interaction in the elastic scattering of electrons. Equations (4) and (5) correspond to the diffusion regime, which is realized in the objects under study ($l < L_\varphi$, where $L_\varphi = (D\tau_\varphi)^{1/2}$ is the phase relaxation length, and $l < L_H^2/L$, where $L_H = (\hbar/eB)^{1/2}$ is the magnetic length). For the quantum correction to the conduc-

tivity of a film in an in-plane magnetic field in the diffusion regime an expression analogous to Eq. (5) was obtained in Ref. 41; in addition, in that paper solutions for the intermediate and ballistic regimes are also given. According to the analysis in Ref. 42, for bismuth films of small thickness it is sufficient to use expressions (4) and (5).

Two terms in formulas (4) and (5) reflect the spin state of electrons on conjugate trajectories forming the interference contribution to the conductivity. The first term corresponds to a triplet spin state of the electrons (total spin $j=1$), which is characterized by three possible values of the projections of the total moment ($M=0,\pm1$). The latter, as a result of the spin-orbit scattering varies in a random manner. The spin-orbit elastic scattering suppresses the coherence of the electronic states in the triplet channel. The second term corresponds to a singlet spin state of the electrons ($j=0$), which is damped only on account of inelastic scattering processes. Remarkably, the interference term corresponding to the singlet state of the electrons enters with a minus sign and leads to a change of the correction to the conductivity opposite to that of the triplet term.

In the case of a weak spin-orbit interaction ($\tau_{so}\gg\tau_\varphi$), τ_φ and τ_φ^* are close in value, and formulas (4) and (5) give negative magnetoresistance. For a strong spin-orbit interaction ($\tau_{so}<\tau_\varphi$) the value of σ_B^{WL} is governed by the second term in Eqs. (4) and (5), which leads to an anomalous positive magnetoresistance. If τ_{so} is less than but close to τ_φ, then τ_φ will be more than twice as large as τ_φ^*, and so initially, with increasing field, one observes positive MR owing to the singlet term, but then, on account of the triplet term, which has a coefficient of 3/2, the MR becomes negative. The position of the resulting maximum of $\Delta\sigma_B$ on the magnetic field scale is close to the value of the characteristic field $B_0^{WL}=\hbar c/4eD\tau_\varphi$.

In analyzing the weak localization effect in bismuth thin films in a *perpendicular* magnetic field it was found[34] that at helium temperatures the time τ_{so} is an order of magnitude (or more) shorter than τ_φ. This was determined by observation of positive MR at all temperatures and for films of all the thicknesses studied. With increasing film thickness one observes a tendency for the value of τ_{so} to increase while τ_φ remains practically unchanged. As a result, the difference of these times decreases. The difference of the times τ_φ and τ_{so} also decreases with increasing temperature, on account of the decrease of τ_φ,

The picture described above for the transformation of the MR curves in a *parallel* field can be explained qualitatively if it is assumed that with increasing magnetic field strength the spin relaxation time τ_{so} decreases. Indeed, only in the case when the times τ_φ and τ_{so} become close will a MR curve with a maximum be formed, and when the inequality $\tau_{so}>\tau_\varphi$ is realized the MR will be negative. This crossover should occur more easily when the initial values of τ_φ and τ_{so} are already close, i.e., in thicker films or at higher temperatures. In more perfect films the time τ_φ is somewhat longer than in otherwise similar samples (see Fig. 4 in Ref. 34), and therefore the formation of a maximum of the MR is delayed on the thickness and temperature scales.

The above-described variations of the shape of the parallel-field MR curves upon variation of the temperature and film thickness are in complete accordance with our stated assumption.

II. RESULTS OF CALCULATIONS

If it is assumed that the observed changes of the resistance of the samples in magnetic field are due to magnetic-field-induced changes of the classical (Drude) contribution and quantum corrections to the conductivity,

$$\sigma(B)=\sigma_0^D+\Delta\sigma^D(B)+\Delta\sigma^{WL}(B)+\Delta\sigma^{EEI}, \tag{6}$$

to determine $\Delta\sigma^{WL}(B)$ one should separate these contributions. The transition from the "total correction to the resistance" to the correction to the conductivity is made with the use of the relation

$$-\Delta\sigma=[\rho(B)-\rho(0)]/\rho(B)\rho(0),$$

where ρ is the "resistance per square" of the film. To separate the Drude contribution, as in Ref. 34, we use the procedure proposed in Ref. 43, namely: in the logarithmic saturation region of the functions (4) and (5), find the values of the mobility μ such that the function $\sigma^D(B)=\sigma_0^D/(1+\mu^2B^2)$ describing the magnetic-field-induced change of the classical contribution agrees with the experimental dependence $\sigma(B)$. The mobility values obtained are in good agreement with the values of the averaged mobility of the electrons and holes calculated from the system of equations for the conductivity, magnetoresistance, and Hall coefficient of bismuth films.[44–46] The difference between the experimental curve $\sigma(B)$ and the "Drude" curve $\sigma_0+\Delta\sigma^D(B)$ is the sum of the corrections $\Delta\sigma^{WL}(B)+\Delta\sigma^{EEI}$. The correction $\Delta\sigma^{EEI}$ due to the electron interaction effect is independent of magnetic field and is automatically eliminated under the requirement that the magnetic-field-induced change of the conductivity due to the localization correction vanish at $B=0$. The inferred magnetic field dependence of the localized correction $\Delta\sigma^{WL}(B)$ is compared with the theoretical formulas (4) and (5). In such a procedure the coincidences of the times τ_φ and τ_{so} are the fitting parameters. The diffusion coefficients were calculated, as in Ref. 34, by the Einstein formula from the conductivity and electron density of states on the Fermi surface, with the changes of the charge carrier density and Fermi energy with decreasing thickness of the bismuth film taken into account.[45]

The values of τ_φ and τ_{so} obtained in the analysis of the weak localization effect in a *perpendicular* magnetic field on the whole reproduce the results of the previous calculations.[34] These values were used as the starting values in the search for values of the times τ_φ and τ_{so} for description of the curves $\Delta\sigma^{WL}(B_\parallel)$ in a parallel magnetic field. We assumed that these relaxation characteristics cannot change appreciably when the orientation of the magnetic field changes.

The influence of a *parallel* magnetic field on the characteristics of spin-orbit scattering in bismuth thin films was manifested in an extremely peculiar but completely unambiguous way. It turned out that the $\Delta\sigma^{WL}(B)$ curves cannot be described by a pair of magnetic-field-independent times τ_φ and τ_{so}. It is impossible to describe the experimental curves by formula (5) obtained on the assumption of magnetic-field-

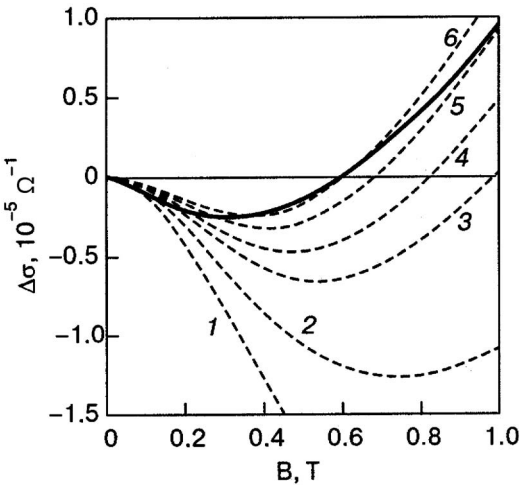

FIG. 4. Illustration of the description of the magnetic-field-induced change of the conductivity according to Eq. (5) for a sample 380 Å thick at a temperature of 4.2 K and for values of the phase relaxation time $\tau_\varphi = 4.6 \times 10^{-12}$ s and spin-orbit interaction time τ_{so} [s]: 2×10^{-13} (*1*), 7×10^{-13} (*2*), 1.1×10^{-12} (*3*), 1.3×10^{-12} (*4*), 1.52×10^{-12} (*5*), and 1.67×10^{-12} (*6*); the experimental dependence is shown by a solid curve (——).

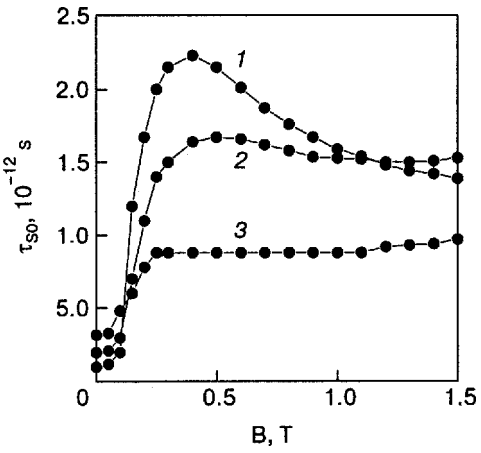

FIG. 5. Magnetic field dependence of the spin-orbit interaction time τ_{so} for a sample 380 Å thick at T [K]: 2 (*1*), 4.2 (*2*), 20 (*3*).

independent times τ_φ and τ_{so}. In the motion along the experimental curve each successive point requires changing the value of at least one of these times. Since it seemed unlikely for τ_φ to be influenced by a weak parallel magnetic field,[47,48] we assumed that it remains unchanged. It was found that in the low-field region the values of τ_{so} calculated according to formula (5) increase. Let us illustrate this by a transparent construction (Fig. 4). If the values τ_φ and τ_{so} known from experiments in a perpendicular field are used, then the calculated curve describes only a very short initial segment of the $\Delta\sigma^{WL}(B)$ curve, and then it moves away sharply into the region of negative $\Delta\sigma^{WL}$ values (Fig. 4, curve *1*). Increasing the values of τ_{so} allows one to obtain values of $\Delta\sigma^{WL}$ on successive small segments of the $\Delta\sigma^{WL}(B)$ curve all the way up to $B\sim0.5$ T (Fig. 4, curves *2*-*6*). However, at higher fields it is necessary in some cases to subsequently decrease τ_{so} to get the experimental values of $\Delta\sigma^{WL}$. We note that changing the initial values of τ_φ only degrades the computational situation.

Thus only values of τ_{so} that vary with magnetic field allow one to describe the experimental $\Delta\sigma^{WL}(B)$ curves with the aid of formula (5). Examples of the curves obtained for the time τ_{so} as a function of the parallel magnetic field are illustrated in Fig. 5. An important feature of these curves is that τ_{so} increases strongly at fields up to $B\sim0.3$–0.5 T and then remains constant or even decreases. It can be noted in Fig. 5 that with increasing temperature, as the inequality $k_BT > g^*\mu_BB$ begins to hold and becomes stronger, the rise of τ_{so} becomes less.

The physical reason for the growth of τ_{so} in a parallel magnetic field consists in the following. In the absence of magnetic field the spins of the electrons near the surface become oriented parallel to the surface under the influence of the potential gradient (see formula (2)) but with a random distribution in the azimuthal direction. The spin relaxation is of a diffusional character and in the general case must be described by the D'yakonov–Perel' mechanism,[49] which is characterized by a spin relaxation rate proportional to $2/\tau_{so}$. We note that in Refs. 50 and 51 it was shown that in narrow semiconductor quantum wells, in which spin splitting occurs, under weak localization conditions the rate of electron spin relaxation by the D'yakonov–Perel' mechanism decreases. As a result, there is a certain level of spin relaxation rate in the absence of magnetic field.

Radical changes of the spin relaxation,[47] and, in some cases, of the phase relaxation as well,[52]—occur when a magnetic field is turned on. The main result of Ref. 52 (see also 53) pertains to the influence on the phase relaxation process by a magnetic field applied parallel to a two-dimensional layer of free carriers in a quantum well in a semiconductor with the zinc blende structure. It was shown that the Zeeman interaction leads to additional dephasing of the electron wave function which can be described by a temperature-independent time $\tau_H(B)$. From the relation $1/\tau_\varphi(B) = 1/\tau_\varphi(0) + 1/\tau_H(B)$ one gets the expression $\tau_\varphi(B)/\tau_\varphi(0) = [1+\tau_\varphi(0)/\tau_H(B)]^{-1}$, which characterizes the influence of magnetic field on τ_φ. This influence is appreciable under the condition $g^*\mu_BB \geqslant \hbar/(\tau_{so}\tau_\varphi)^{-1/2}$. In our objects in fields $B \leqslant 1$ T the inequality is strongly the opposite. If one nevertheless uses the hypothetical expression $1/\tau_H = \tau_{so}(0)g^{*2}\mu_B^2B^2/\hbar^2$, as was done in the experimental paper,[54] then estimates show that the change of τ_φ in a field $B=1$ T is only 20%. Thus our assumption that the time τ_φ remains unchanged with increasing magnetic field is admissible.

A magnetic field parallel to the surface tends to order the orientation of the electron spins interacting with the surface. In a system with an ordered polarization of spins the character of the spin relaxation is no longer diffusional, and it differs from the D'yakonov–Perel' mechanism.[47] In Ref. 47 (using as an example a quantum well in a semiconductor of the GaAs type) the evolution of the spin density after the creation of local polarization of the spins in the system was calculated as a function of the coordinate and time at different values of the parallel magnetic field. The calculations showed that the turning on of even a weak magnetic field ($g^*\mu_BB \ll \hbar/\tau_{so}$) results in a manyfold decrease of the spin density $S(r,t)$ [with units of $1/(2\pi D\tau_{so})$], which characterizes the distribution of the spin polarization in space and time. This is indicative of an increase of τ_{so} under the influence of a parallel magnetic field, or, more precisely, it attests

to the appearance in the disordered spin system of some new evolutionary mechanism of spin relaxation upon partial (or total) polarization of the spin system in place of the D'yakonov–Perel' diffusion mechanism.

Experimental observations of an increase of the time τ_{so} under conditions of spin-orbit interaction by the Rashba mechanism in quantum well in semiconductors under the influence of a parallel magnetic field were made in Refs. 48 and 54. In Ref. 48 an analysis of the recording of the MR of an InP/InGaAs/InP quantum well in tilted magnetic fields showed that the manifestation of spin-orbit interaction in the form of a maximum of the MR at low fields is determined by the normal component of the field; here τ_{φ} is independent of magnetic field, while τ_{so} increases with increasing parallel component of the magnetic field. The authors of Ref. 54 recorded the resistance of an InAlAs/InGaAs/InAlAs quantum well as a function of the value of the perpendicular magnetic field $B_{\perp}$ for different values of the parallel magnetic field $B_{\parallel}$, and from an analysis of quantum corrections to the conductivity the changes of τ_{φ} and τ_{so} as functions of $B_{\parallel}$ were determined. In Refs. 48 and 54 it was found that the change of τ_{so} occurs in accordance with the hypothetical dependence $\tau_{so} \propto 1+\alpha B^2$. The analogy between our problem of the spin-orbital processes near the surface in bismuth films and processes occurring in a semiconductor quantum well is only very rough. Moreover, in our case the rapid growth of τ_{so} occurs at relatively low fields, and because of the smallness of the interval our construction procedure (see Fig. 5) does not permit determination of the functional form of the rise of τ_{so} with field.

III. ROLE OF THE QUANTUM INTERACTION CORRECTION TO THE MAGNETORESISTANCE

The interaction correction to the conductivity is independent of magnetic field (in the region where Zeeman splitting is absent), but it has a temperature dependence of the form[9–11]

$$\Delta\sigma^{EEI}(T) = \frac{e^2}{2\pi^2\hbar}\lambda \ln\left(\frac{k_B T\tau}{\hbar}\right), \tag{7}$$

where λ is the interaction constant.

In the separation of the Drude contribution and quantum corrections to the conductivity, according to Eq. (6), the correction $\Delta\sigma^{EEI}$ is determined by the shift of the reference point of the conductivity necessary in order to satisfy the requirement that the magnetic-field-induced change of the conductivity due to the localization correction must vanish at $B=0$. The value of this shift agrees in order of magnitude with the estimate from formula (7) and varies strictly by a $\ln T$ law [for estimation we used the value $\lambda=0.9$, which follows from the temperature dependence of the quantum correction to the conductivity of bismuth films at a high magnetic field ($B \geqslant 1.5$ T) in which the weak-localization contribution is destroyed[55]].

Meanwhile, in a magnetic field, dependence of the resistance on the field can appear due to the interaction correction $\Delta\sigma^{EEI}$. As was shown in Ref. 56, when the conductivity tensor is inverted to the magnetoresistance tensor, the correction to the resistance acquires a factor of $\{-(1-\omega_c^2\tau^2)\}$, where $\omega_c = eB/m^*$ is the cyclotron frequency. This transformation takes into account the fact that the correction to the Hall conductivity can be neglected in the diffusion regime. The change of the resistance due to $\Delta\sigma^{EEI}$ is described by the expression[57,58]

$$\rho_{xx}(B,T) = \frac{1}{\sigma_0} - \frac{1}{\sigma_0^2}[1-(\omega_c\tau)^2]\Delta\sigma_{xx}^{EEI}(T). \tag{8}$$

In Eq. (7) the argument of the logarithm $k_B T\tau/\hbar < 1$ and, hence, the interaction correction $\Delta\sigma^{EEI}$ is negative and can lead to negative quadratic magnetoresistance. However, since $(\omega_c\tau)^2 \ll 1$ in our objects, the magnetic-field-induced change of the resistance is insignificant. Consequently, the interaction correction determines only a shift of the magnetoresistance curve to lower values of the resistance in zero field, and it is not reflected in the $\Delta\rho(B)$ curves in Figs. 1–3. The absolute value of the change of resistance due to the interaction correction at helium temperatures is of the same order as the characteristic change of the resistance under the influence of the weak localization effect in the magnetic field interval shown. With increasing temperature, both corrections decrease, and at room temperature the weak positive magnetoresistance is the classical (Drude) magnetic-field-induced change of the resistance under conditions where the weak localization and electron interaction effects are absent.

CONCLUSION

The quantum corrections to the conductivity due to the weak localization effect in bismuth thin films in a parallel magnetic field are actually manifested only at helium temperatures and are responsible for the maximum of the magnetoresistance, which falls off rapidly in amplitude as the temperature increases in the interval 1.5–10 K (see Figs. 1 and 2). Analysis of the magnetic-field-induced change of the localization correction showed that with increasing strength of the parallel magnetic field in the interval 0–0.5 T the spin-orbit interaction time τ_{so} grows. This result should be interpreted as a transition from a diffusional character of the spin relaxation according to the D'yakonov-Perel' mechanism to a different (evolutionary) character of the spin relaxation in a system with an ordered orientation of the spins that arises under the influence of a parallel magnetic field. Such an interpretation supports the assumption that the surface scattering that is dominant in thin films is characterized by strong spin-orbit interaction due to the existence of a gradient of the internal potential near the surface.

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