

Radiation of electromagnetic waves from edge dislocations moving in ionic crystal

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Abstract

The emission of electromagnetic waves by a straight edge dislocation moving in ionic crystals with a NaCl-type lattice has been investigated theoretically. It is shown that the motion of a dislocation in the ionic lattice results in the appearance of a specific alternative polarization current, generated by uncompensated valence bonds along the edge of the extra plane of a dislocation. For a dislocation moving with a constant average velocity V , the frequency of this current is of the order of $\omega \sim 2\pi V/b$, where b is Burgers vector. The fax of electromagnetic energy in the far field and the radiation friction force acting per unit length of the dislocation line are calculated. The emission of electromagnetic waves by an edge dislocation oscillating in the glide plane as well as by dislocation moving with a constant average acceleration is studied. © 1997 Elsevier Science S.A.

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1. Introduction

The motion of dislocations in solids is accompanied by elastic and electromagnetic excitations arising as a result of the perturbation of the atomic and electronic subsystems of the crystal. The main physical mechanisms of the acoustic emission arising in the process of plastic deformation and fracture are now completely understood [1]. Electromagnetic effects in solids, of course, have also been well-studied in many respects, but at the same time there are only a few published papers on the specific problem of electromagnetic emission from dislocations and cracks. Here we call attention first to the experimental investigations of electromagnetic noise generated in the process of development of slip bands and cracks in ionic crystals [2]. For all practical purposes, a detailed and consistent theoretical analysis of the corresponding phenomena has never been made.

A mechanism for the generation of an electromagnetic wave by a straight dislocation moving in an ionic crystal was proposed a rather long time ago in [3]. This mechanism is based on the electroelastic effects, which for most nonpiezoelectric dielectric crystals are small to the extent that the electroelastic moduli are small [4].

Our objective in the present paper is to propose and work out an alternative mechanism for the electromagnetic emission by edge dislocations. The new mechanism is associated with the excitation of microcurrents in the core of a dislocation moving in a nonpiezoelectric ionic crystal.

2. Dislocation as a source of electromagnetic fields in crystal

Let us consider an ionic crystal with the NaCl structure. In such a crystal, the core of a straight edge dislocation, whose line coincides with the [001] direction, consists of a chain of sign-alternating charges that bounds the extra plane of the dislocation [5]. Let the Z -axis be directed along the dislocation line and let the plane $y = 0$ be the glide plane. Then the charge density on the dislocation line equals

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$$\rho(r, t) = e^* F(x) \delta(x - x_0(t)) \delta(y) \sum_{m=-\infty}^{\infty} \{\delta(z - 2ma) - \delta[z - (2m + 1)a]\} \quad (1)$$

Here $2a$ is the lattice period along the dislocation line with coordinate $x(t)$ (the positive and the negative charges are located at the positions $z = 2ma$ and $z = (2m + 1)a$, respectively), e^* is the effective charge of the edge site on the extra plane, and the function $F(x)$ is periodic with period $2b$ in the x -direction (b is the distance between the neighboring minima of the Peierls relief in the direction of the X -axis), $|F(x)| \leq 1$.

The co-factor $F(x)$ appears Eq. (1) because when the dislocation crosses over into a neighboring valley of the Peierls relief (advancing by one interatomic spacing b along the X -axis) the m -th site, initially possessing the charge $\pm e^*$, on the dislocation line undergoes a change in polarity acquiring in the new position the charge $\mp e^*$ [5]. The function $F(x)$ can be written as

$$F(x) = \sum_{m=-\infty}^{\infty} F_m \exp\left[\frac{i\pi m x}{b}\right] \quad (2)$$

We shall assume below that the coefficients F_m are given. For example, in the simplest model case, where $F(x) = \cos(\pi x/b)$, $F_1 = F_{-1} = 1$ and all other $F_n = 0$.

For the further calculations, we shall apply the scheme employed in classical electrodynamics [6]. Let the average velocity of the dislocation $V(t) = \partial x_0 / \partial t$ be a known function of time. We transform to a locally inertial coordinate system moving together with the dislocation along the X -axis with the velocity $V(t)$ and we write the equation of continuity for the charge, fluctuating on the dislocation line,

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = -\operatorname{div} \mathbf{j}(\mathbf{r}, t) \quad (3)$$

where $\mathbf{j}(\mathbf{r}, t)$ is the current density in the dislocation core. We are interested in the fields which vary little over distances of the order of the interatomic spacing. To obtain them, we average the microcurrents as done in the electrodynamics of continuous media [7]. Thus,

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = -\frac{\partial}{\partial z} \left[a e^* V(t) F(x) \delta(y) \frac{\partial}{\partial x} \delta(x - x_0(t)) \sum_{m=-\infty}^{\infty} \delta(z - 2ma) \right] = -\frac{\partial}{\partial z} j_z(\mathbf{R}, z, t). \quad (4)$$

Here \mathbf{R} is the two-dimensional position vector in the $z = 0$ plane. After averaging of $j_z(\mathbf{R}, z, t)$ over the coordinate z , we represent the corresponding component of the current density in the form

$$j_z(\mathbf{R}, z, t) = \frac{e^*}{2} V(t) F(x) \delta'(x - x_0(t)) \delta(y) \quad (5)$$

Therefore the expression Eq. (5) describes the macroscopic current density in the core of an edge dislocation gliding with velocity V .

3. Electromagnetic radiation from an edge dislocation

In calculating the electromagnetic fields \mathbf{E} and \mathbf{H} , we shall assume that the dislocation moves in an infinite homogeneous isotropic nonmagnetic ($\mu = 1$) and nondispersive ($\epsilon = \text{const}$) medium. The polarizability of the medium is not of fundamental importance in our problem, so that without loss of generality we set $\epsilon = 1$. We employ the electrodynamic potentials in the standard definition [6]

$$\mathbf{H} = \operatorname{rot} \mathbf{A}, \quad \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \operatorname{grad} \varphi \quad (6)$$

in the Lorentz gauge $\partial \varphi / \partial t + c \operatorname{div} \mathbf{A} = 0$ after which for the only nonzero component $A_z(\mathbf{R}, t)$ of the vector potential we obtain d'Alembert's equation

$$\left[\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right] A_z(\mathbf{R}, t) = \frac{4\pi}{c} j_z(\mathbf{R}, t) \quad (7)$$

The dislocation radiation fields are determined by the relations Eq. (6), where the asymptotic forms of $A_z(\mathbf{R}, t)$ in the wave zone must be substituted.

The spectral components \mathbf{A}^ω (Fourier transformation over time) of the function \mathbf{A} in dipole approximation can be obtained in the form

$$A_z^\omega(\mathbf{R}) = \left(\frac{2\pi}{i\omega R c} \right)^{1/2} (i\omega D_z^\omega) \exp\left(-i\omega \frac{R}{c} \right) \quad (8)$$

Here D_z^ω are the spectral components of the electric dipole moment density $D_z(t)$ of a straight dislocation (electric dipole moment per unit length of dislocation line):

$$\frac{\partial D_z(t)}{\partial t} = \int d^2 R' j_z(\mathbf{R}', t) = \frac{e^*}{2} V(t) \Phi(x_0(t)); \quad \Phi(x) = \frac{\partial}{\partial x} F(x) \quad (9)$$

$$i\omega D_z^\omega = \frac{e^*}{4\pi} \int_{-\infty}^{\infty} d\omega' v(\omega - \omega') \phi(\omega') \quad (10)$$

Here $v(\omega)$ and $\phi(\omega)$ are the spectral components of the functions V and Φ , respectively.

In a cylindrical coordinate system \mathbf{R} , φ , and z (the angle φ is measured from the positive direction of the X -axis) the electromagnetic field emitted by the dislocation have two nonzero components:

$$H_\varphi^\omega(\mathbf{R}, \varphi) = \frac{1}{c} \left(\frac{2\pi}{i\omega R c} \right)^{1/2} [(i\omega)^2 D_z^\omega] \exp\left(-i\omega \frac{R}{c} \right); \quad E_z^\omega(\mathbf{R}, \varphi) = -H_\varphi^\omega(\mathbf{R}, \varphi) \quad (11)$$

Expression Eq. (11) describes a cylindrical linearly polarized electromagnetic wave propagating away from the dislocation line. We obtain the space-time distribution of the radiation by inverting the time Fourier transform in Eq. (11):

$$H_\varphi(R, \varphi, t) = -\frac{1}{c} \left(\frac{2}{Rc} \right)^{1/2} \int_0^\infty \frac{d\tau}{\sqrt{\tau}} \frac{\partial^2}{\partial t^2} D_z \left(t - \tau - \frac{R}{c} \right);$$

$$E_z(R, \varphi, t) = -H_\varphi(R, \varphi, t) \quad (12)$$

Naturally, the amplitude of the bremsstrahlung in the dipole approximation [7] is proportional to the second time derivative of the dipole moment of the radiating system

$$\frac{\partial^2}{\partial t^2} D_z(t) = \frac{e^*}{2} \left[W(t)\Phi(x_0(t)) + V^2(t) \frac{\partial}{\partial x_0} \Phi(x_0(t)) \right] \quad (13)$$

where $W(t) = \partial V(t)/\partial t$ is the average acceleration of the dislocation. Both terms in Eq. (13) refer to bremsstrahlung of the system. The first term is the high-frequency carrier (with a frequency of the order of $V/2b$), which is frequency-modulated (to the extent that the second derivative differs from zero) and amplitude-modulated (on account of the co-factor $V(t)$ varying slowly over times $\sim 2b/V$). The second term in Eq. (13) does not depend on the acceleration of the dislocation and leads to the appearance of radiation even when the dislocation moves with a constant average velocity.

4. Radiation from an uniformly moving edge dislocation

Consider an edge dislocation gliding in the plane $y = 0$ with constant average velocity V . Then $x_0(t) = Vt$ and, correspondingly, $v(\omega) = 2\pi V\delta(\omega)$ and

$$\phi(\omega) = \frac{2i\pi^2}{b} \sum_{n=-\infty}^{\infty} nF_n\delta(\omega - n\omega_0) \quad (14)$$

where $\omega_0 = \pi V/b$. We obtain the spectral components of the radiation fields by substituting expression Eq. (14) into Eq. (10):

$$H_\varphi^\omega(R, \varphi) = \frac{i\pi e^* \omega_0}{c} \left(\frac{2\pi i \omega}{Rc} \right)^{1/2} \exp\left(-i\omega \frac{R}{c}\right) \sum_{n=-\infty}^{\infty} nF_n\delta(\omega - n\omega_0) \quad (15)$$

The spectral components of the electric field are determined by relation Eq. (11). The radiation from a uniformly moving dislocation consists of a collection of cylindrical harmonics with frequencies which are multiples of ω_0 . Inverting the Fourier transform in Eq. (14), we obtain for the space-time distribution of the electromagnetic radiation fields:

$$H_\varphi(R, \varphi, t) = -\frac{e^*}{c} \left(\frac{2\pi}{Rc|\omega_0|} \right)^{1/2} \frac{\partial^2}{\partial t^2} \sum_{n=1}^{\infty} n^{-1/2} F_n \sin \left[n|\omega_0| \left(t - \frac{R}{c} \right) + \frac{\pi}{4} \right] \quad (16)$$

In writing Eq. (16), we employed the fact that the function $F(x)$ is even. The intensity of the electric field is determined, once again, by the formula Eq. (12). A straight edge dislocation radiates cylindrical waves, whose amplitude decreases with distance away from the dislocation line as $R^{-1/2}$.

An estimate of the electric field intensity in the emitted wave follows from the expressions Eq. (16) and Eq. (12):

$$E \simeq \frac{e^* \omega_0}{c} \left(\frac{2\pi \omega_0}{Rc} \right)^{1/2} \quad (17)$$

At distances of the order of 1 cm from the dislocation, the field intensity equals $E \simeq 10^{-2} \mu\text{V/m}$, if $V \simeq 10 \div 100 \text{ cm/s}$. In real plastic deformation processes up to 10^5 dislocations in a slip band move simultaneously, so that in a ionic crystal the electromagnetic radiation will be appreciable.

5. Radiation from an oscillating dislocation

Let a straight edge dislocation with the geometry described above oscillates with frequency Ω and amplitude u_0 in the slip plane $X(t) = u_0 \sin \Omega t$. We obtain from Eq. (8) the spectral components of vector potential for the radiation fields of an oscillating dislocation, after which we find the spectral components of the magnetic field

$$H_\varphi^\omega(R, \varphi) = 8\pi i \omega e^* \frac{\Omega}{c} \left(\frac{\pi i}{2\omega Rc} \right)^{1/2} \exp\left(-i\omega \frac{R}{c}\right) \sum_{n=1}^{\infty} F_n \sum_{k=1}^{\infty} k J_{2k} \left(\frac{\pi n u_0}{b} \right) [\delta(\omega - 2k\Omega) - \delta(\omega + 2k\Omega)] \quad (18)$$

where J_m is a Bessel function of integer order m . The electric field has one nonzero component Eq. (11). Inverting the Fourier transform over time in Eq. (18), we obtain the space-time form of the radiation fields

$$H_\varphi(R, \varphi, t) = -\frac{2e^*}{c^2} \left(\frac{\pi c}{\Omega R} \right)^{1/2} \frac{\partial^2}{\partial t^2} \sum_{n=1}^{\infty} F_n \sum_{k=1}^{\infty} k^{-1/2} J_{2k} \left(\frac{\pi n u_0}{b} \right) \cos \left[2k\Omega \left(t - \frac{R}{c} \right) - \frac{\pi}{4} \right] \quad (19)$$

The intensity of electric field is determined from Eq. (12).

One can see from Eq. (18) and Eq. (19) that the radiation of an oscillating straight edge dislocation consists of a superposition of cylindrical harmonics

with frequencies which are multiples of twice frequency Ω of the dislocation oscillations. We obtain an estimate of the intensity of the electric field in the wave emitted by an oscillating edge dislocation

$$E \simeq e^* \left(\frac{\Omega}{c} \right)^{3/2} \left(\frac{2b}{\pi u_0 R} \right)^{1/2} \quad (20)$$

For the typical values $\Omega \simeq 10^{11} \text{ s}^{-1}$, $b/x_0 \simeq 0.01$, and $R \simeq 1 \text{ cm}$, we find $E \simeq 10^{-1} - 10^{-2} \text{ } \mu\text{V/m}$, which agrees with the estimate presented in the preceding section.

6. Radiation from an accelerated dislocation

Let the dislocation starts to move at time $t = 0$ at the point $x = 0$ and moves along the X -axis with constant average acceleration w

$$X(t) = wt^2\theta(t)/2, \quad V(t) = wt\theta(t) \quad (21)$$

Here $\theta(t)$ is the Heaviside function. Then space-time distribution in this case as

$$\begin{aligned} H_\varphi(R, \varphi, t) &= -\frac{\pi e^* |w|}{cb} \left(\frac{1}{cR} \right)^{1/2} \frac{\partial}{\partial t} \sum_{n=1}^{\infty} n F_n \int_0^{\infty} \tau \sin\left(\frac{\pi n |w| \tau^2}{2b} \right) \\ &\quad \frac{\theta(t - \tau - R/c)}{(t - \tau - R/c)} d\tau \end{aligned} \quad (22)$$

The electric field is determined by Eq. (12).

One can see from Eq. (22) that an accelerated edge dislocation emits cylindrical waves. This is bremsstrahlung, but the acceleration dependence of the fields (via the co-factor $\sim \sin(\pi n |w| \tau^2 / 2b)$ in the integrand in Eq. (22)) is more complicated than in the case of bremsstrahlung from a single free charge. It is obvious, however, that because the function $\sim (\pi n |w| \tau^2 / 2b)$ in the integrand is bounded, the intensities of the radiation fields are mainly proportional to the accelera-

tion $|w|$ of the dislocation, as should be for the dipole component of bremsstrahlung [6].

7. Conclusions

The results obtained in the present paper show that the detection of electromagnetic radiation emitted by dislocations moving in ionic crystals can be an effective tool for investigating the dynamical parameters of defects of this kind in the process of plastic deformation. Analysis of the spectral composition of the radiation in principle yields important information about the structure of the Peierls relief in the crystal. Indeed, if several harmonics of the fundamental frequency and a_0 can be recorded experimentally in the radiation, then the amplitudes F_n of the Fourier harmonics of the function $F(x)$ can be found and the form of this function can be reconstructed. It must be underscored that this cannot be done by any of the currently available methods for investigating dislocations. It will ultimately make it possible to develop new methods of nondestructive testing of materials; such methods which will find many applications in both fundamental solid-state physics and engineering problems.

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