Université Lille Nord de France

Ecole doctorale régionale Sciences Pour l'Ingénieur Lille Nord-de-France - 072

Cours spécialisé du Collège Doctoral Lille Nord-de-France

Titre : **Riemann-Hilbert problems and integrable nonlinear partial differential equations**

Intervenant : Dmitry Shepelsky

Affiliation : Insitute for Low Temperature Physics & Kharazin Kharkiv National University, Ukraine

Lieu : Université Lille - Faculté des Sciences et Technologies,
Cité Scientifique – bât. M1 salle 302, 59655 Villeneuve-d'Ascq

Dates et l'horaire: le 20 novembre, 13 h 30 - 15 h 30 et 15 h 50 - 17 h 50
le 21 novembre, 13 h 30 - 15 h 30 et 15 h 50 - 17 h 50
le 22 novembre, 13 h 30 - 15 h 30 et 15 h 50 - 17 h 50

Ce cours permet aux doctorants de bénéficier de 6 CFD

Descriptif : A vast array of problems in mathematics and mathematical physics (diffraction, radiation, elasticity, hydrodynamic problems, orthogonal polynomials, random matrix theory etc.) can be posed as Riemann-Hilbert problems formulated as boundary value problems for analytic functions in the complex plane. The data for a RH problem depend on external parameters, which are physical variables (space, time, matrix size, etc), and, in turn, the solution depends on these parameters as well.

In this course we will present the Nonlinear Schrödinger (NLS) equation as a prototype model for using the Inverse Scattering Transform method to obtain a representation of a solution of the Cauchy problem in terms of a solution of the associated RH problem. This approach provides means to efficiently study the existence and uniqueness problems, to derive asymptotics of solutions and to evaluate the solutions inside as well as outside asymptotic regimes. The plan of the course is the following :

- Solvability of the scalar and matrix multiplicative RH problem (2 h)
- Lax pair representation of a nonlinear PDE, Jost solutions and scattering matrix for NSE, reduction of the Cauchy problem to the RH problem, soliton solutions by the RH problem approach (4 h)
- the *nonlinear* steepest descent method and long-time asymptotic analysis of the NLS solutions (4 h)
- the RH problem approach in the case of non-decaying solutions using the "g-function mechanism", reduction to the model problem solvable in terms of Riemann theta-functions and Abel integrals (2 h)