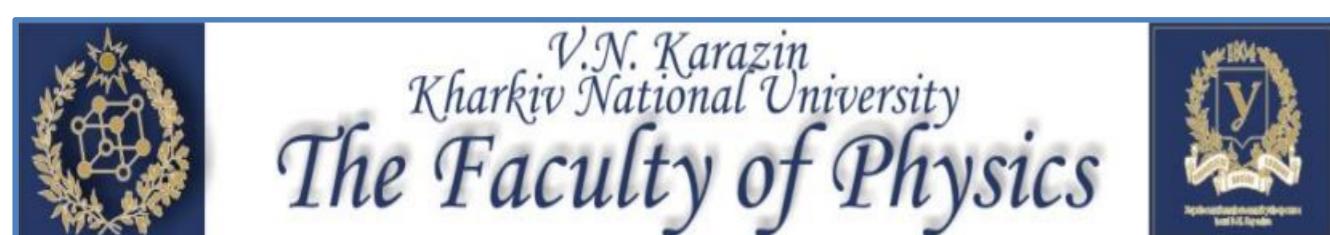
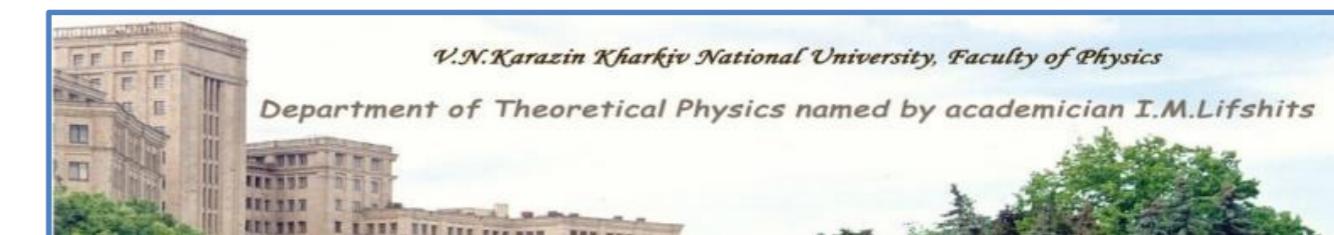


# Magnetic Properties of Two Finite spin-1/2 XX Chains Connected through Two Ising spins



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We propose exactly solvable quantum model based on finite spin-1/2 XX chains with two bridging Ising spins, connecting XX chains at two intermediate lattice sites.

Model Hamiltonian has the form:

$$\mathbf{H} = -g_1\mu_B H \sum_{n=1}^{N_1} S_{1,n}^z - J_1 \sum_{n=1}^{N_1-1} (S_{1,n}^x S_{1,n+1}^x + S_{1,n}^y S_{1,n+1}^y) - g_2\mu_B H \sum_{n=1}^{N_2} S_{2,n}^z - J_2 \sum_{n=1}^{N_2-1} (S_{2,n}^x S_{2,n+1}^x + S_{2,n}^y S_{2,n+1}^y) - g_{01}\mu_B H \sigma_1^z - g_{02}\mu_B H \sigma_2^z - J_{01}\sigma_1^z (S_{1,n_1}^z + S_{2,n_3}^z) + J_{02}\sigma_2^z (S_{1,n_2}^z + S_{2,n_4}^z).$$

Z-projections of Ising additional spins commute with model Hamiltonian and are the good quantum numbers. This property permits us to consider Hamiltonian (1), as the Hamiltonian of the finite XX-chain with an effective impurity spin  $S = 1/2$  at lattice sites  $1, n_1; 1, n_2; 2, n_3; 2, n_4$ . Spin-1/2 XX chain is the well known example of exactly solvable model [1]. The XX model Hamiltonian one can rewrite as the Hamiltonian of ideal gas of spinless fermions.

## Dispersion relations

$$\left(1 + \beta_1 x_1 \frac{1 - x_1^{-2n_1}}{x_1^2 - 1}\right) \left(1 + \beta_2 x_1 \frac{(1 - x_1^{-2(N_1+1-n_2)})}{x_1^2 - 1}\right) x_1^{N_1+1} - \left(1 + \beta_1 x_1 \frac{(1 - x_1^{2n_1})}{1 - x_1^2}\right) \left(1 + \beta_2 x_1 \frac{(1 - x_1^{2(N_1+1-n_2)})}{1 - x_1^2}\right) x_1^{-(N_1+1)} = 0;$$

$$\left(1 + \beta_1 x_2 \frac{1 - x_2^{-2n_3}}{x_2^2 - 1}\right) \left(1 + \beta_2 x_2 \frac{(1 - x_2^{-2(N_2+1-n_4)})}{x_2^2 - 1}\right) x_2^{N_2+1} - \left(1 + \beta_1 x_2 \frac{(1 - x_2^{2n_3})}{1 - x_2^2}\right) \left(1 + \beta_2 x_2 \frac{(1 - x_2^{2(N_2+1-n_4)})}{1 - x_2^2}\right) x_2^{-(N_2+1)} = 0;$$

## Low temperature thermodynamics

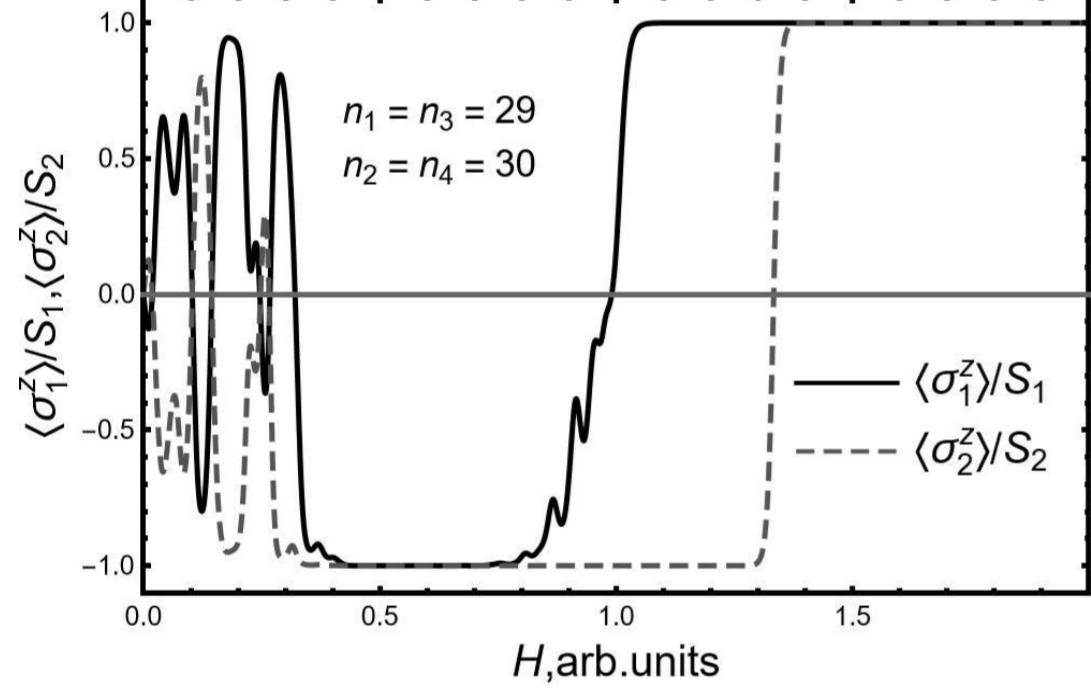
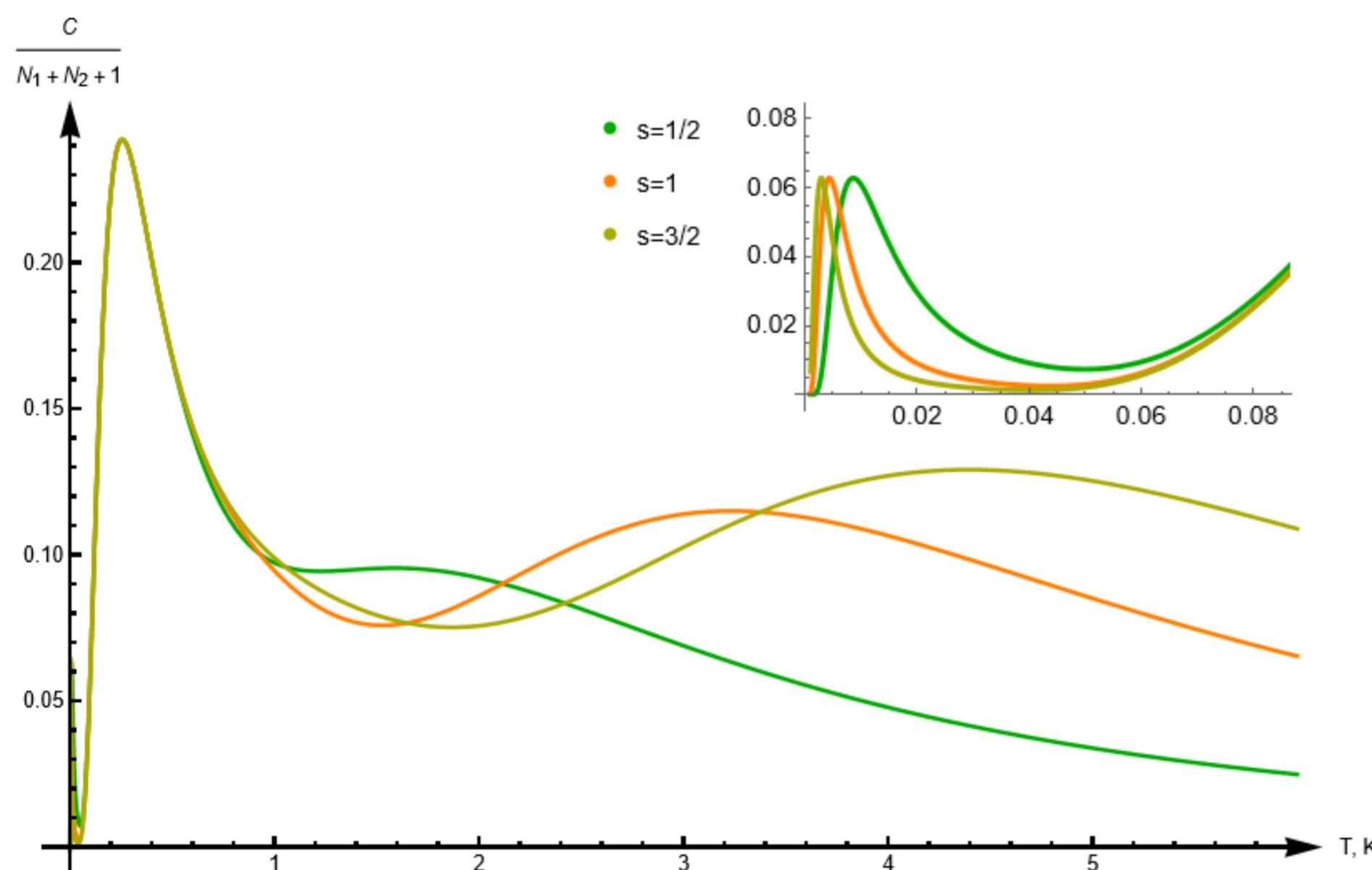


Fig. 1. Field dependence of  $\langle \sigma_i^z \rangle / S_i$ ,  $i=1,2$ ; at  $T = 0.01$ ,  $g_1 = 1$ ,  $g_2 = 2$ ,  $g_{01} = 1$ ,  $g_{02} = 1.5$ ,  $J_1/J_2 = 2$ ,  $J_{01}/J_2 = -1$ ,  $J_{02}/J_2 = -2$ ,  $N_1 = N_2 = 50$

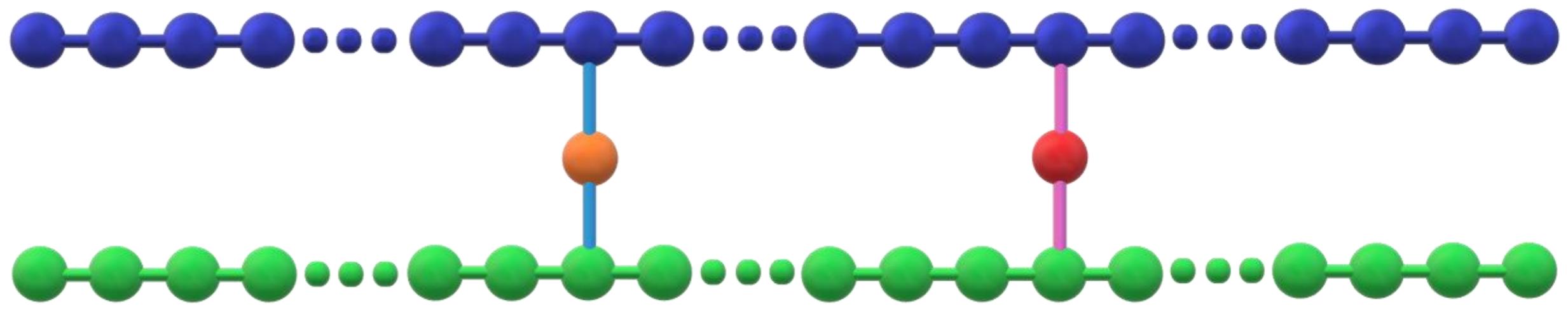


## Summary:

- We derive exact dispersion equations for the stationary states with one inverted spin. The spectrum consists of two quasi-continuous zones and several localized impurity levels.
- The peculiarities of field and temperature dependences of the thermodynamic characteristics of the model were investigated numerically.
- The behavior of the average  $z$ -projection of Ising spins and longitudinal Ising impurity spin-spin correlation functions at low temperatures have been studied numerically.
- It was shown that under certain conditions, the average  $z$ -spin projection for impurity spins may have the finite jumps and non-monotonic dependence on the magnetic field at very low temperatures.
- Field dependence of magnetization may demonstrate two intermediate plateaus for strong AF Ising interactions at very low temperatures.
- We found numerically three maxima behavior of zero field temperature dependence of specific heat at some values of model parameters.

## References

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- [2] P.M. Duxbury, J. Oitmaa, M.N. Barber, A. van der Bilt, K.O. Joung, R.L. Carlin, Phys. Rev. B 24, 5149 (1981). DOI: <https://doi.org/10.1103/PhysRevB.24.5149>
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$$\left(1 + \beta_1 x_2 \frac{1 - x_2^{-2n_3}}{x_2^2 - 1}\right) \left(1 + \beta_2 x_2 \frac{(1 - x_2^{-2(N_2+1-n_4)})}{x_2^2 - 1}\right) x_2^{N_2+1} - \left(1 + \beta_1 x_2 \frac{(1 - x_2^{2n_3})}{1 - x_2^2}\right) \left(1 + \beta_2 x_2 \frac{(1 - x_2^{2(N_2+1-n_4)})}{1 - x_2^2}\right) x_2^{-(N_2+1)} = 0;$$

## Low temperature thermodynamics

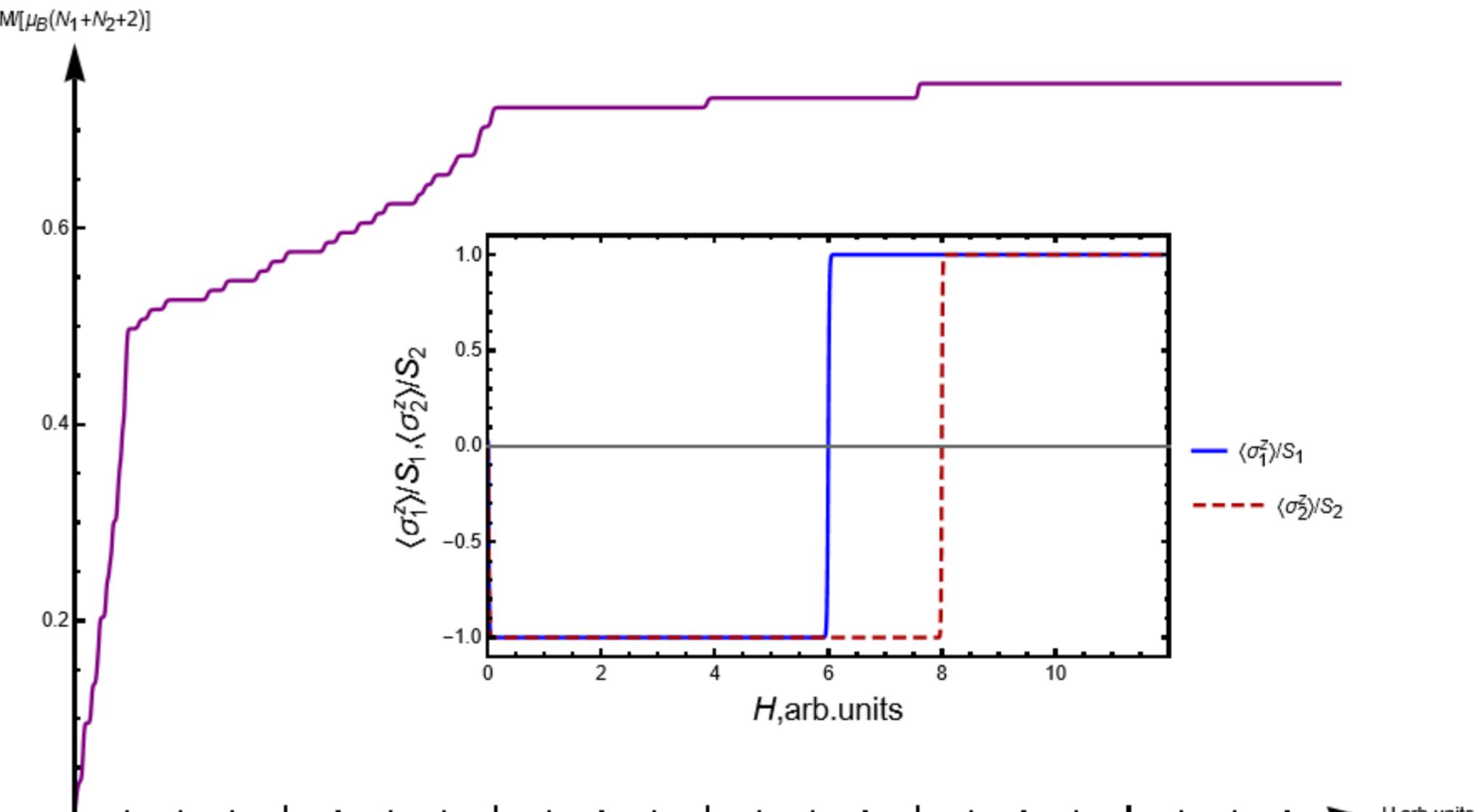


Fig. 2. Field dependence of magnetization per spin at  $T = 0.01$ ,  $g_1 = 1$ ,  $g_2 = 2$ ,  $g_{01} = 1$ ,  $g_{02} = 1.5$ ,  $J_1/J_2 = 4$ ,  $J_{01}/J_2 = -6$ ,  $J_{02}/J_2 = -12$ ,  $N_1 = N_2 = 50$ ,  $n_1 = n_3 = 20$ ,  $n_2 = n_4 = 40$

Fig. 3. Temperature dependence of heat capacity at different spin values at  $H = 0$ ,  $g_1 = 1$ ,  $g_2 = 1$ ,  $g_{01} = 1.2$ ,  $g_{02} = 1.3$ ,  $J_1/J_2 = 1$ ,  $J_{01}/J_2 = 10$ ,  $J_{02}/J_2 = 2$ ,  $N_1 = N_2 = 6$ ,  $n_1 = n_2 = n_3 = n_4 = 3$ ,

