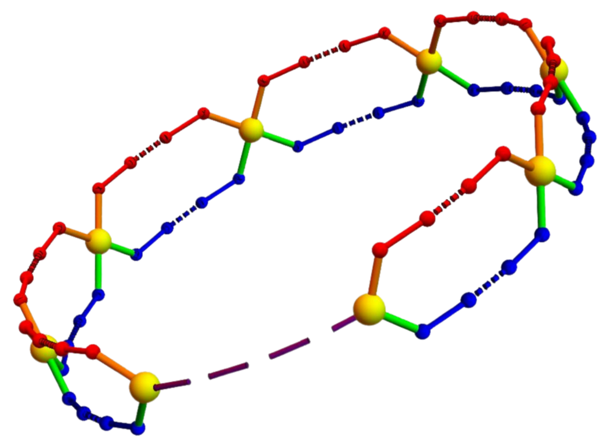
Low-temperature Thermodynamics of Branched Spin-1/2 System Formed by XX Chains Connected through Ising Spins

K. S. Dzhenzherova, E. V. Ezerskaya, V. O. Kovalenko
V. N. Karazin Kharkiv National University 4 Svoboda Sq, Kharkiv, 61022, Ukraine
e-mail: xa13346957@student.karazin.ua

We propose mesoscopic spin-ring model formed by finite spin-1/2 XX chains "pinned" periodically via additional Ising spins.



The model Hamiltonian is

$$\hat{\mathbf{H}} = -\sum_{l=1}^{L}\{\sum_{i=1}^{2}\left[g_i\mu_B H\sum_{n-1}^{N_i} S^z_{i,l,n} + J_i\sum_{n-1}^{N_i-1}\left(S^x_{i,l,n}S^x_{i,l,n+1}+S^y_{i,l,n}S^y_{i,l,n+1}\right)\right] - g_0\mu_B H\sigma^z_l + I_i\left(\sigma^z_l S^z_{i,l,1}+\sigma^z_{l+1}S^z_{i,l,N_i}\right)\}$$

Taking into account the periodicity and commutation relation $[\hat{\mathbf{H}},\sigma^z_l]=0$, one can rewrite the Hamiltonian as

$$\hat{H}(\sigma_1,\ldots,\sigma_L)=\sum_{l=1}^{L}\left[\hat{H}_1(\sigma_l,\sigma_{l+1})+\hat{H}_2(\sigma_l,\sigma_{l+1})\right],\quad \sigma_{L+1}\to\sigma_1$$

$$\hat{H}_i(\sigma_l,\sigma_{l+1}) = E_0^{(i)} + (g_i\mu_B H + I_{\sigma_l})a^\dagger_{i,l,1}a_{i,l,1} + (g_i\mu_B H + I_{\sigma_{l+1}})a^\dagger_{i,l,N}a_{i,l,N} + g_i\mu_B H\sum_{n=2}^{N-1}a^\dagger_{i,l,n}a_{i,l,n} - \frac{J_i}{2}\sum_{n=1}^{N-1}\left(a^\dagger_{i,l,n+1}a_{i,l,n}+a^\dagger_{i,l,n}a_{i,l,n+1}\right),\quad i=1,2$$

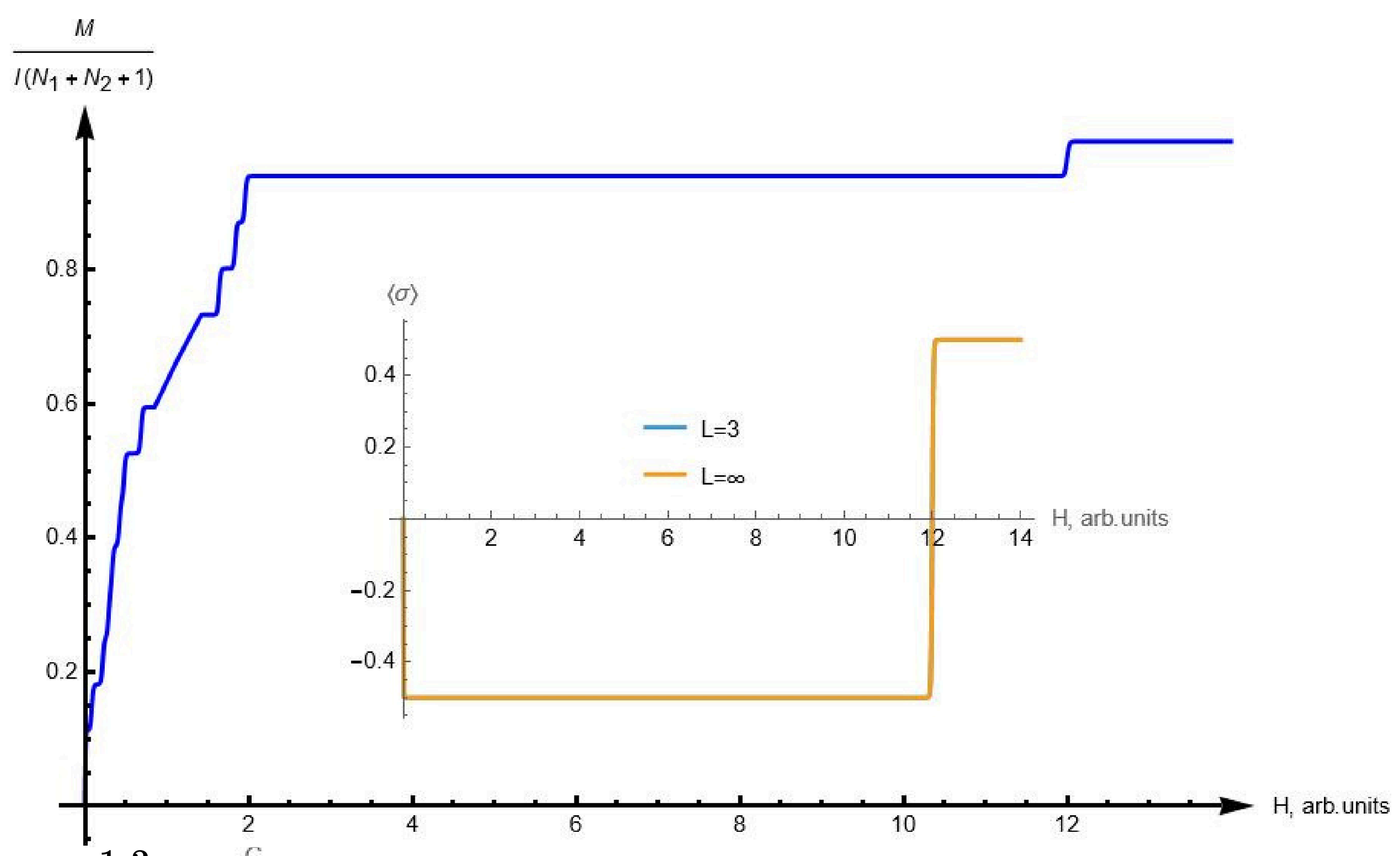
$\hat{H}_i(\sigma_l,\sigma_{l+1})$ are the Hamiltonians of finite spin-1/2 XX chain with two "impurities" at both ends in terms of spinless fermions [1,2] with dirspersion relations:

$$(1+\beta_i x)(1+\alpha_i x) - x^{2(N_i+1)}(1+\alpha_i x^{-1})(1+\beta_i x^{-1}) = 0,\quad \alpha_i=\frac{2I_i\sigma_l}{J_i},\quad \beta_i=\frac{2I_i\sigma_{l+1}}{J_i}$$

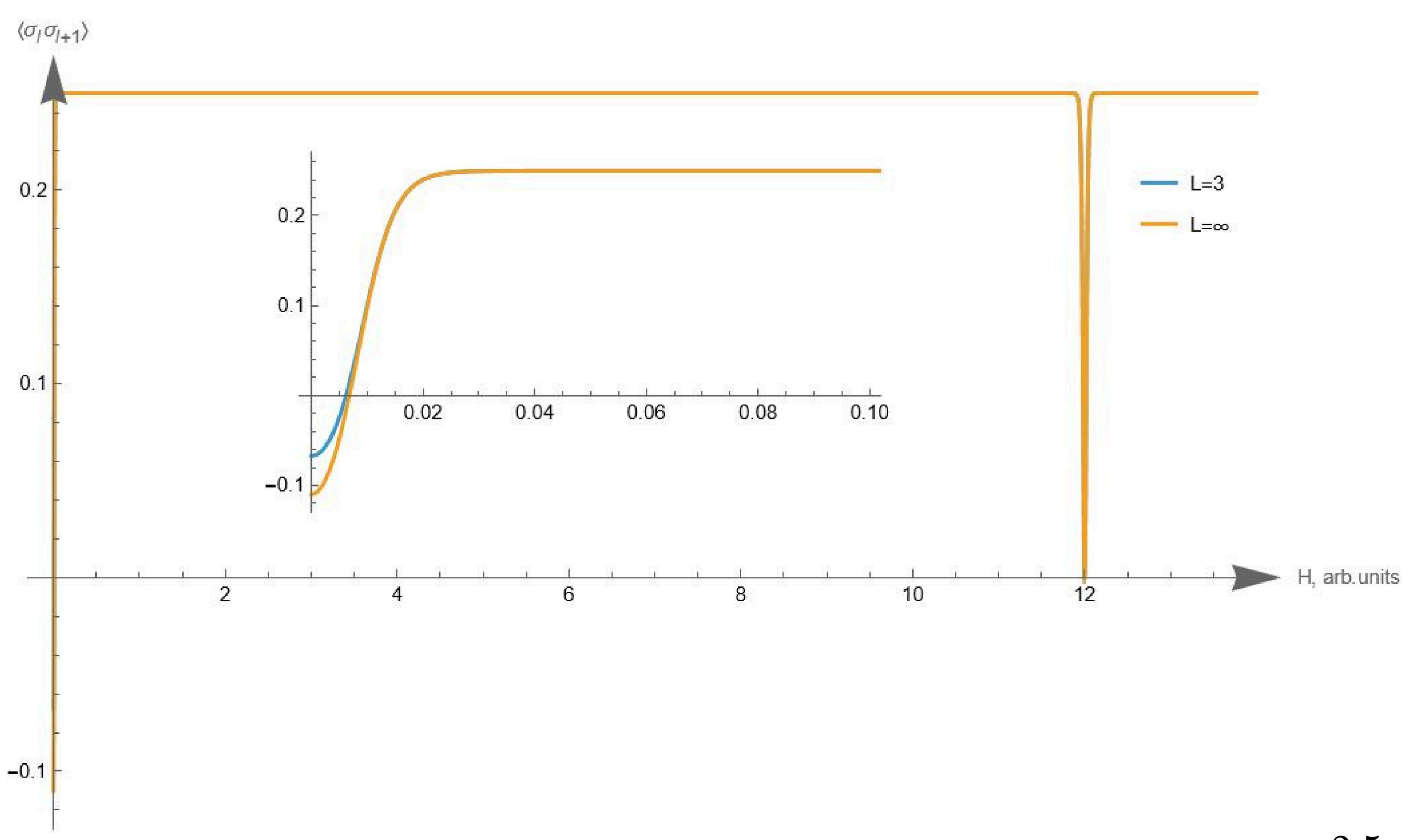
By means of a standard transfer-matrix scheme, we obtained the exact partition function of the above system

$$Z=\mathrm{Tr}\,T^L,\ \ T(\sigma_l,\sigma_{l+1})=\exp\left[-\frac{E_0(\sigma_l,\sigma_{l+1})}{T}\right]\prod_\lambda\left[1+\exp\left(-\frac{\varepsilon_\lambda(\sigma_l,\sigma_{l+1})}{T}\right)\right]\prod_\kappa\left[1+\exp\left(-\frac{\varepsilon_\kappa(\sigma_l,\sigma_{l+1})}{T}\right)\right]$$

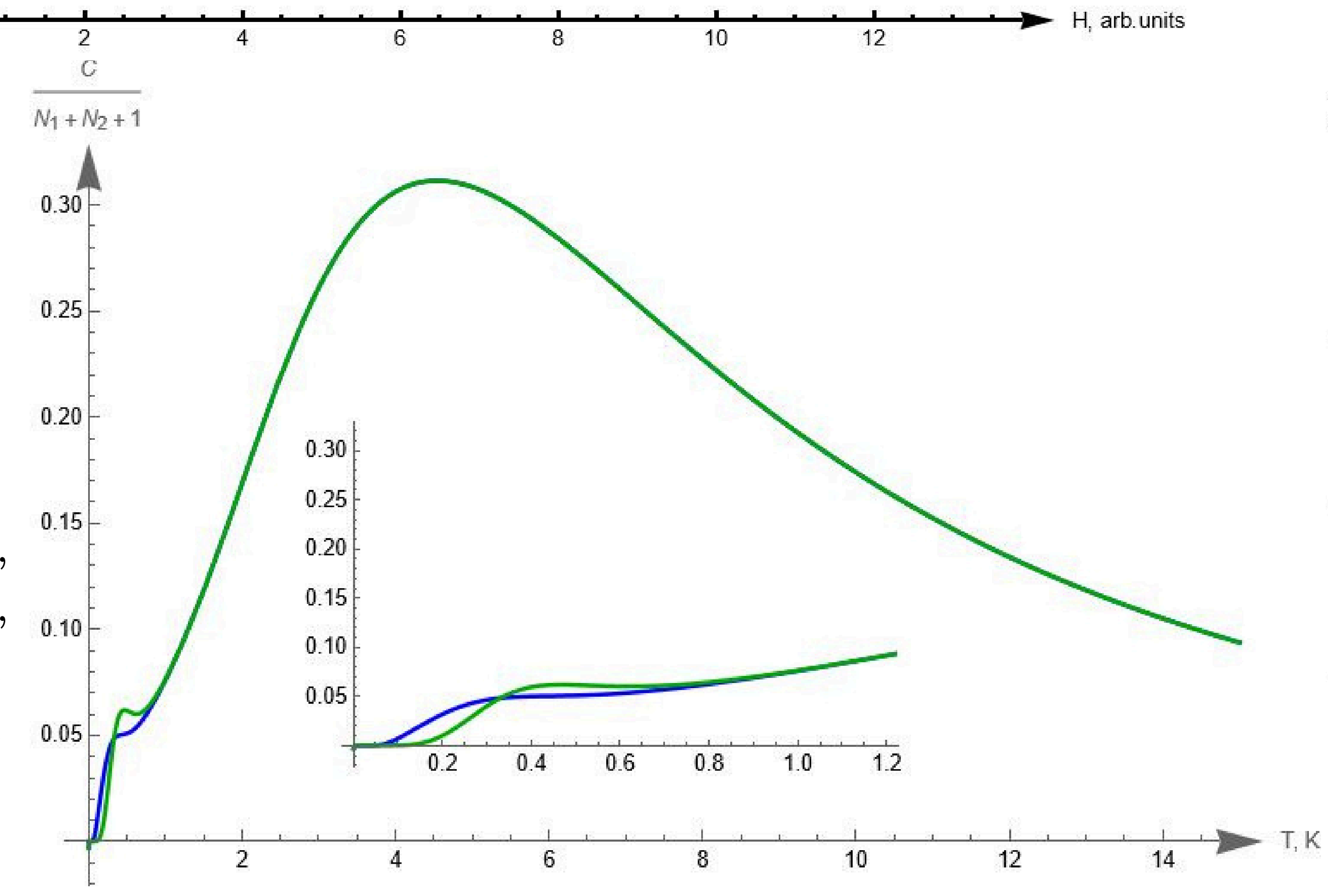
Modeling

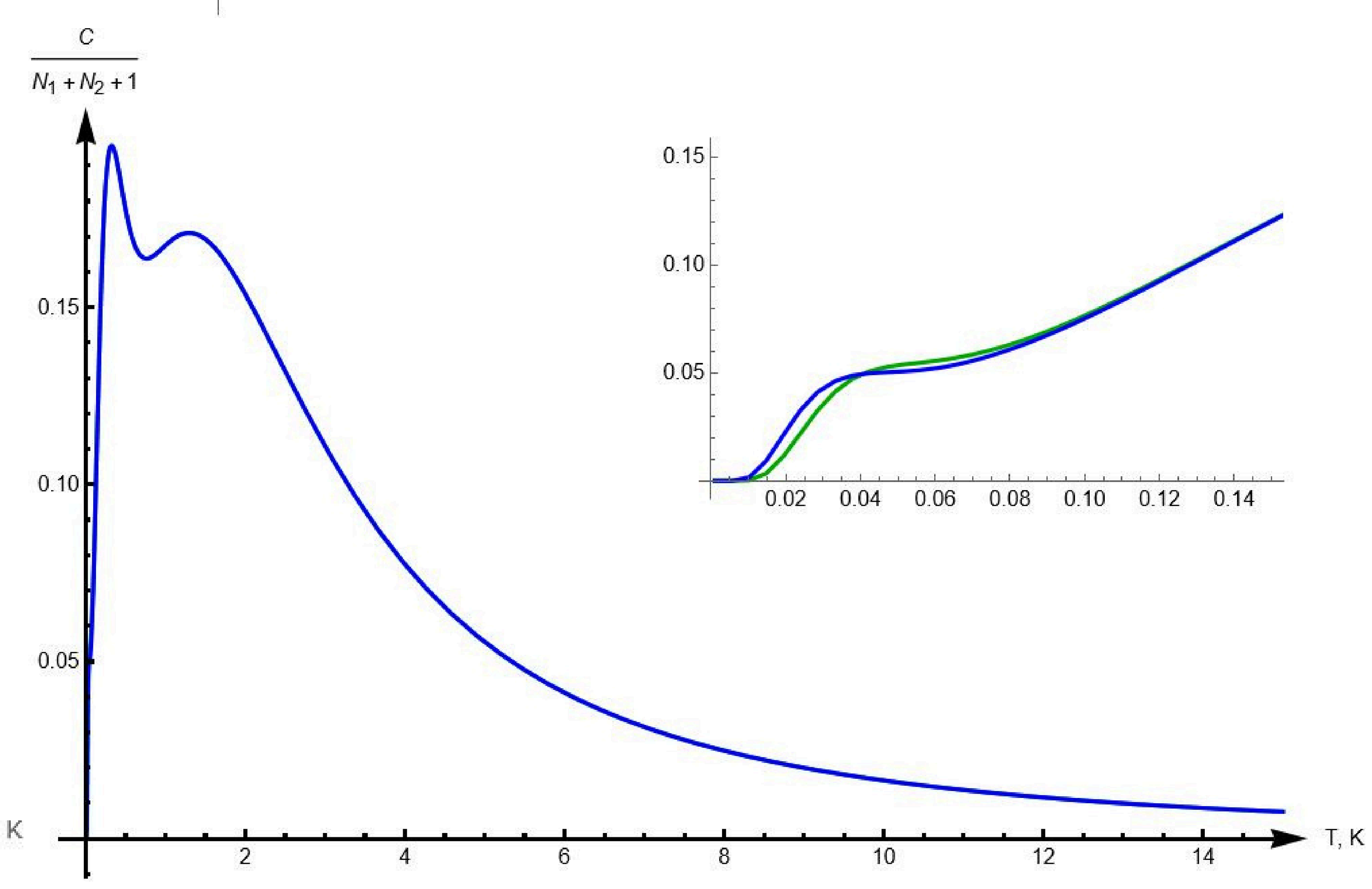


$g_0=1.5$, $\mu_B=1$, $g_1=2$, $g_2=2$, $J_1=1$, $J_2=4$, $I_1=-6$, $I_2=-12$, $S_1=1/2$, $S_2=1/2$, $N_1=12$, $N_2=16$.



$g_0=1.3$, $\mu_B=1$, $g_1=1$, $g_2=2$, $J_1=15$, $J_2=13$, $I_1=10$, $I_2=20$, $S_1=1/2$, $S_2=1/2$, $N_1=16$, $N_2=10$, $H=0$, $l=2$;





$g_0=2.5$, $\mu_B=1$, $g_1=1$, $g_2=2$, $J_1=1$, $J_2=5$, $I_1=6$, $I_2=10$, $S_1=1/2$, $S_2=1/2$, $N_1=16$, $N_2=10$, $H=0$, $l=4$.

Summury:

We investigated the energy spectra and low-temperature magnetic behavior of mesoscopic spin-ring model characterized by their specific lattice topology.

We performed numerical simulations of the low-temperature thermodynamics. The field dependence of magnetization may exhibit a finite jump due to antiferromagnetic Ising impurities, and the temperature dependence of specific heat may display several maxima at zero magnetic field.

[1] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. 16, 407 (1961).

[2] A.A. Zvyagin, Quantum Theory of One-Dimensional Spin Systems, Cambridge Scientific Publishers, Cambridge, 2010. – 330 p.