TWO-PHOTON INTERACTION IN A SUPERCONDUCTING CIRCUIT WITH SQUID-MEDIATED COUPLING

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INTRODUCTION

In this work, we study interactions between a resonator and an rf SQUID (Superconducting QUantum Interference Device) coupled via a Josephson element. The latter is a symmetrical dc SQUID, which induces two-photon and other inductive interactions. By changing the SQUID magnetic bias, one can control two- and single-photon interaction rates [1].

In addition to the two-photon interaction mediated by the SQUID coupler, we consider cross-Kerr, linear, and optomechanical interactions. We focus on the regime of resonant two-photon interaction. We calculate the rates of relevant interactions, including renormalizations near the two-photon resonance.

MODEL

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_c + \overbrace{\hat{\mathcal{H}}_r + \hat{\mathcal{H}}_a}^{\text{Resonator and atom Hamiltonians}}$$

$$\hat{\mathcal{H}}_{r,a} = \omega^p_{r,a}\left(m^\dagger m + \tfrac{1}{2}\right) - X_{r,a}\left(m^\dagger + m\right)^3 - Y_{r,a}\left(m^\dagger + m\right)^4, \quad m \in \{a, b\}$$

qubic and quartic self-nonlinearities

Coupler Hamiltonian (under two-photon resonance)

$$\hat{\mathcal{H}}_c = \underbrace{-g_2\left(a^{\dagger 2} b + b^\dagger a^2\right)}_{\text{two-photon interaction}} - \underbrace{K_0 a^\dagger a b^\dagger b}_{\text{cross-Kerr interaction}} + \hat{\mathcal{H}}_{n.r.}$$

Non-resonant interactions, which we treat as perturbation

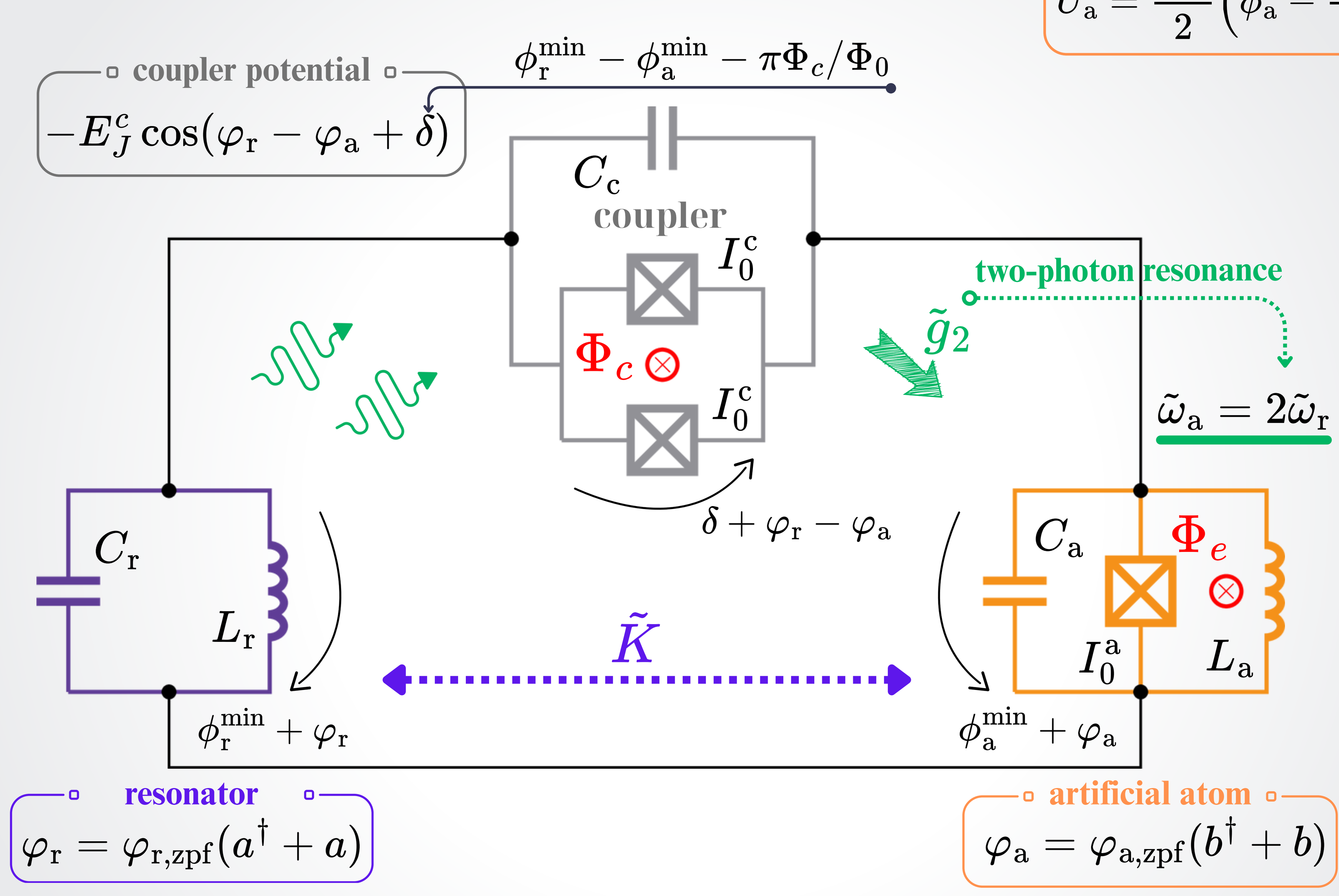
$$\hat{\mathcal{H}}_{n.r.} = \underbrace{-g_1^c\left(a^\dagger - a\right)\left(b^\dagger - b\right)}_{\text{capacitive interaction}} - \underbrace{g_1^i\left(a^\dagger + a\right)\left(b^\dagger + b\right)}_{\text{inductive interaction}}$$

$$-g_2\left(2a^\dagger a + 1\right)\left(b^\dagger + b\right) - G_2\left(2b^\dagger b + 1\right)\left(a^\dagger + a\right) + \cdots$$

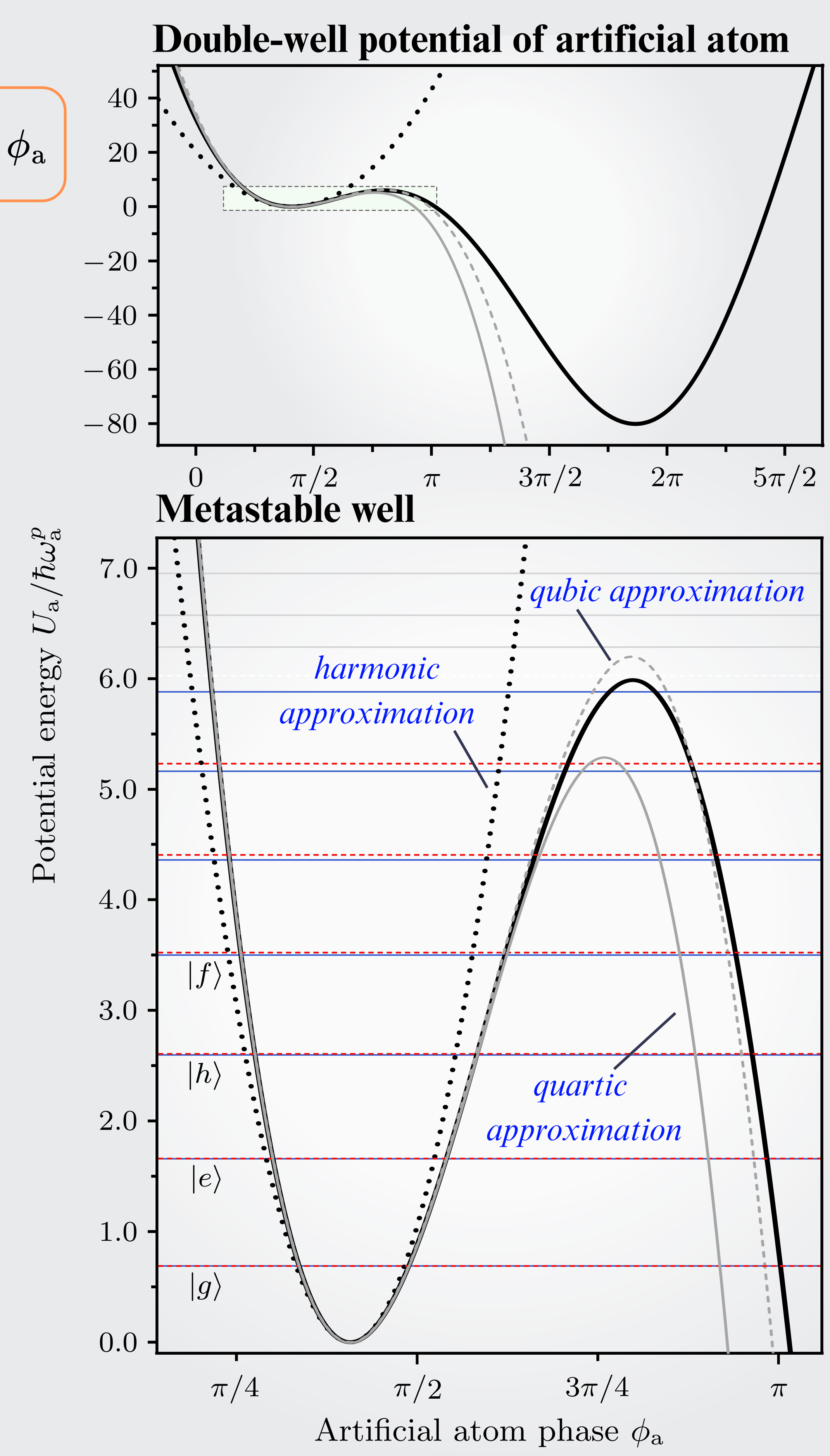
optomechanical-type interactions

Artificial atom potential

$$U_a = \frac{E_L^a}{2}\left(\phi_a - \frac{2\pi\Phi_e}{\Phi_0}\right) - E_J^a \cos\phi_a$$



Circuit parameters	resonator	coupler	artificial atom
capacitance, fF	692.8	5.0	300.0
inductance, nH	0.84686	—	1.0000
critical current, nA	—	30.0	1100.0

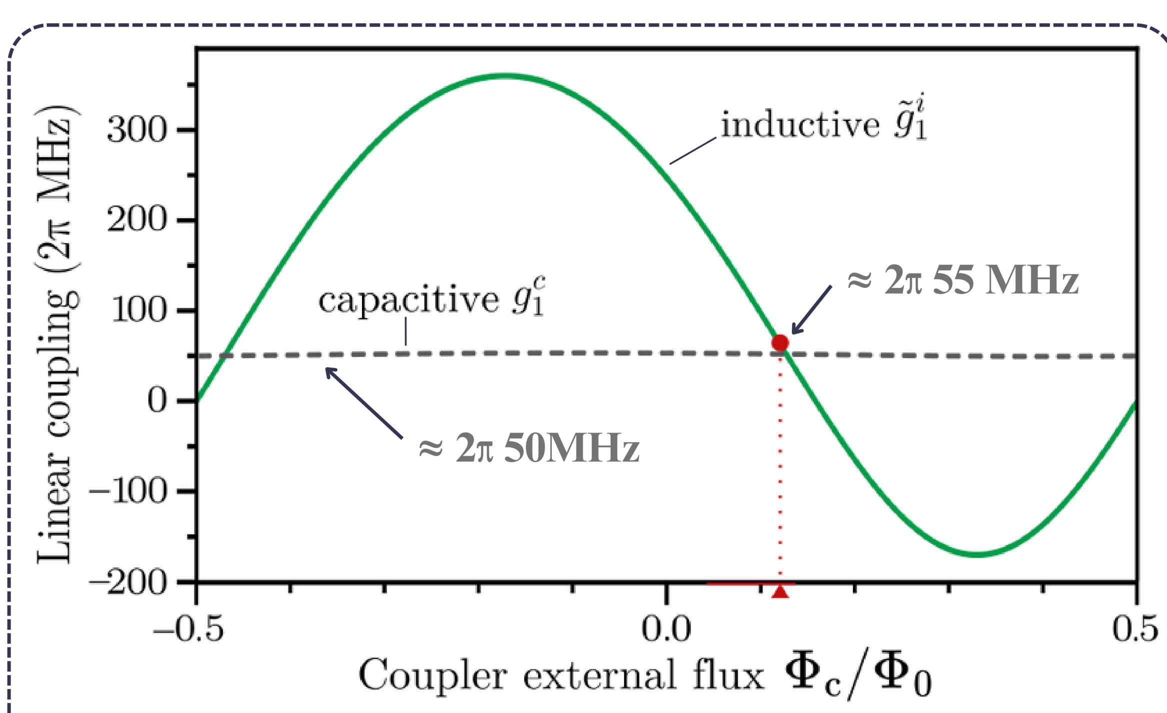


RESULTS

To obtain our results, we focus on the regime of two-photon resonance $\tilde\omega_a = 2\tilde\omega_r$. Then we have used a series of Schrieffer-Wolff transformations*, to diagonalize resonator and atom Hamiltonians, and eliminate non-resonant linear and optomechanical interactions.

Hamiltonian close to the two-photon resonance

$$\frac{\hat{\mathcal{H}}}{\hbar} \approx \tilde\omega_r a^\dagger a + \tilde\omega_a b^\dagger b - \underbrace{\frac{\Xi_a}{2}}_{\text{self-Kerr nonlinearity}} b^{\dagger 2} b^2 - \overbrace{\tilde K a^\dagger a b^\dagger b - \tilde g_2\left(a^{\dagger 2} b + b^\dagger a^2\right)}^{\text{dominant interactions}}$$



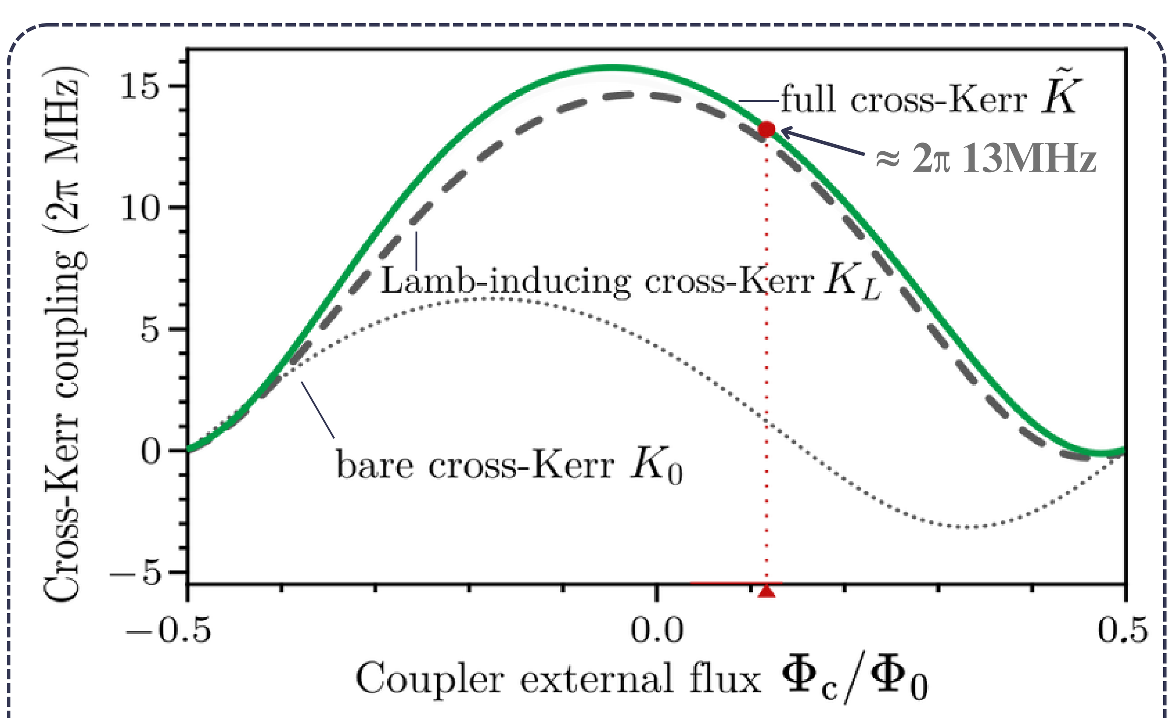
bare inductive coupling

$$\tilde g_1^i \approx g_1^i - \frac{20 G_2 X_a}{\omega_a^p} + \cdots$$

small corections

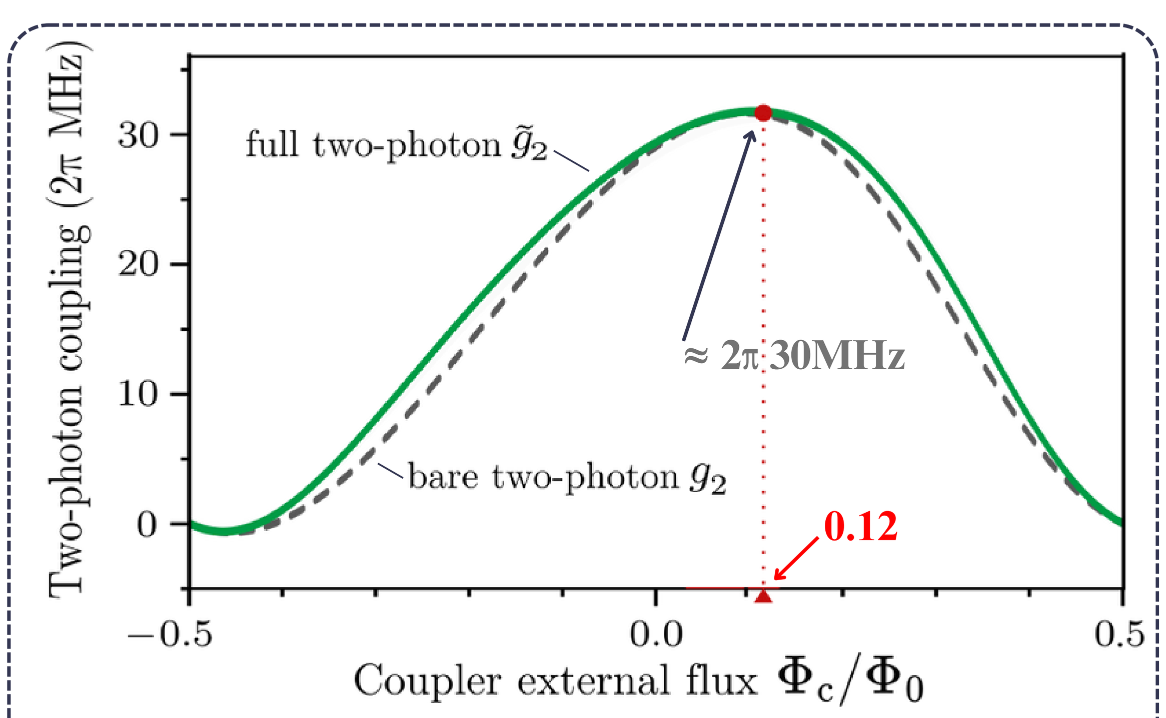
arise due to the dressing of the interaction G_2 by the qubic nonlinearity in the atom potential

$$g_\pm = g_1^c \pm \tilde g_1^i$$



leading renormalization

$$\tilde K = K_0 + \overbrace{\frac{24 g_2 X_a}{\omega_a^p}}^{K_L} + \frac{2\Xi_a g_-^2}{(\tilde\omega_a - \tilde\omega_r)(\tilde\omega_a - \tilde\omega_r - \Xi_a)} + \frac{2\Xi_a g_+^2}{(\tilde\omega_a + \tilde\omega_r)(\tilde\omega_a + \tilde\omega_r - \Xi_a)} + \frac{16\Xi_a g_2^2}{\tilde\omega_a(\tilde\omega_a - \Xi_a)} + \cdots$$



dressed optomechanical coupling

$$\tilde g_2 = g_2 + \frac{2g_-}{\tilde\omega_r}\overbrace{\left(G_2 + \frac{6 g_1^i X_a}{\omega_a^p}\right)} - \frac{4}{3\tilde\omega_r}\left(G_2 + \frac{1}{6}\mu\varphi_{a,zpf} g_1^c\right)\left(g_+ - \frac{5}{3}\mu\varphi_{a,zpf} G_2\right) + \cdots$$

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[1] arXiv:2412.05016