Breathers dumping by extended moving domain walls in highly dispersive nonlinear systems

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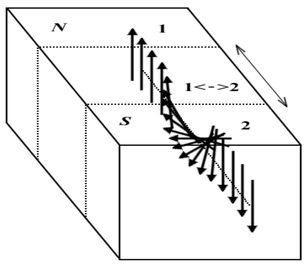
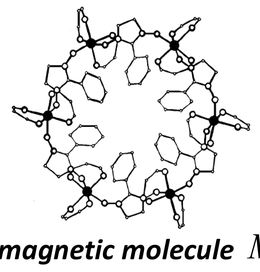
1. Topological excitations in condensed matter physics.

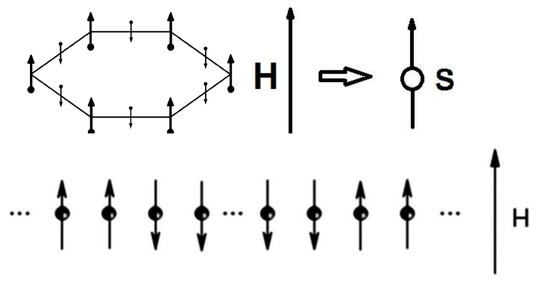


Domain wall in a ferromagnet

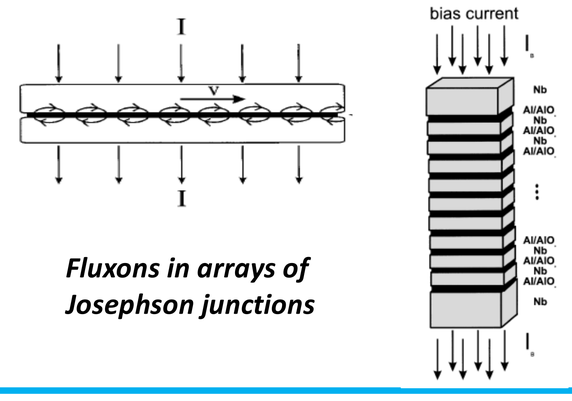


Ferrimagnetic molecule Mn_6R_6

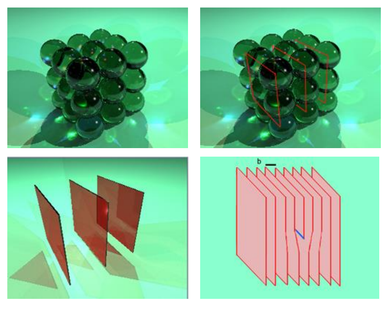
V.V. Kostyuchenko, I.M. Markevtsev, et al., *Phys. Rev. B: Condens. Matter* **67**, 184412 (2003)



1D metamaterial from magnetic molecules with S>>1



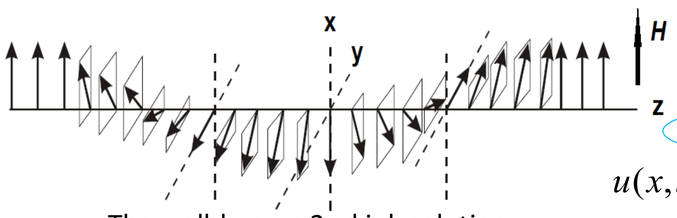
Fluxons in arrays of Josephson junctions



Dislocations in crystals

2. Theoretical task: Breather generation by moving nonequilibrium domain wall in 1D ferromagnet described by the sine-Gordon equations.

The purpose of the work is to build a theory of the evolution of a non-equilibrium moving domain wall in the 1D easy-plane ferromagnet placed in a strong magnetic field.



The well-known 2π-kink solution, corresponding to the stationary moving 360°domain wall with $k = 1/\sqrt{1-V^2}$

$$\mathcal{H} = -J\sum_n \mathbf{S}_n\mathbf{S}_{n+1} + \frac{1}{2}\sum_n D(S_n^z)^2 - g\mu_B H\sum_n S_n^x$$

$t = \sqrt{h}\tau\,,\ x = \sqrt{h}X$ ⇨ $u_{tt} - u_{xx} + \sin u = 0$

$u(x,t)$ - the spin azimuthal angle

The integrable SG equation

$$E = \int_{-\infty}^{\infty}\left[\frac{1}{2}\left(u_\tau^2 + u_X^2\right) + 1 - \cos u\right]dX \quad\Rightarrow\quad u_{\tau\tau} - u_{XX} + \sin u = 0$$

The energy and momentum integrals: $E_{SG} = \int_{-\infty}^{\infty}\left[\frac{1}{2}\left(u_t^2 + u_x^2\right) + 1 - \cos u\right]dx$

$$u_{2\pi}(x,t) = 4\arctan\exp\left[k(x - Vt)\right] \quad\Rightarrow\quad \textbf{The initial problem:} \quad \begin{cases} u(x,0) = 4\arctan\exp(kx) \\ u_t(x,0) = -kv/\cosh kx \end{cases}$$

$$P_{SG} = -\int_{-\infty}^{\infty} u_t u_x dx$$

3. Solving the evolution problem of nonequilibrium moving DW.

We start from the integrable SG equation, in which non-equilibrium moving domain wall (DW) dumps the excess energy by generating the unmovable breathers

$E_{in} = 4\left[k\left(1+v^2\right) + k^{-1}\right]$ is the energy of initial spin configuration

$P_{in} = 8kv$ is the total momentum, v is the initial velocity

The question is how will an extended initial profile behave and evolve?

From the solution of ***the direct scattering problem associated with the SG equation*** It follows that the non-equilibrium moving extented domain wall dumps unmovable breathers and then stationary moves, having the parameters:

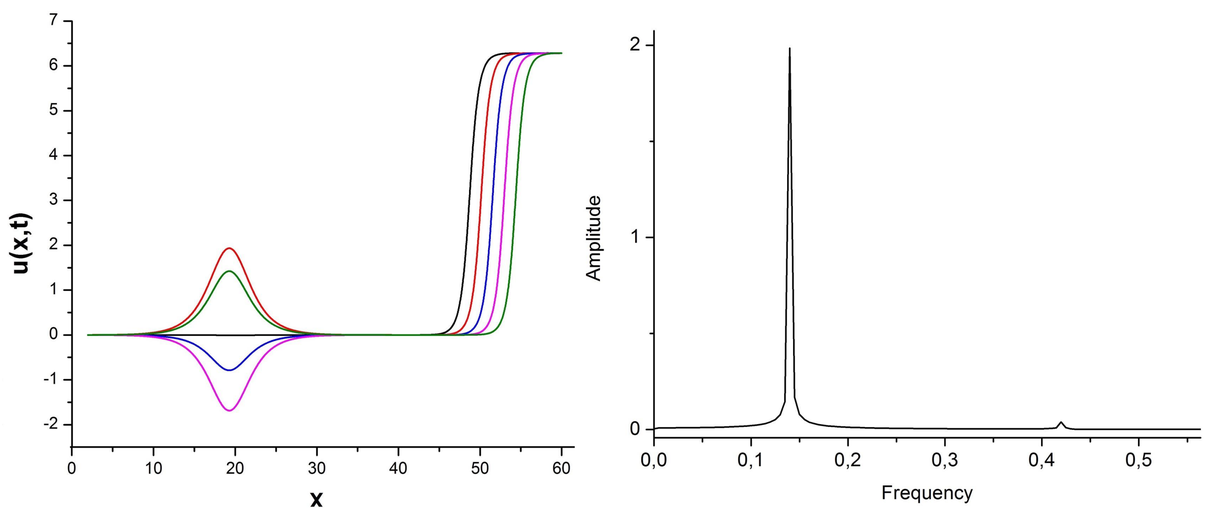
The DW final velocity is $V = kv/\sqrt{1+k^2v^2}$ in the the absence radiation, $E_{SGK} = 8/\sqrt{1-V^2}$

Every breather parameters are the following: the frequency $\omega = |\cos\chi|$

measure of amplitude $\varepsilon = |\sin\chi|$, the breather energy $E_{br} = 16\varepsilon$

The no-radiation condition requires $k_N = 1/\sqrt{(2N+1)^2 - v^2}$, where N is a number of breathers.

Then the initial energy is equal to the sum of energies of the stationary moving DW and breathers



The one-breather dumping by the moving DW and the breather frequency spectrum

The regularized sine-Gordon equation and the influence of a strong dispersion:

$$u_{tt} - u_{xx} - \beta u_{xxtt} + \sin u = 0$$

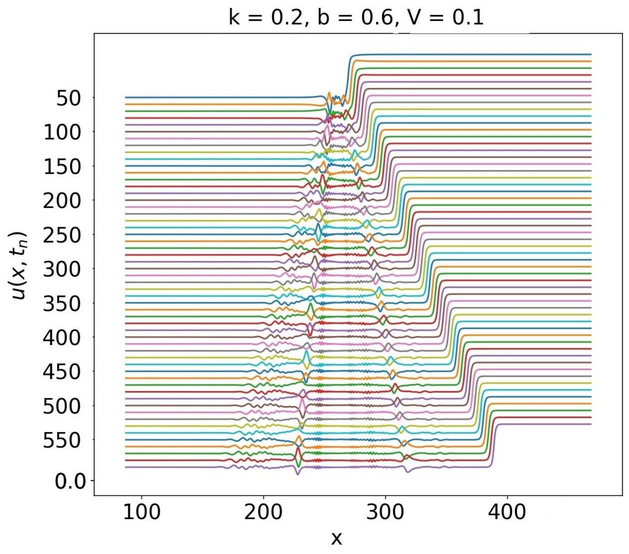
The principal differences in the breathers dumping are the following: (i) all breathers move; (ii) the breathers are metastable but long-living; (iii) they are identified by their spatial localization and the FFT spectrum with the main harmonic frequencies lying in the forbidden gap.

$$E_{RSG} = E_{SG} + \frac{\beta}{2}\int_{-\infty}^{\infty} u_{tx}^2 dx \qquad E_{RSG}^{in} = E_{in} + \frac{8}{3}\beta k^3 v^2$$

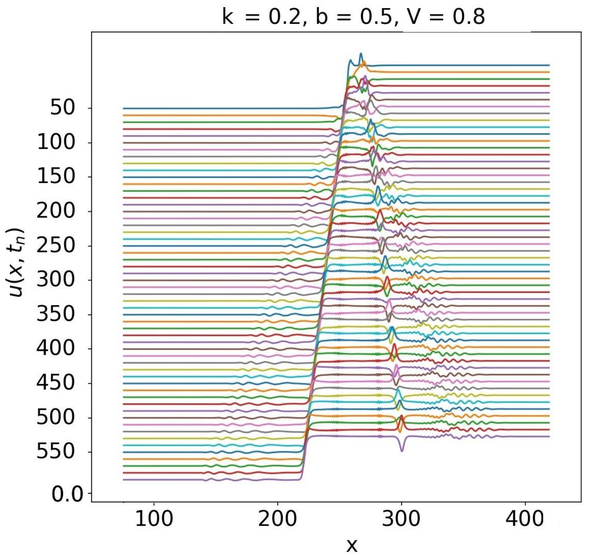
$$P_{RSG} = P_{SG} - \beta\int_{-\infty}^{\infty} u_{tx}u_{xx}dx \qquad P_{RSG}^{in} = P_{in}\left(1 + \frac{\beta}{3}k^2\right)$$

The regimes of almost radiationless motion are possible and the first integrals are distributed among nonlinear excitations quite well.

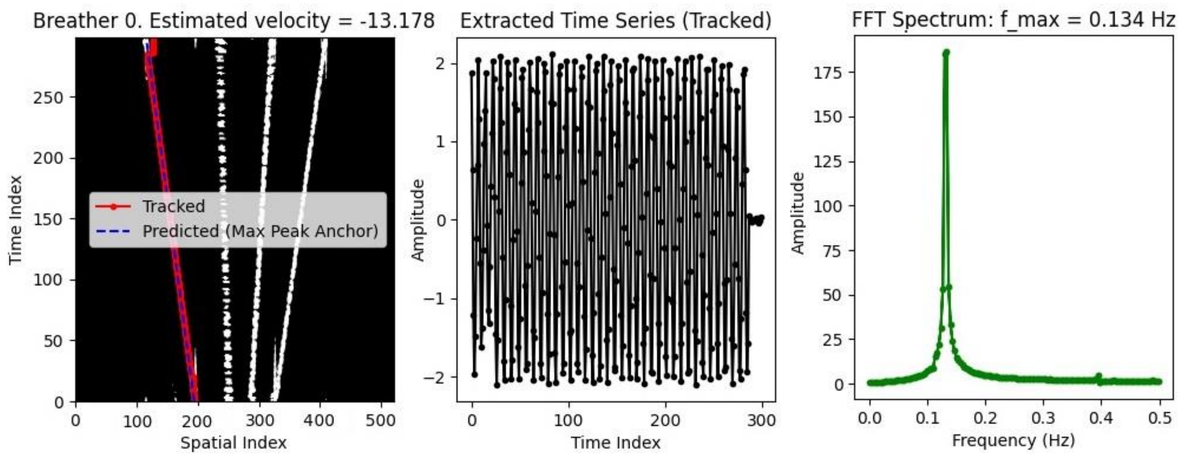
4. Breathers dumping and reactive scattering in the RSGE.



Two-breather dumping by DW moving ahead, resulting in two breathers motion in its wake.



The shooting by two breathers ahead and a backward recoil of DW first moving ahead.



Tracking the four breathers and determination of their velocities; the time series of the left breather oscillation; the FFT spectrum in the moving-with-breather reference frame.

5. Conclusion.

1. We analytically described the evolution of the nonequilibrium moving domain wall in the framework of the integrable SGE for the spin angular variable. We calculated exactly the energy distribution among all the nonlinear excitations as a result of the dissociation process, using the exact solution of the direct scattering problem associated with the SGE in the absence of radiation.
2. We studied numerically the effects of the strong dispersion of 1D ferromagnets and magnetic metamaterials, coming from their discreteness, in the framework of the regularized sine-Gordon equation having the fourth mixed derivative. We have extended the breather concept on this non-integrable equation and investigated the breather dumping process by the nonequilibrium moving domain wall.
3. We revealed that due to the higher dispersion all resulting breathers began to move and we found a large variety of the unusual reactive motions of all nonlinear excitations with their ahead and backward mutual scattering. In the one-breather dumping process the domain wall moved ahead and left the breather in its wake. In the case of two- and three-breathers dumping the initially fast domain wall could shoot the large breather ahead and was stopped itself and then moved backward. The rest breathers could move forward or backward as single excitations, but sometimes, they formed the breather molecules.
4. We calculated the frequencies of the single breathers in the reference frames moving with them, analyzed the FFT spectra of oscillations of the breathers and their “molecules” and estimated the energy and field moment distributions among all the nonlinear excitations.

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