Fluctuation pinning/depinning as a result of transmutation of diffusive gas-fluctuation modes into opposite propagating ones at the formation of two non-Hermitian topological phases originated by gas scattering on impurity center

Section 8: Theory of Condensed Matter Physics

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Introduction

problem statement

Gas Scattering on an Impurity Center:
Setting and Motivation

Main points:

- Study of non-equilibrium steady states formed from gas scattering on an impurity center.
- The system is non-Hermitian with periodic boundary conditions.
- Emergence of two spatially separated topological bulk phases.

Methods / Procedures / Model

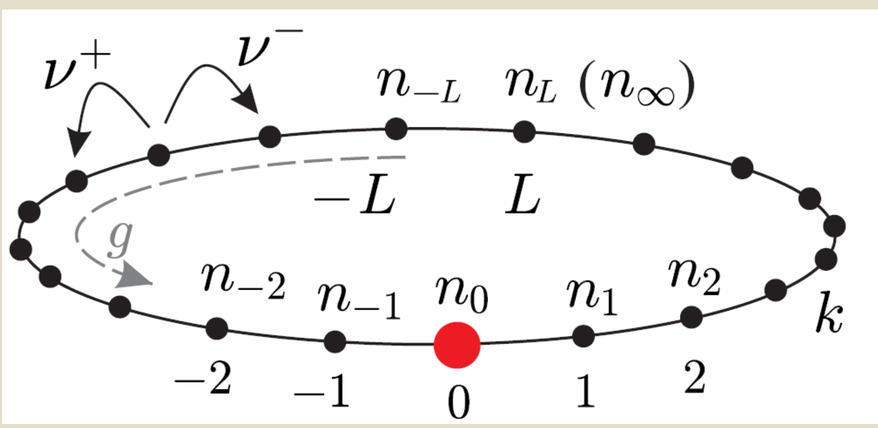
Kinetics of the mean occupation numbers $n_k(t)$ corresponds to mean-field Smoluchowsky Eqs.:

$$\partial_t n_k = f_k^g(\boldsymbol{n}) = \sum_{j=k\pm 1} \left(\nu_{jk} n_j h_k - \nu_{kj} n_k h_j\right),$$

$$\nu_{k,k\pm1} = \nu(1 \pm g)$$

$$\partial_t \delta n_k = \sum_j \left[\nu_{jk}\left(h_k^s\, \delta n_j - n_j^s\, \delta n_k\right) - (k \leftrightarrow j)\right] + \delta\tilde{I}_k$$

$$\left\langle \delta\tilde{I}_k(t)\delta\tilde{I}_{k'}(t')\right\rangle \approx 2\delta(t-t')\sum_j \nu_{kj} n_k^s\, h_j^s (\delta_{kk'} - \delta_{jk'})$$



We are interested in the possible spectrum of the system fluctuations $n_k(t) = n_k^g + \delta n_k(t)$ near its nonequilibrium steady state $\vec{n}^g$ satisfying the equations $J_{k-1,k}^g - J_{k,k+1}^g = 0$ for given constant field $g(t) = g$. Assuming δn_k and δg are small enough, the long-time dynamics of gas density fluctuations δn_k induced by the external drive noise δg is governed by the linear Langevin Eqs.

Results / Findings

In the simplified case, such gas fluctuations behavior can be reduced to the Hatano-Nelson model [3] for two different interacting topological phases with the same complex spectrum but opposite winding numbers. Despite the fluctuation eigenspectrum remains complex, the system demonstrates a non-Hermitian skin effect in each phase, and undergoes several spectral transitions, revealing an interesting interplay between non-Hermitian topology and nonequilibrium dynamics of gas fluctuations.

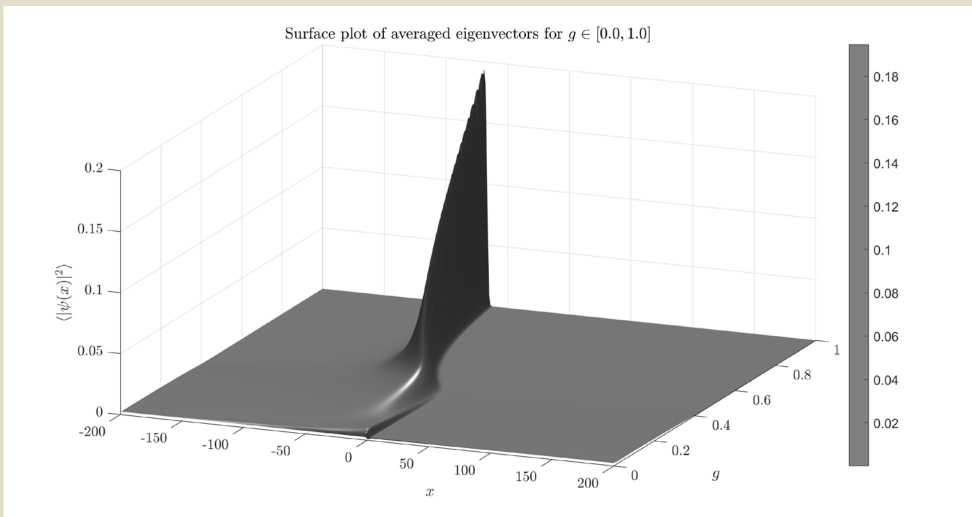
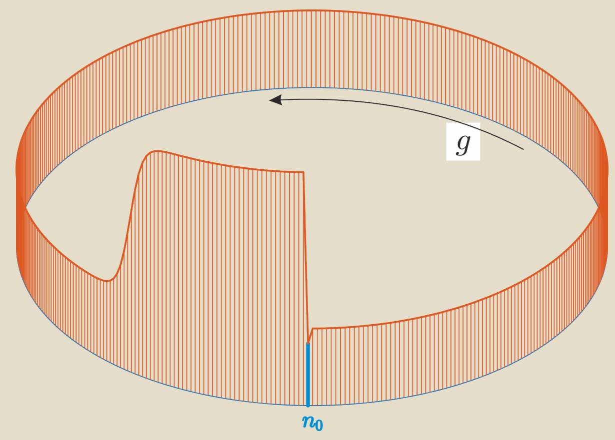


Figure 2: Localisation & Delocalisation of averaged eigenvectors as the driving field g increases.

Conclusion / Discussions

We considered the emergence of spatially separated topological phases, as a formed non-equilibrium steady state, originating from compressible gas scattering on an impurity center, which corresponds to the non-Hermitian system obeying periodical boundary conditions. The formation of such two bulk phases is accompanied by the transmutation of gas fluctuation diffusive modes into propagating ones with opposite directions in the different phases, and by a Liouvillian gap onset for fluctuation spectrum. These result in the pinning of gas fluctuations near the one of inter-phase boundary and their "repulsion" from the other one corresponding to the impurity center. This mechanism leads to the emergence of an adiabatic invariant – the sub-system (impurity-center state or the boundary one) effective protection against perturbations or fluctuations in gas.



Mode transmutation

Mechanism of Phase Formation

From Diffusion to Propagation:
Topological Transmutation of Fluctuations

Main points:

- Diffusive gas fluctuation modes become propagating in the two phases.
- Directions of propagation are opposite in each phase.
- Liouvillian gap opens in the fluctuation spectrum.

$$\delta J \sim 2g(1 - 2n^s)\delta n$$

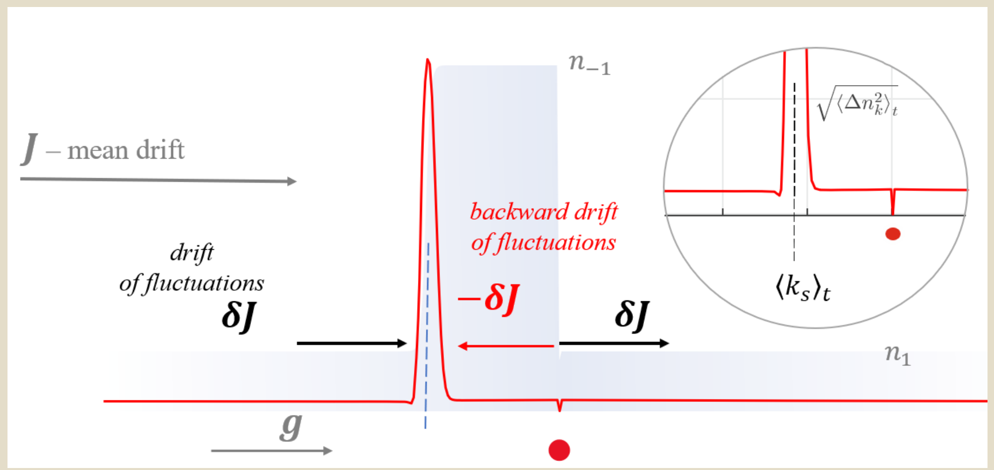


Figure 3: Repulsion of fluctuations from the impurity. Localization of fluctuations near the domain boundary of dense phase.

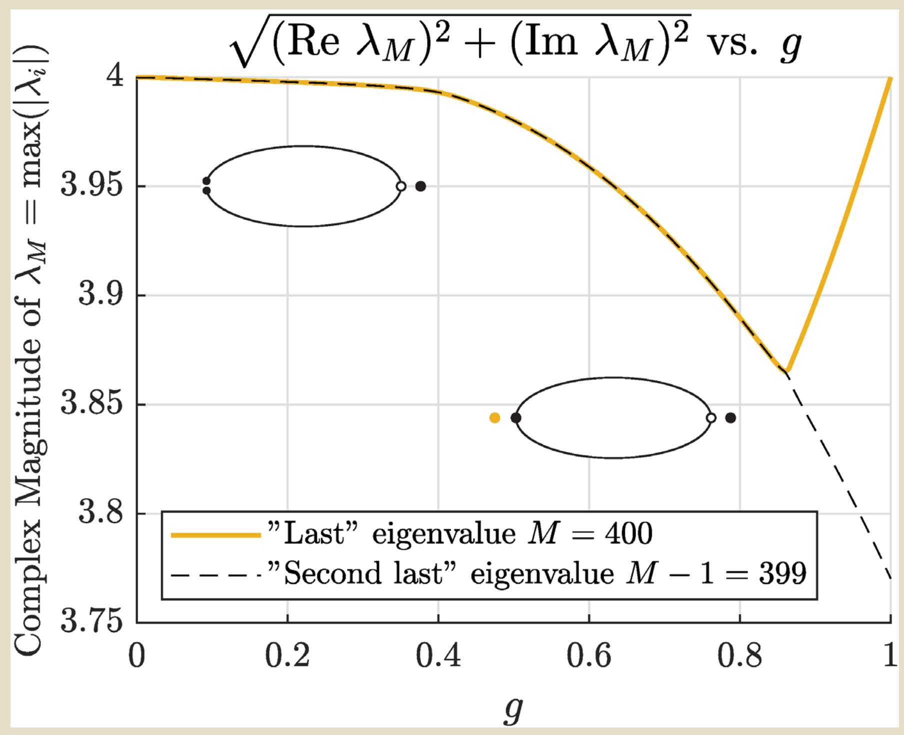


Figure 4: Spectrum of eigenvalues of the Lyapunov matrix for fluctuations.

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Condensed Matter & Low Temperature Physics 2025
2th - 6th June 2025 | online | Kharkiv, Ukraine

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