Edge states at the boundary of hexagonal and Lieb lattices in a quantizing magnetic field

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A boundary of two media with different topological order can be characterized by series of properties, the existence of which is protected. In the present work, the properties of such a boundary of two conductors are studied in a quantizing magnetic field: with charge carriers of the Dirac type and so-called spin-1 fermions, which are realized in graphene-like and Lieb lattices [1] respectively. Both lattices are characterized by the presence of a cone in the electron energy spectrum, the Klein paradox, but differ in Berry phase, which is equal to π in the graphene-like lattice, and trivial in the Lieb lattice. Which also attracts attention due to the presence of the flat band (Fig.1).

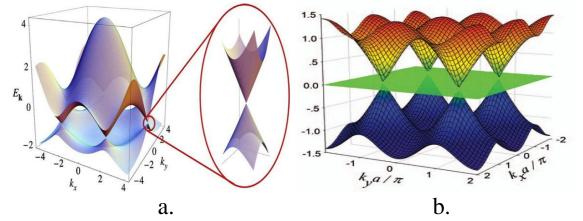


Fig.1 The electron energy spectrum of grapheme-like (a) and Lieb lattices.

Problem formulation.

Let us consider the contact between the Lieb lattice and a graphenelike lattice, as shown in Fig. 2. within the standard nearest neighbor approximation for the electronic energy spectrum

$$E = \sum_{i,j} t_{i,j} \overline{\phi}_i \phi_j, \qquad (1)$$

where $t_{i,j} = t_L$ for Lieb lattice, $t_{i,j} = t_G$ for the graphene-like lattice, and $t_{i,j} = t_M$ between atoms of two types, the summation is carried out over all pairs of neighboring atoms, ϕ_i -- amplitude of the probability of finding an electron near a site i. The energy of edge states is considered positive and small enough so that deviations from the linearity of the energy spectrum can be neglected,

$$0 < E << t_{G,L,M}$$
 (2)

It is shown that edge states arise that join the properties of conductors to the trivial and nontrivial Berry phase. The energy of edge states is calculated numerically.

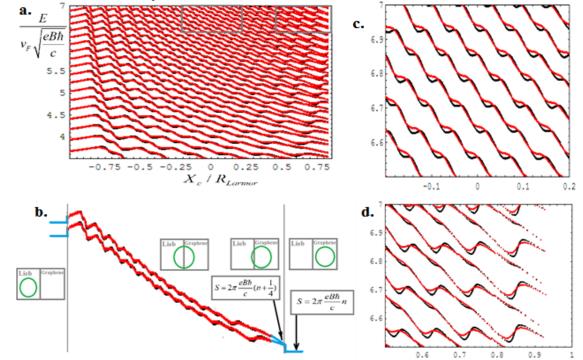


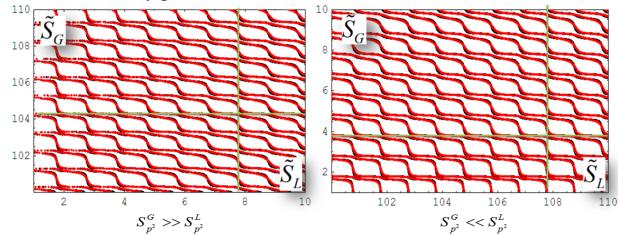
Fig.4. The energy of the edge state as a function of the coordinate of its center (a) and the qualitative behavior of the dependence (b). Increased fragments (c,d).

An analogue of the area quantization rule is obtained:

$$\tilde{S}_{L} = 2\pi n - \frac{\pi}{2} + 2Arg\left(\sqrt{3}\left(\sin\theta + \sin\tilde{S}_{G}\right) + \cos\tilde{S}_{G}\cos\theta + \frac{2i}{\sqrt{3}}\frac{t_{L}t_{G}}{t_{M}^{2}}\cos\tilde{S}_{G}\cos^{2}\theta\right), (3)$$

where where S_G and S_L are areas of segments of the edge state in the graphene-like lattice and the Lieb lattice respectively, $\tilde{S}_{L,G} = \frac{c}{|e|B\hbar} S_{p^2}^{L,G}$.

Asymptotic behavior of energy levels contains information about the influence of the Berry phase on edge states. It is thus easy to see that the increase in $S_{p^2}^G$ leads to an additional phase shift equal to the effect of the Berry phase, while for $S_{p^2}^L$ such there is none (Fig.5), illustrating the absence of a Berry phase in the Lieb lattice.



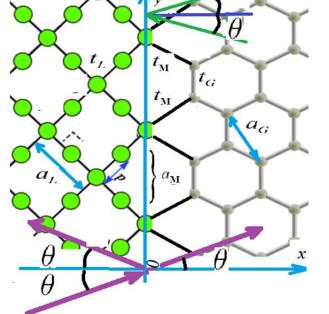


Fig.1 The boundary of two lattices and some of their parameters.

Results:

A transmission coefficient is calculated. Spin-nonconserving Klein tunneling cannot occur (Fig.3), in contrast to the model proposed in [2].

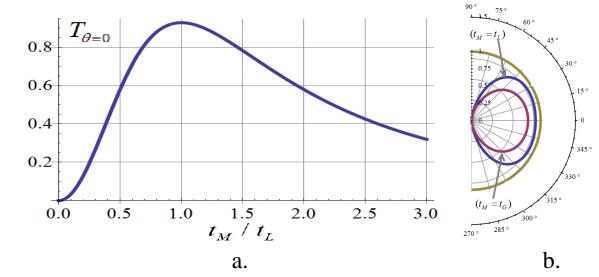
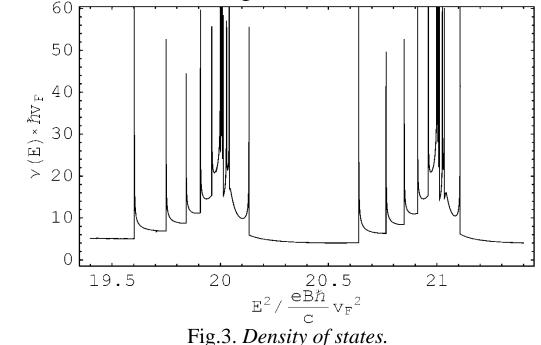


Fig.3. Dependence of the transmission coefficient on the overlap integral at normal incidence (a) and its angular dependence (b).

Fig.5. Edge states for $t_M = t_L$ (red), $t_M = t_G$ (black) and asymptotics (green).

Edge states lead to the appearance of a characteristic series of root singularities in the density of states v(E), which are shown in the Fig.6.



[1] Elliott H. Lieb, Phys. Rev. Lett. 62, 10, 1201 (1989). https://doi.org/10.1103/PhysRevLett.62.1927.5

[2] Luyang Wang and Dao-Xin Yao, Phys. Rev. B 98, 161403 (2018). https://doi.org/10.1103/PhysRevB.98.161403