Quantum capacitance of qubit-based systems

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Introduction

If a classical resonator is coupled to a quantum system, its capacitance, inductance and resistance change [1-3]. We investigate how different qubit-based systems (Fig. 1) interact with classical electric circuits and how to replace them with equivalent impedance which can be measured directly. The problem can be solved for the two-level approximation as well as for multi-level systems.

Our approach demonstrates how to strictly introduce quantum capacitance by quantizing the system and by applying the Krylov-Bogolyubov formalism [4]. The equations of motion for the classical and quantum subsystems then can be obtained and solved, so the correction terms for the circuit eigenfrequency as well as the effective impedance can be found.

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Krylov-Bogolyubov asymptotic method

We consider qubit-resonator system which consists of a quantum subsystem and a classical one (resonator) with weak coupling between them. Equation for "displacement" of the resonator is:

$$\ddot{x} + \omega_0^2 x = \varepsilon f(x, \dot{x}) + \varepsilon f_p \sin \omega_p t \quad (1)$$

Nonlinear oscillator equation for classical subsystem

Weak qubit-resonator coupling — Probing signal — ε — small parameter

In the first-order ε approximation including probing signal we can rewrite Eq. (1) in the form

$$m\ddot{x} + \lambda_{\text{eff}}(a)\dot{x} + k_{\text{eff}}(a)x = \varepsilon F_p \sin \omega_p t \quad (2)$$

Here effective elasticity and damping factor are:

$$\begin{cases} k_{\text{eff}}(a) = k_0 + k_q \\ \lambda_{\text{eff}}(a) = \lambda_0 + \lambda_q \end{cases}$$

$$k_{\text{eff}}(a) = k_0 - \frac{\varepsilon}{\pi a} \int_0^{2\pi} F_0(a, \psi) \cos \psi \, d\psi$$

$$\lambda_{\text{eff}}(a) = \lambda_0 + \frac{\varepsilon}{\pi a \omega_0} \int_0^{2\pi} F_0(a, \psi) \sin \psi \, d\psi$$

$$\omega_{\text{eff}} = \sqrt{\frac{k_{\text{eff}}}{m}}$$

The solution of Eq. (2) is $x = a \cos(\omega_p t + \delta)$ with

$$\begin{cases} \dfrac{da}{dt} = -\dfrac{\lambda_{\text{eff}}}{2m} a - \dfrac{\varepsilon F_p}{m(\omega_0 + \omega_p)} \cos \delta \\ \dfrac{d\delta}{dt} = \omega_{\text{eff}} - \omega_p + \dfrac{\varepsilon F_p}{ma(\omega_0 + \omega_p)} \sin \delta \end{cases} \quad (3)$$

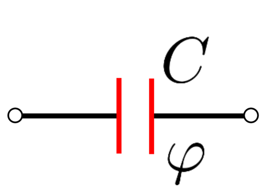
To be able to apply Krylov-Bogolyubov asymptotic method we must know exact form of qubit-resonator coupling $f(x, \dot{x})$. So equations of motion for classical and quantum subsystems need to be obtained.

Lagrangians of basic elements

Capacitance

Node phase will be the generalized coordinate for Lagrangian approach

$$\varphi = \frac{2e}{\hbar} \int V dt$$

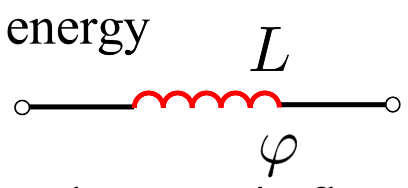


φ — difference of node phases on the element

$$L_C = \left(\frac{\hbar}{2e}\right)^2 \frac{C}{2} \dot{\varphi}^2$$

Inductance

Inductance is described as an element with potential energy



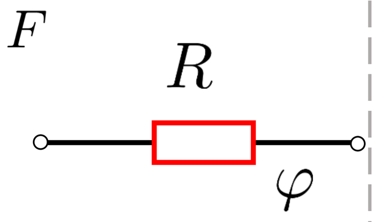
In the presence of external magnetic flux through inductance, the Lagrangian is

$$L_L = -\left(\frac{\hbar}{2e}\right)^2 \frac{c^2}{2L} \left(\varphi + 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0}\right)^2$$

Resistance

Dissipative elements can be included via dissipation function F

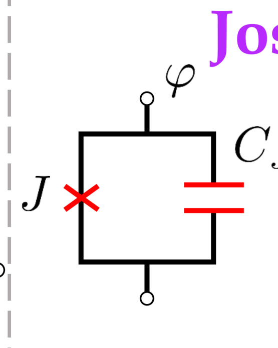
$$F = \left(\frac{\hbar}{2e}\right)^2 \frac{1}{2R} \dot{\varphi}^2$$



Lagrange equation including dissipation function

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = -\frac{\partial F}{\partial \dot{\varphi}}$$

Josephson junction



Josephson junction is considered as a capacitance C_J and the Josephson element J connected in parallel

$$L_J = \left(\frac{\hbar}{2e}\right)^2 \frac{C_J}{2} \dot{\varphi}^2 + E_J \cos \varphi$$

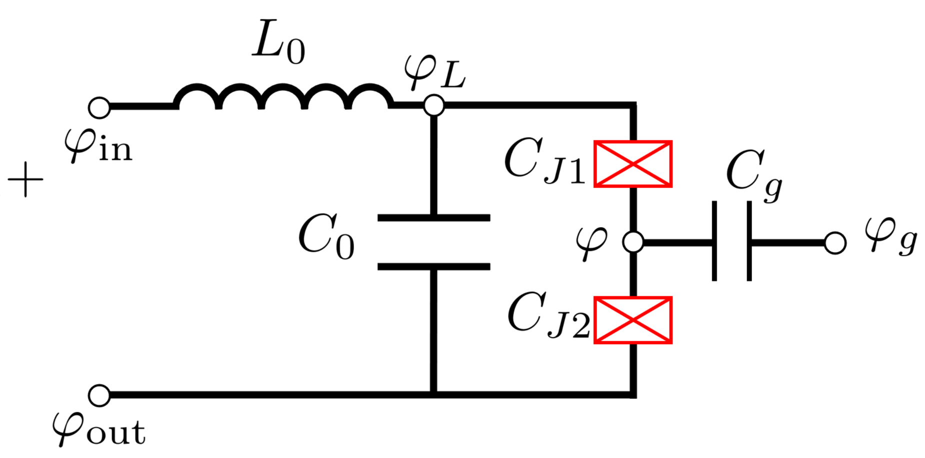
Lagrangian of Single-electron transistor (SET)

Let us consider dynamics of SET [3]. Lagrangian of the system is

$$\left(\frac{2e}{\hbar}\right)^2 \mathcal{L} = \frac{C_0}{2} (\dot{\varphi}_L - \dot{\varphi}_{\text{out}})^2 - \frac{1}{2L_0} (\varphi_L - \varphi_{\text{in}})^2 + \frac{C_g}{2} (\dot{\varphi} - \dot{\varphi}_g)^2 + \frac{C_{J1}}{2} (\dot{\varphi} - \dot{\varphi}_L)^2 + \frac{C_{J2}}{2} (\dot{\varphi} - \dot{\varphi}_{\text{out}})^2 + E_{J1} \cos(\varphi - \varphi_L) + E_{J2} \cos(\varphi - \varphi_{\text{out}})$$

Classical equations of motion for φ_L and φ_{out}

$$\begin{cases} (C_0 + C_{J1})\ddot{\varphi}_L + \frac{\varphi_L - \varphi_{\text{in}}}{L_0} = C_0 \ddot{\varphi}_{\text{out}} + \\ + C_{J1}\ddot{\varphi} + \frac{4e^2 E_{J1}}{\hbar^2} \sin(\varphi - \varphi_L), \\ (C_0 + C_{J2})\ddot{\varphi}_{\text{out}} = C_0 \ddot{\varphi}_L + \\ + C_{J2}\ddot{\varphi} + \frac{4e^2 E_{J2}}{\hbar^2} \sin(\varphi - \varphi_{\text{out}}). \end{cases}$$



Schematics of electric contour for SET

Variable φ needs to be quantized, so we have to construct the Hamiltonian

Equations contain $\ddot{\varphi}$ and $\sin\varphi$, $\cos\varphi$ which must be changed to quantum expectation values $\langle\hat{\ddot{\varphi}}\rangle$, $\langle\sin\hat{\varphi}\rangle$, $\langle\cos\hat{\varphi}\rangle$.

Quantization procedure

For the quantum variable φ Hamiltonian $\mathcal{H} = \dot{\varphi}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \mathcal{L}$ needs to be quantized $\mathcal{H}(\varphi, p) \to \hat{\mathcal{H}}(\hat{\varphi}, \hat{p})$.

So the Hamiltonian is

$$\hat{\mathcal{H}} = 4E_C(\hat{n} - n_g)^2 - E_{J1}\cos(\hat{\varphi} - \varphi_L) - E_{J2}\cos(\hat{\varphi} - \varphi_{\text{out}})$$

with charging energy $E_C = \frac{e^2}{2C_\Sigma}$, total capacitance $C_\Sigma = C_g + C_{J1} + C_{J2}$, charge operator $\hat{n} = \hat{p}/\hbar$, and dimensionless voltage $n_g = -\frac{\hbar(C_g\dot{\varphi}_g + C_{J1}\dot{\varphi}_L + C_{J2}\dot{\varphi}_{\text{out}})}{8E_C C_\Sigma}$.

In two-level approximation

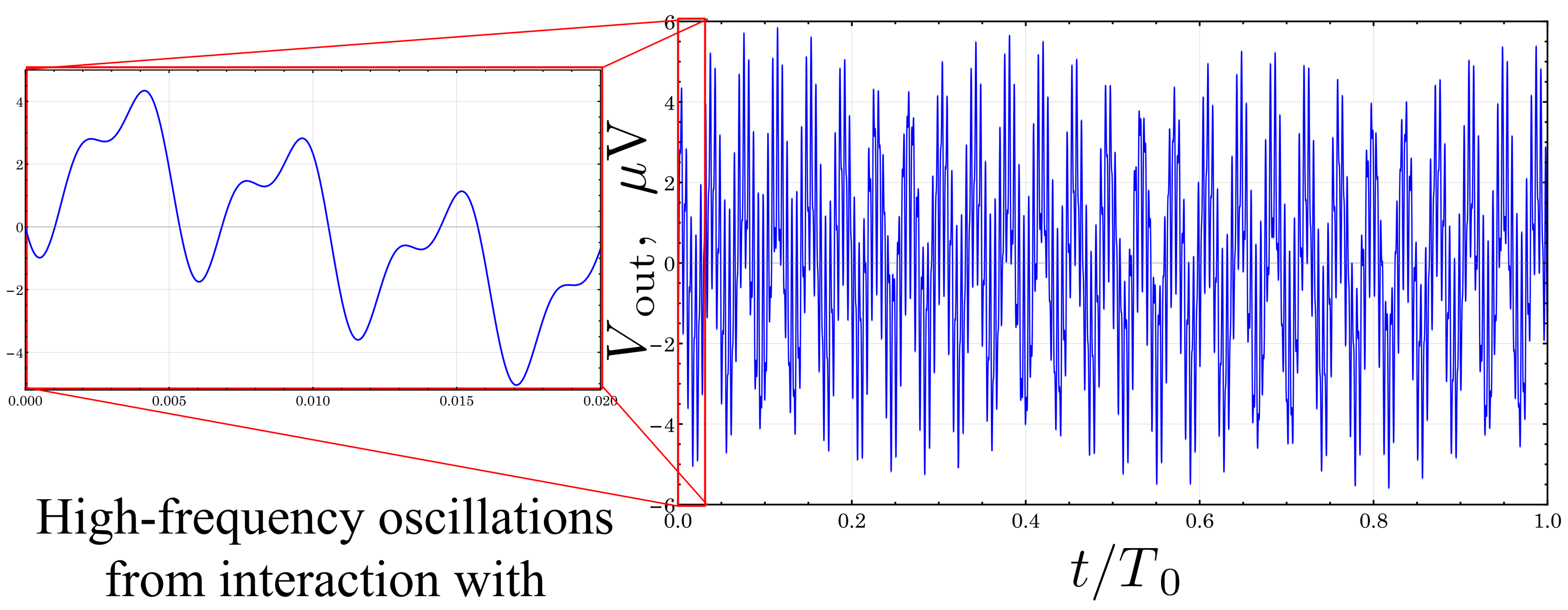
$$\mathcal{H} = -\frac{\Delta}{2}\sigma_x - \frac{\eta}{2}\sigma_y - \frac{\varepsilon}{2}\sigma_z$$

with

$$\begin{cases} \varepsilon = 4E_C(1 - 2n_g) \\ \Delta = E_{J1}\cos\varphi_L + E_{J2}\cos\varphi_{\text{out}} \\ \eta = -E_{J1}\sin\varphi_L - E_{J2}\sin\varphi_{\text{out}} \end{cases}$$

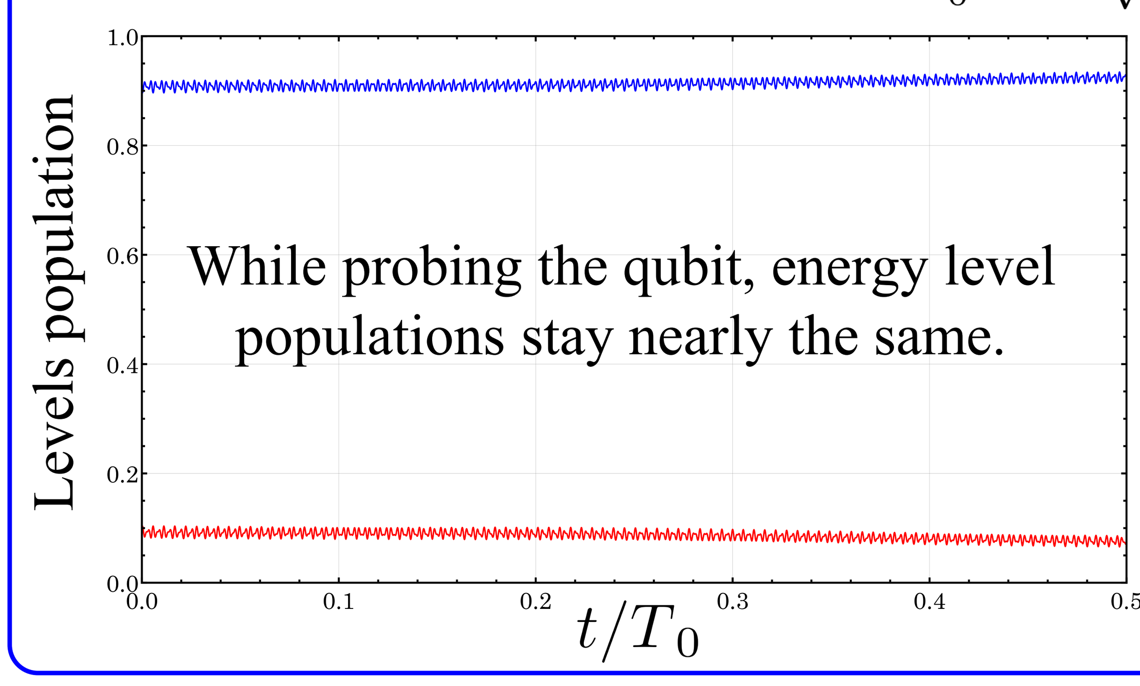
Schrodinger equation coupled with classical equations forms a closed system of equations for dynamics!

Dynamics of SET



High-frequency oscillations from interaction with quantum subsystem

$T_0 = 2\pi\sqrt{L_0 C_0}$ — period of classical oscillations



Parameters of the system

$C_0 = 440$ fF, $L_0 = 490$ nH

$\omega_0 = 2.15$ GHz

$C_g = C_{J1} = C_{J2} = 10^{-15}$ fF

$E_{J1}/h = E_{J2}/h = 2.9$ GHz

$V_{\text{in}} = V_{\text{SD}} + V_0 \sin\omega_0 t$

$V_{\text{SD}} = 130\mu\text{V}$, $V_0 = 0.2\mu\text{V}$

Conclusions

-Our approach demonstrates how to strictly obtain Lagrangian, Hamiltonian and equations of motion for classical and quantum subsystems. We obtained the Hamiltonians and equations of motion for an electrical resonator coupled to superconducting charge qubit and other systems.

-Our approach should demonstrate how to strictly introduce quantum capacitance by quantizing the system and by applying the Krylov-Bogolyubov formalism.

-We plan to study dynamical behaviour of qubit-resonator systems and study respective corrections to phenomenological quantum capacitance.