Magnetic Properties of Low-dimensional Spin System Formed by Spin-1/2 XX Chains Coupled through Ising Spins

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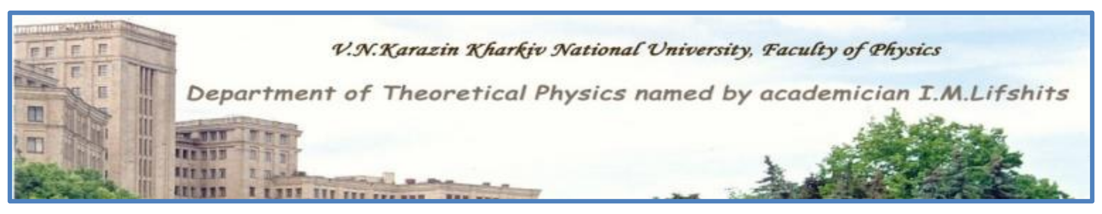
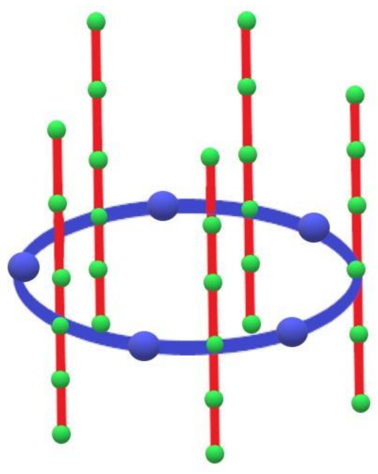
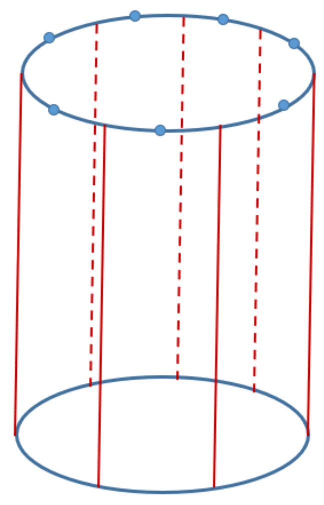


This work is devoted to the theoretical study of quantum stationary states and thermodynamics of two exactly solvable quantum models based on open spin-1/2 XX chain [1, 2]. We consider a set of finite XX chains connected by Ising spins-S into the "bunch" via one intermediate site with the same number for each XX chain, and a "cylinder" of XX chains connected by Ising spins S_1, S_2 at both ends, describing by the Hamiltonians



$$\hat{\mathbf{H}}_1 = -\left[\sum_{l=1}^{L} g_0\mu_B H\sigma^z_{l,n_0} + J_0\left(\sigma^z_{l,n_0}+\sigma^z_{l+1,n_0}\right)S^z_{l,n_0} + \sum_{n=1}^{N} g\mu_B H S^z_{l,n} + J\sum_{n=1}^{N-1}\left(S^x_{l,n}S^x_{l,n+1}+S^y_{l,n}S^y_{l,n+1}\right)\right] \qquad (1)$$



$$\hat{\mathbf{H}}_2 = -\sum_{l=1}^{L}\Big[g_0\mu_B H\sigma^z_{l,1} + g_0'\mu_B H\sigma^z_{l,N} + J_0\left(\sigma^z_{l,1}+\sigma^z_{l+1,1}\right)S^z_{l,1} + J_0'\left(\sigma^z_{l,N}+\sigma^z_{l+1,N}\right)S^z_{l,N} + \sum_{n=1}^{N} g\mu_B H S^z_{l,n} + J\sum_{n=1}^{N-1}\left(S^x_{l,n}S^x_{l,n+1}+S^y_{l,n}S^y_{l,n+1}\right)\Big] \qquad (2)$$

The eigenvalues of all Ising spins operators are the parameters of the Hamiltonians (1) or (2) due to the commutation relations of Ising spins and model Hamiltonians. These Hamiltonians have a simple block form, which permits us to use standard transfer-matrix technique for numerical simulation of the model thermodynamics.

$$\hat{\mathbf{H}}_1 = \sum_{l=1}^{L}\mathbf{H}(\sigma_{l,n_0},\sigma_{l+1,n_0});\qquad \hat{\mathbf{H}}_2 = \sum_{l=1}^{L}\mathbf{H}(\sigma_{l,1},\sigma_{l+1,1},\sigma_{l,N},\sigma_{l+1,N})$$

Jourdan-Wigner Transformation + diagonalization for XX chains

$$\hat{\mathbf{H}}(\sigma_l,\sigma_{l+1}) = E_0(\sigma_l,\sigma_{l+1}) + \sum_{k_{\sigma_l,\sigma_{l+1}}}\varepsilon(k_{\sigma_l,\sigma_{l+1}})a^\dagger_{k_{\sigma_l,\sigma_{l+1}}}a_{k_{\sigma_l,\sigma_{l+1}}}$$

$$\hat{\mathbf{H}}(\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}) = E_0(\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}) + \sum_{k_{\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}}}\varepsilon(k_{\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}})a^\dagger_{k_{\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}}}a_{k_{\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}}}$$

Transfer matrix method

Partition function of effective XX chain with "impurity"

$$Z_{\sigma_l,\sigma_{l+1}} = \exp\left(-\frac{E_0(\sigma_l,\sigma_{l+1})}{T}\right)\prod_{k_{\sigma_l\sigma_{l+1}}}\left[1+\exp\left(\frac{\varepsilon(k_{\sigma_l\sigma_{l+1}})}{T}\right)\right]$$

Total partition function for (1)

$$Z_L = \mathrm{Tr}\left(\exp\left(-\frac{\hat{\mathbf{H}}_1}{T}\right)\right) = \mathrm{Tr}\left(\exp\left(-\frac{1}{T}\sum_l\hat{\mathbf{H}}(\sigma_l,\sigma_{l+1})\right)\right) = \qquad (3)$$

$$= \mathrm{Tr}\left(\exp\left(-\frac{1}{T}\hat{\mathbf{H}}(\sigma_1,\sigma_2)\right)\cdot\exp\left(-\frac{1}{T}\hat{\mathbf{H}}(\sigma_2,\sigma_3)\right)\ldots\exp\left(-\frac{1}{T}\hat{\mathbf{H}}(\sigma_L,\sigma_1)\right)\right) =$$

$$= \mathrm{Tr}\left(Z^L_{\sigma_l,\sigma_{l+1}}\right) = \sum_{i=1}^{2S+1}\lambda_i^L$$

Partition function of effective XX chain with two "impurities" at both ends

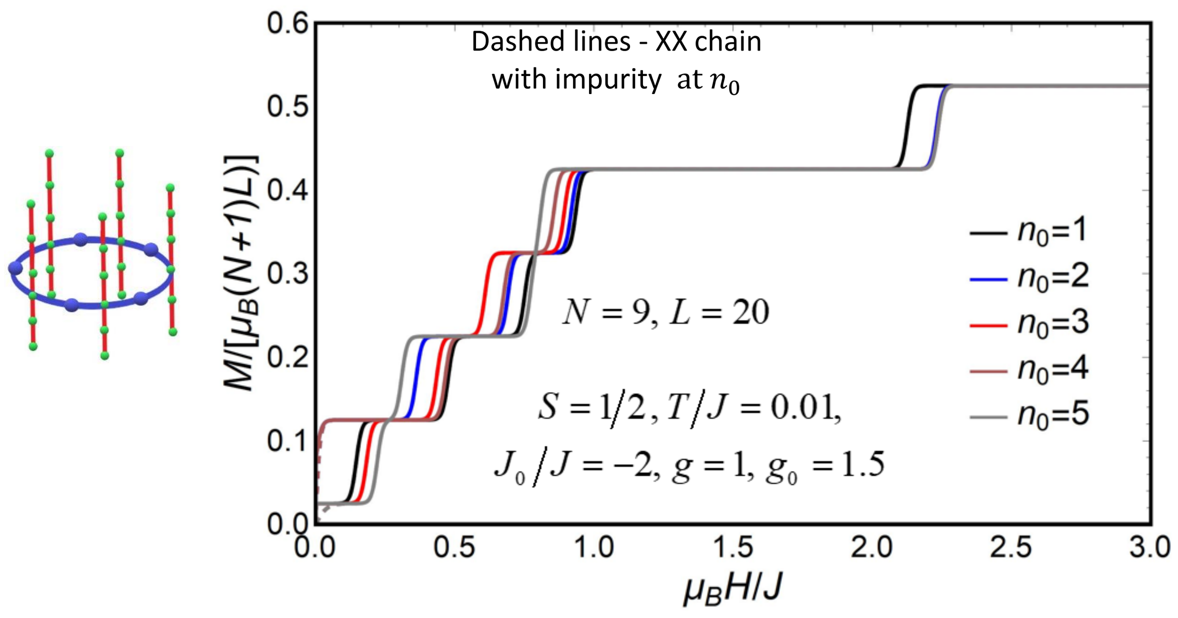
$$Z_{1,\sigma_l,N,\sigma_l,1,\sigma_{l+1},N,\sigma_l+1} = \exp\left(-\frac{E_0\left(1,\sigma_l,N,\sigma_l,1,\sigma_{l+1},N,\sigma_l+1\right)}{T}\right)\prod_{k_{1,\sigma_l,N,\sigma_l,1,\sigma_{l+1},N,\sigma_l+1}}\left[1+\exp\left(\frac{\varepsilon(k_{1,\sigma_l,N,\sigma_l,1,\sigma_{l+1},N,\sigma_l+1})}{T}\right)\right]$$

$$Z_L = \mathrm{Tr}\left(\exp\left(-\frac{\hat{\mathbf{H}}_2}{T}\right)\right) = \mathrm{Tr}\left(\exp\left(-\frac{1}{T}\sum_l\hat{\mathbf{H}}(\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1})\right)\right) =$$

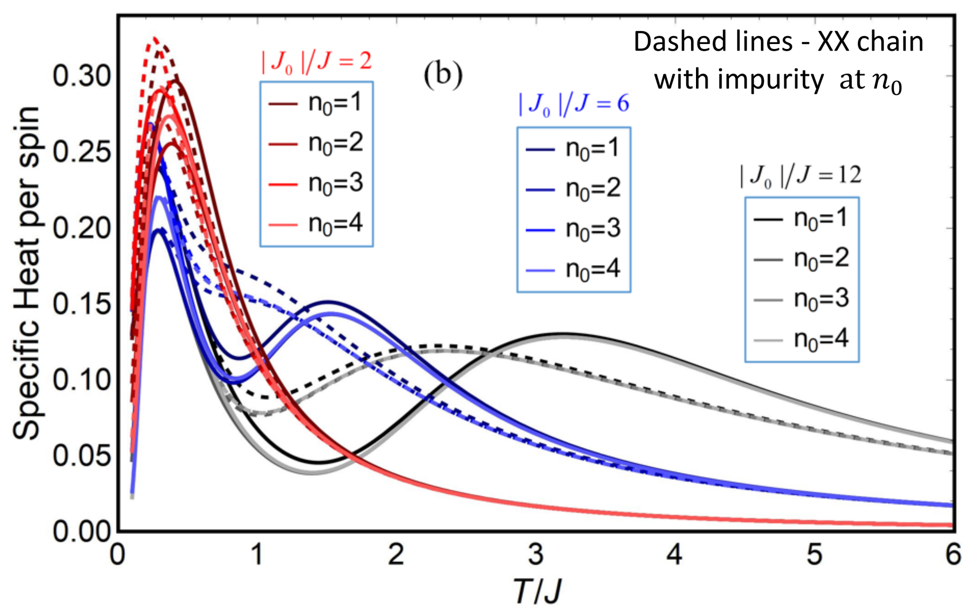
$$= \mathrm{Tr}\left(\exp\left(-\frac{1}{T}\hat{\mathbf{H}}(\sigma_{1,1},\sigma_{N,1},\sigma_{1,2},\sigma_{N,2})\right)\cdot\ldots\cdot\exp\left(-\frac{1}{T}\hat{\mathbf{H}}(\sigma_{1,L},\sigma_{N,L},\sigma_{1,1},\sigma_{N,1})\right)\right) = \qquad (4)$$

$$= \mathrm{Tr}\left(Z^L_{\sigma_{1,l},\sigma_{N,l},\sigma_{1,l+1},\sigma_{N,l+1}}\right) = \sum_{i=1}^{2(S_1+S_2+1)}\lambda_i^L$$

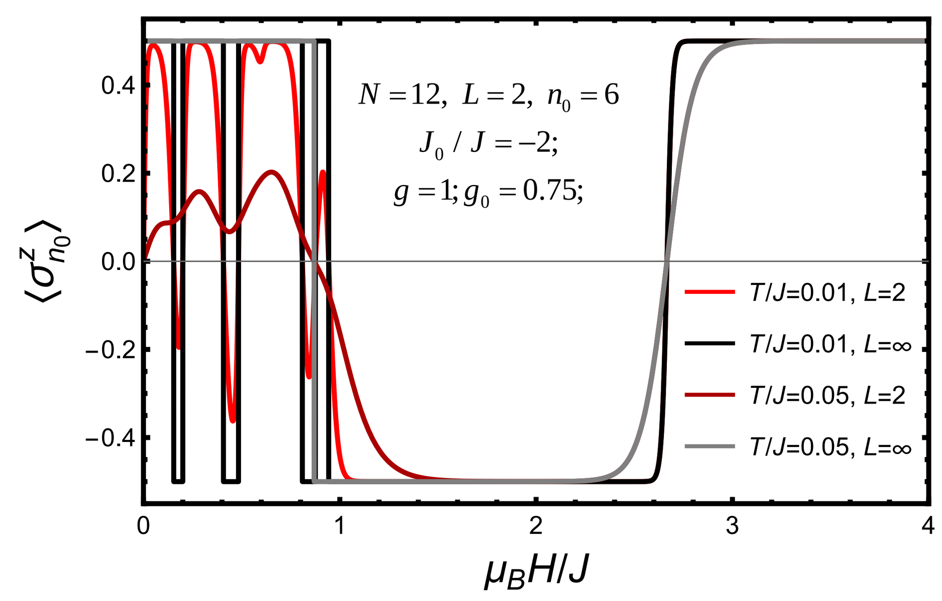
Magnetization

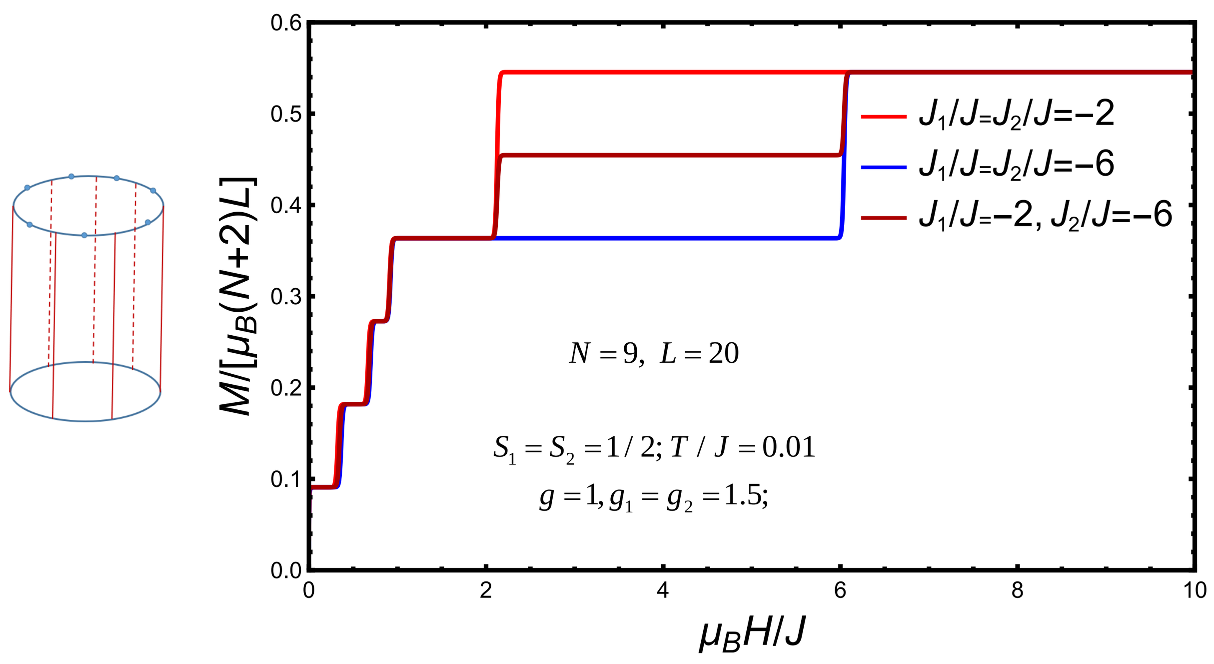


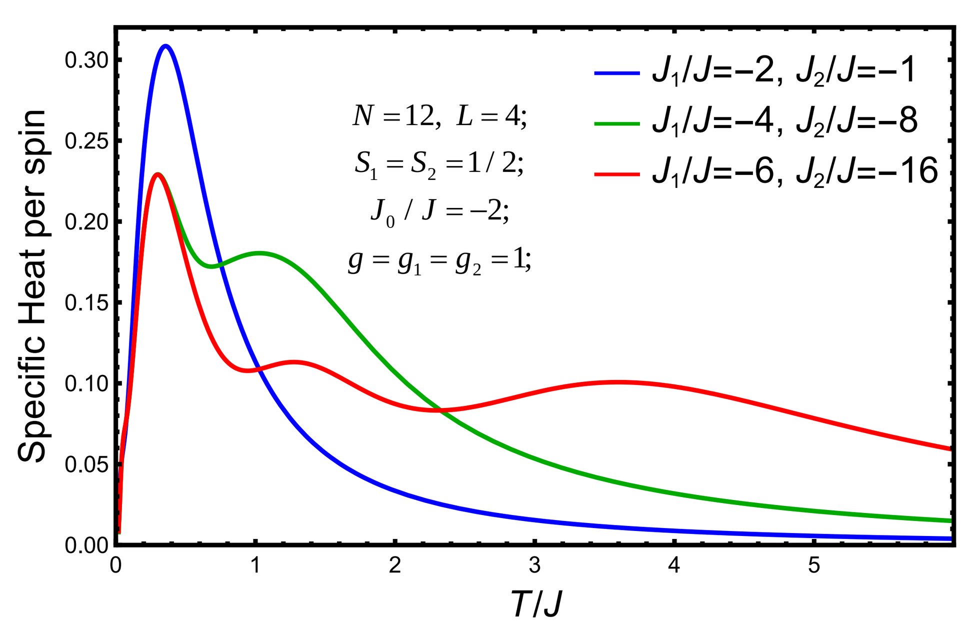
Specific heat at zero magnetic field

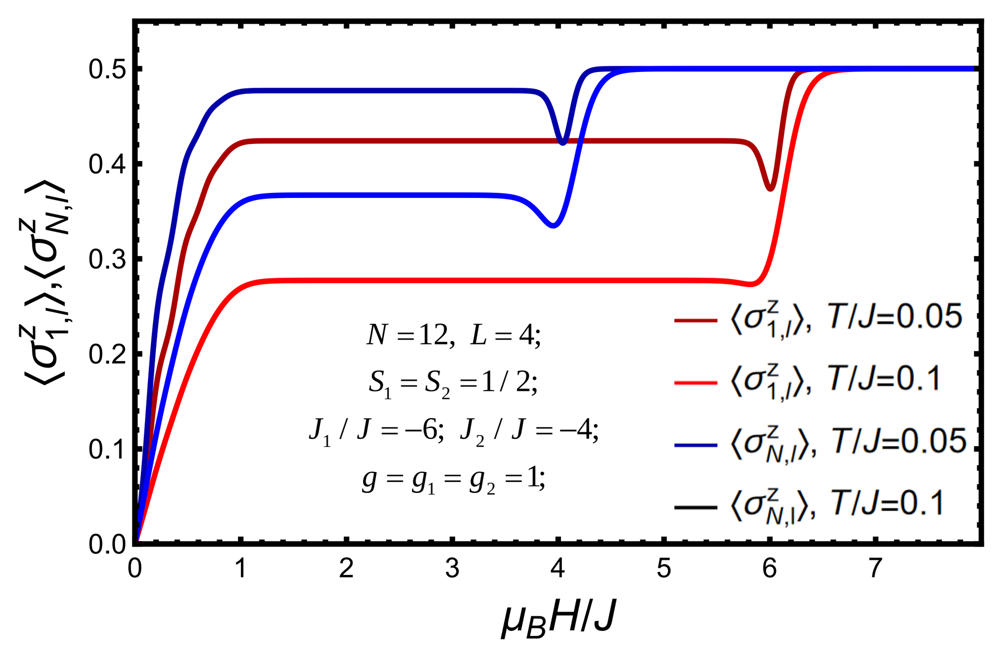


Average values of decorated spins









Summary:

- For strong antiferromagnetic Ising interaction, the field dependence of the magnetization at very low temperatures demonstrates a jump associated with the spin-flip of impurity spins at sufficiently strong magnetic field for (1) and two jumps for (2).
- The possibility of the appearance of two-peak for model (1) and three-peak for model (2) in zero-field temperature dependence of specific heat was found numerically.
- The field dependence of thermodynamic average value z projection of Ising spins for both models may demonstrate the unstable behavior of this quantity in weak fields and low temperatures.

We acknowledge support by IEEE via "Magnetism in Ukraine Initiative" (STCU project No. 9918)

References

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