Four-particle states of low-dimensional fermions with three-body interaction

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Model

Present research deals with the two-channel model of three-component fermionic particles with unequal masses that interact through the three-body forces in fractional dimension. We consider a model of three-component Galilean-invariant fermions with different masses m_σ (σ = 1; 2; 3) in d-dimensional space [1]. All two-body interactions between particles are assumed to be suppressed, and only the three-body potential [2] is switched on between fermions of different sorts

$$H=\sum_\sigma\int_{\mathbf{p}}\varepsilon_\sigma(\mathbf{p})f^\dagger_{\sigma,\mathbf{p}}f_{\sigma,\mathbf{p}}+\int_{\mathbf{p}}\left[\frac{\mathbf{p}^2}{2M}+\delta\nu_\Lambda\right]c^\dagger_{\mathbf{p}}c_{\mathbf{p}}$$
$$+g\int_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3}c^\dagger_{\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3}f_{3,\mathbf{p}_3}f_{2,\mathbf{p}_2}f_{1,\mathbf{p}_1}+\text{h.c.},$$

where $\varepsilon_\sigma(\mathbf{p})=\dfrac{\mathbf{p}^2}{2m_\sigma}$, $\int_{\mathbf{p}}=\int\dfrac{d\mathbf{p}}{(2\pi)^d}$ and $M=m_1+m_2+m_3$ is the mass of composite fermion; g and $\delta\nu_\Lambda$ are the coupling and the detuning.
The anti-commutators of the field operators are fixed as follows

$$\{c_{\mathbf{p}},c^\dagger_{\mathbf{p}'}\}=(2\pi)^d\delta(\mathbf{p}-\mathbf{p}'),\ \{f_{\sigma,\mathbf{p}},f^\dagger_{\sigma',\mathbf{p}'}\}=(2\pi)^d\delta_{\sigma,\sigma'}\delta(\mathbf{p}-\mathbf{p}')$$

and zero all other pairs.
Renormalization leads to the observable three-body coupling

$$g_3^{-1}=-\frac{\Gamma(1-d)}{(2\pi)^d}\left(\frac{m_1m_2m_3}{M}\right)^{d/2}|\epsilon_\infty|^{d-1},$$

where ϵ_∞ is three-particle bound state energy at the broad resonance.

Four-body problem

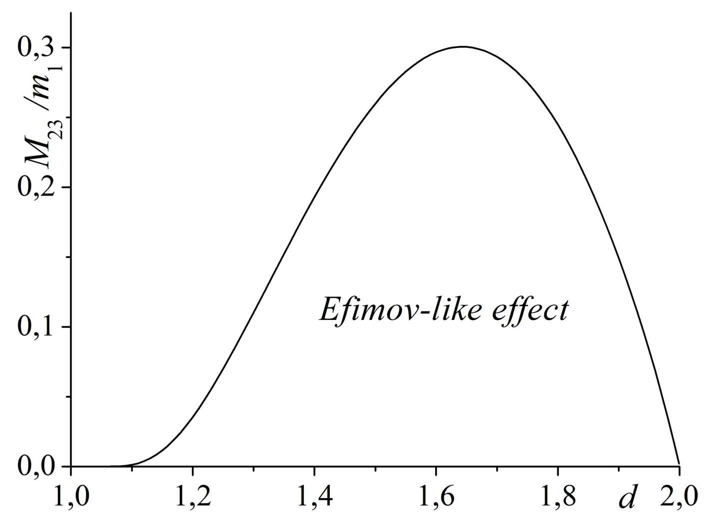
The wave function of an arbitrary four-body (composite fermion $+f_1$-atom) state with zero center-of-mass momentum reads

$$|f_1c\rangle=\int_{\mathbf{p}}B_{\mathbf{p}}f^\dagger_{1,\mathbf{p}}c^\dagger_{-\mathbf{p}}|0\rangle+\int_{\mathbf{p}_1,\mathbf{p}'_1,\mathbf{p}_2}B_{\mathbf{p}_1\mathbf{p}'_1,\mathbf{p}_2,-\mathbf{p}_1-\mathbf{p}'_1-\mathbf{p}_2}$$
$$\times f^\dagger_{1,\mathbf{p}_1}f^\dagger_{1,\mathbf{p}'_1}f^\dagger_{2,\mathbf{p}_2}f^\dagger_{3,-\mathbf{p}_1-\mathbf{p}'_1-\mathbf{p}_2}|0\rangle,$$

where $B_{\mathbf{p}_1\mathbf{p}'_1,\mathbf{p}_2,-\mathbf{p}_1-\mathbf{p}'_1-\mathbf{p}_2}=-B_{\mathbf{p}'_1\mathbf{p}_1,\mathbf{p}_2,-\mathbf{p}_1-\mathbf{p}'_1-\mathbf{p}_2}$ is anti-symmetric function of first two arguments.
Because of the Fermi statistics, the four-body bound states can only occur in the odd-wave channels. The one requiring the deepest potential well is the *p*-wave with the wave function of the following form $B_{\mathbf{p}}=(\mathbf{n}\mathbf{p}/p)B_p$ (here B_p depends on modulus $\mathbf{p}$ with $\mathbf{n}$ being a unit vector).

Efimov-like effect

Considering the limit of broad resonance $g\to\infty$ and disappearing three-body bound state $g_3\to\infty$ allows to investigate the conditions for Efimov-like effect appearance.



Region, where the *p*-wave Efimov-like effect in the four-body sector of the three-component Fermi system with the three-body interaction emerges.

The region under line on the figure above determine a region of the four-body *p*-wave Efimov-like effect in the considered system, with the universal ratio of energy levels

$$\epsilon_4^{(n)}/\epsilon_4^{(n+1)}=e^{2\pi/\eta_0},\quad(n\gg1),$$

η_0 depends only on the spatial dimension and the ratio of total mass of the second and third fermions M_{23} to mass of the first particle m_1.

References

[1] V. Polkanov, V. Patukhov, Phys. Rev. A **109**, 043322 (2024)
[2] M. Valiente, Phys. Rev. A **100**, 013614 (2019)

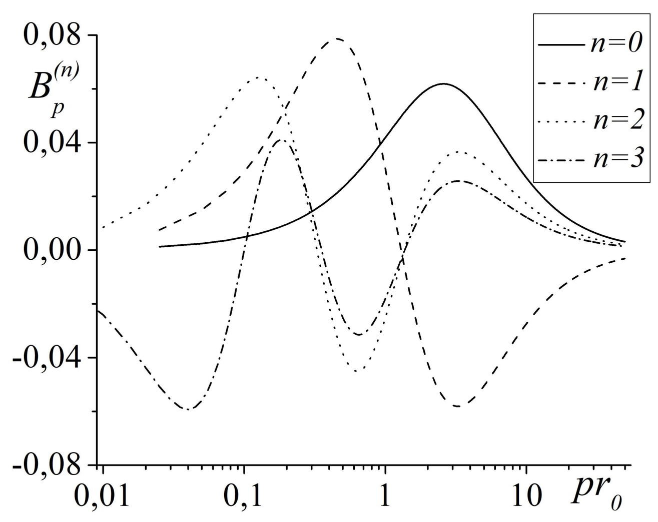
Numerical results for Efimov-like effect

To verify the predictions about the behavior of our model in the four-particle sector, we have numerically solved the integral equation for four-particle wave functions and binding energies in the *p*-wave channel for d=1.5 and mass ratios $M_{23}/m_1=1/50$ small enough to provide a strong induced attractive potential between two f_1 -atoms when the three-body binding energy is sent to zero $\epsilon_g\to0$, and coupling $g=\sqrt{\dfrac{m_1m_2m_3}{M}}\dfrac{1}{r_0^{2-d}}$ is parametrized by scale r_0 which is related to the effective range. The four-body *p*-wave energies $\epsilon_4^{(n)}$ are measured in units of $\dfrac{m_1+M}{2m_1Mr_0^2}$, and the numerical prefactors for the first five levels are gathered in the table below.

n	$\lvert\epsilon_4^{(n)}\rvert$	$\epsilon_4^{(n-1)}/\epsilon_4^{(n)}$
0	4.2250	
1	3.4802×10^{-1}	12.141
2	3.5671×10^{-2}	9.7564
3	3.8086×10^{-3}	9.3668
4	4.0949×10^{-4}	9.3008
⋮	⋮	⋮
∞	0	9.2909

The first five eigenvalues $\epsilon_4^{(n)}$ (in units of $\dfrac{m_1+M}{2m_1Mr_0^2}$) of the four-body problem in the *p*-wave channel. Numerical calculations were performed in d=1.5 and for mass ratios $M_{23}=m_1/50$.

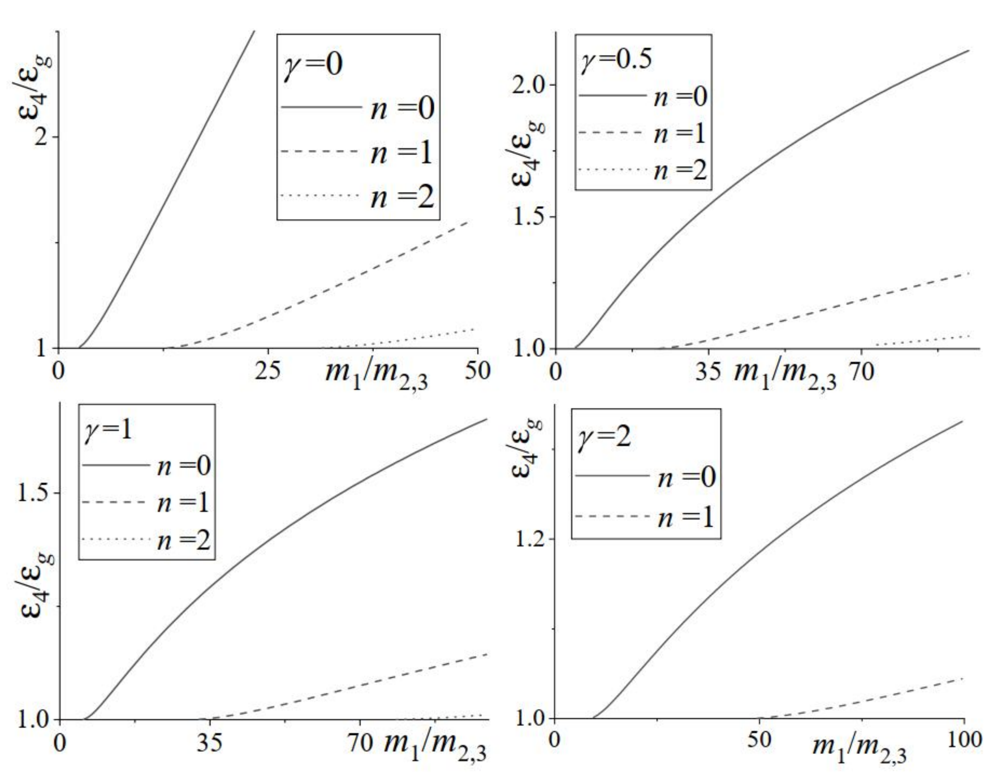
In the third column of above table, the ratio of neighboring eigenenergies which should tend to $e^{2\pi/\eta_0}=9.290926...$ at large n.



The ground state and the first few excited states wave functions $B_p^{(n)}$ (unnormalized) of the four-body problem in the *p*-wave channel.

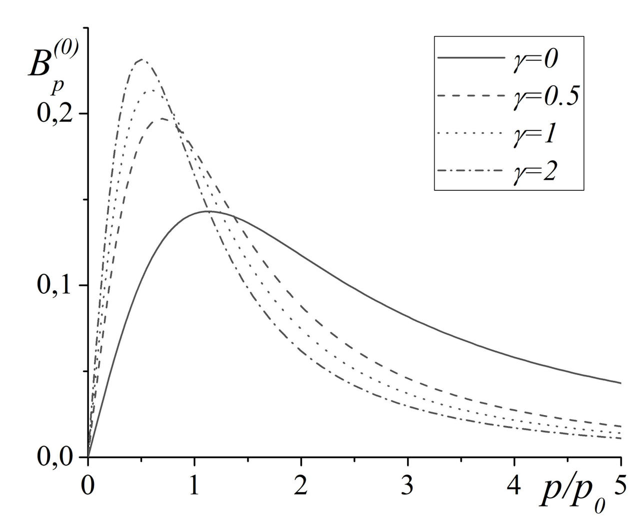
d=1

For solving d = 1 case of problem the measurement units of ϵ_4 was chosen as three-body bound-state energy ϵ_g , then effective range is determined by closeness of the three-body bound state energy to its broad-resonance value $\gamma=\ln\left(\dfrac{\epsilon_\infty}{\epsilon_g}\right)$. Then $\gamma=0$ corresponds to zero range $r_0=0$ case, while $\gamma>0$ refers to the model with narrow resonance.



ϵ_g as functions of mass ratio $m_1/m_{2,3}$ at the different effective ranges (parameterized by $\gamma=\ln\left(\dfrac{\epsilon_\infty}{\epsilon_g}\right)$) of the three-body interaction.

The calculations demonstrate that even at the broad resonance $\gamma=0$, which is the most favorable limit for the existence of the four-body bound states, the first level emerges at $m_1/m_2\approx2.5$ (the second and the third ones at ≈13.0 and ≈31.5, respectively).



The four-body ground state wave functions $B_p^{(0)}$ (unnormalized) at different values of the effective range and fixed mass ratio $m_1/M_{23}=7.75$ in 1D.