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Enhancement of critical current by microwave irradiation in wide superconducting films

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Abstract

The temperature dependences of the enhanced critical current in wide and thin Sn films exposed to a microwave field have been investigated experimentally and analysed. It was found that the microwave field stabilizes the current state of a wide film with respect to the entry of Abrikosov vortices. The stabilizing effect of irradiation increases with frequency. Using the similarity between the effects of microwave enhancement of superconductivity observed for homogeneous (narrow films) and inhomogeneous (wide films) distributions of the superconducting current over the film width, we have succeeded in partially extending the Eliashberg theory to the case of wide films.

1. Introduction

During the last few decades, current states in wide superconducting films in the absence of external magnetic and microwave fields have been studied in considerable detail. The main property of wide films, which distinguishes them from narrow channels, is an inhomogeneous distribution of the transport current over the film width. This distribution is characterized by an increase in the current density towards the film edges due to the Meissner screening of the current-induced magnetic field. It should be emphasized that the current state of a wide film is qualitatively different from the Meissner state of a bulk current-carrying superconductor, despite their seeming resemblance. Indeed, whereas the transport current in the bulk superconductor flows only within a thin surface layer and vanishes exponentially at a distance of the London penetration depth $\lambda(T)$ from the surface, the current in the wide film is distributed over its width w more uniformly, according to the approximate power-like law $[x(w-x)]^{-1/2}$ [1, 2], where x is the transversal coordinate. Thus the characteristic length $\lambda_\perp(T) = 2\lambda^2(T)/d$ (where d is the film thickness), which is commonly referred to as the penetration depth of the perpendicular magnetic field, has nothing to do with any spatial scale of the current decay with the distance from the edges. In fact, the quantity $\lambda_\perp(T)$ plays the role of a 'cutoff factor' in the above-mentioned law of the current distribution at the distances $x, w-x \sim \lambda_\perp$ from the film edges and thereby determines the magnitude of the edge current density. In films whose width is much larger than $\lambda_\perp(T)$ and the coherence length $\xi(T)$, the edge current density approaches the value $j_0 = I/d\sqrt{\pi w\lambda_\perp}$ [1] if the total current I does not exceed the homogeneous pair-breaking current I_c^{GL}.

In such an inhomogeneous situation, the mechanism of superconductivity breaking by the transport current differs from homogeneous Ginzburg–Landau (GL) pair-breaking in narrow channels. In wide films this mechanism is associated with the disappearance of the edge barrier for the vortex entry into the film when the current density at the film edges approaches the value of the order of the GL pair-breaking current density j_c^{GL} [1–4]. Using a qualitative estimate of $j_0 \approx j_c^{GL}$ for the current density which suppresses the edge barrier, one thus derives the expression $I_c(T) \approx j_c^{GL}(T)d\sqrt{\pi w\lambda_\perp(T)}$ for the critical current of a wide film. This equation imposes a linear temperature dependence of the critical current, $I_c(T) \propto (1 - T/T_c)$, near the critical temperature T_c, and it is widely used in the analysis of experimental data (see, e.g., [5]). A quantitative theory for the resistive states of wide films by Aslamazov and Lempitskiy (AL) [2] also predicts the linear temperature dependence of I_c but gives the magnitude of the critical current larger by a factor of 1.5 than the above estimate of I_c. This result was supported by recent experiments on the critical current in wide films [6].

Since the parameters $\xi(T)$ and $\lambda_\perp(T)$ grow infinitely while the temperature approaches T_c, any film reveals the features of a narrow channel in the immediate vicinity of T_c; in particular, its critical current is due to the uniform pair-breaking (narrow channel regime), thus showing the temperature dependence of the GL pair-breaking current $I_c^{GL}(T) \propto (1 - T/T_c)^{3/2}$. As the temperature decreases, the film exhibits a crossover to an essentially inhomogeneous current state, in which vortex nucleation is responsible for the resistive transition; in what follows, this regime will be referred to as a wide film regime. A following quantitative criterion of such a crossover was formulated in [6] on the basis of careful measurements: if the temperature T satisfies an implicit condition $w < 4\lambda_\perp(T)$, the superconducting film can be treated as a narrow channel, whereas at $w > 4\lambda_\perp(T)$ it behaves as a wide film. The physical interpretation of this criterion is quite simple: the existence of the resistive vortex state requires at least two opposite vortices (vortex and anti-vortex), with a diameter $2\lambda_\perp$ each, to be placed across the film of width w. At the same time, it was noted in [6] that, after entering the wide film regime, the dependence $I_c(T) \propto (1 - T/T_c)^{3/2}$, which is typical for narrow channels, still holds within a rather wide temperature range, although the absolute value of I_c is lower than the pair-breaking current $I_c^{GL}(T)$. In practice, the linear temperature dependence [2] of the critical current becomes apparent only at low enough temperatures, when $\lambda_\perp(T)$ becomes 10–20 times smaller than the film width.

Whereas the equilibrium critical current of wide films has been quite well studied, an interesting question about current states in the films under nonequilibrium conditions has been much less investigated. In recent papers [7, 8] it was first reported that wide superconducting films, similar to short bridges [9] and narrow channels [10], exhibit an increase in the critical current under microwave irradiation (superconductivity enhancement). In this paper, we present results of systematic investigations of the enhanced critical current in wide superconducting films. We argue that all essential features of the enhancement effect in wide films with an inhomogeneous current distribution are very similar to that observed before in narrow channels [10]. We found that microwave irradiation stabilizes the current state with respect to the vortex nucleation and thus considerably extends the temperature region of the narrow channel regime. The relative moderateness of the current inhomogeneity in wide films enables us to exploit the theory of superconductivity stimulation in spatially homogeneous systems with minor modifications for a quantitative treatment of our experimental data.

2. Nonequilibrium critical current of superconducting channels in microwave field

The theory of superconductivity enhancement under microwave irradiation was created by Eliashberg [11–13] for superconducting systems, in which the equilibrium energy gap Δ and the superconducting current density j_s are distributed homogeneously over the sample cross-section. The theory applies to rather narrow and thin films $[w, d \ll \xi(T), \lambda_\perp(T)]$ with an homogeneous spatial distribution of microwave power and, correspondingly, of the enhanced gap. The length of electron scattering by impurities, l_i, is assumed to be small compared to the coherence length. According to this theory, the effect of the microwave irradiation on the energy gap Δ of a superconductor carrying transport current with a density j_s is described by the generalized GL equation:

$$\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k T_c)^2} - \frac{2kT_c\hbar}{\pi e^2 D\Delta^4 N^2(0)} j_s^2 + \Phi(\Delta) = 0. \quad (1)$$

Here, $N(0)$ is the density of states at the Fermi level, $D = v_F l_i/3$ is the diffusion coefficient, v_F is the Fermi velocity, and $\Phi(\Delta)$ is a nonequilibrium term arising from the nonequilibrium addition to the electron distribution function [11, 12, 14],

$$\Phi(\Delta) = -\frac{\pi\alpha}{2kT_c}\left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k T_c} - \frac{(\hbar\omega)^2}{2\pi\gamma\Delta}\left(\ln\frac{8\Delta}{\hbar\omega} - 1\right)\right],$$
$$\hbar\omega < \Delta. \quad (2)$$

In this equation, $\alpha = Dp_s^2/\hbar$ is the quantity proportional to the irradiation power P, p_s is the amplitude of the superfluid momentum excited by the microwave field, $\gamma = \hbar/\tau_\varepsilon$, and τ_ε is the energy relaxation time.

In our studies of the superconductivity enhancement, we usually measure the critical current rather than the energy gap. Thus, in order to compare our experimental data with the Eliashberg theory, we should express the superconducting current density, j_s, through a function of the energy gap, temperature and irradiation power, using (1) and (2):

$$j_s = \eta\Delta^2\left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta^2}{8(\pi k T_c)^2} + \Phi(\Delta)\right]^{1/2}, \quad (3)$$

$$\eta = eN(0)\sqrt{\frac{\pi D}{2\hbar k T_c}}. \quad (4)$$

The extremum condition for the superconducting current, $\partial j_s/\partial\Delta = 0$, at a given temperature and irradiation power results in a transcendental equation for the energy gap Δ,

$$\frac{T_c - T}{T_c} - \frac{21\zeta(3)\Delta^2}{(4\pi k T_c)^2} - \frac{\pi\alpha}{2kT_c} \times\left[1 + 0.11\frac{(\hbar\omega)^2}{\gamma k T_c} - \frac{(\hbar\omega)^2}{4\pi\gamma\Delta}\left(\frac{3}{2}\ln\frac{8\Delta}{\hbar\omega} - 1\right)\right] = 0. \quad (5)$$

The solution of this equation, $\Delta = \Delta_m$, being substituted into (3), determines the maximum value of j_s, i.e. the critical current [15],

$$I_c^P(T) = \eta d w \Delta_m^2\left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta_m^2}{8(\pi k T_c)^2} + \Phi(\Delta_m)\right]^{1/2}. \quad (6)$$

This basic equation for the enhanced critical current will be used throughout this paper. With no microwave field applied ($\alpha = 0$), equation (6) transforms into the following expression for the equilibrium pair-breaking current:

$$I_c(T) = I_c^{GL}(T) = \eta d w \Delta_m^2\left[\frac{T_c - T}{T_c} - \frac{7\zeta(3)\Delta_m^2}{8(\pi k T_c)^2}\right]^{1/2}, \quad (7)$$

where $\Delta_m = \sqrt{2/3}\Delta_0$, and

$$\Delta_0 = \pi k T_c \sqrt{8(T_c - T)/7\zeta(3)T_c} = 3.062 k T_c \sqrt{1 - T/T_c} \tag{8}$$

is the equilibrium value of the gap at zero transport current.

When using equation (4) for the quantity η with the density of states given by the free-electron model, $N(0) = m^2 v_F/\pi^2\hbar^3$, in the calculation of the equilibrium critical current (7), we are faced with a considerable discrepancy between (7) and the experimental values of $I_c^{GL}(T)$. This implies that such an estimate of $N(0)$ for the metal (Sn) used in our experiments is rather rough. The way to overcome such an inconsistency is to express $N(0)$ through an experimentally measured quantity, viz film resistance per square, $R^{\square} = R_{4.2}w/L$, where $R_{4.2}$ is the total film resistance at $T = 4.2$ K and L is the film length. Then equation (4) transforms to the relation

$$\eta = (edR^{\square})^{-1}\sqrt{3\pi/2kT_c v_F l_i \hbar}, \tag{9}$$

which provides good agreement of equation (7) with both the experimental values of equilibrium pair-breaking current and the values calculated through the microscopic parameters of the film, ξ_0 and $\lambda_\perp(0)$ (see (10)).

Curiously, as far as we know, the temperature dependences of the enhanced critical current following from (6) have been never compared quantitatively with experimental data. However, some qualitative attempts to compare the experimental dependences $I_c^P(T)$ with the Eliashberg theory have been made. For instance, the authors of [14] interpreted their experimental results on the enhancement effect in narrow films in the following way. First, using the relation (8) for the equilibrium gap, they presented the GL pair-breaking current as

$$I_c^{GL}(T) = \frac{c\Phi_0 w}{6\sqrt{3}\pi^2\xi(0)\lambda_\perp(0)}(1 - T/T_c)^{3/2} = K_1\Delta_0^3(T), \tag{10}$$

where $\Phi_0 = hc/2e$ is the magnetic flux quantum. Then, using an empirical fact that the temperature dependence of the enhanced critical current in the narrow film is close to the equilibrium current, $I_c^P(T) \propto (1 - T/T_c^P)^{3/2}$, where T_c^P is the superconducting transition temperature in a microwave field, this dependence was modelled by an equation similar to (10):

$$I_c^P(T) = K_2\Delta_P^3(T). \tag{11}$$

The enhanced energy gap $\Delta_P(T)$ in (11) was calculated by the Eliashberg theory for zero superconducting current (equation (1) at $j_s = 0$). Assuming that $K_1 = K_2$ and using the magnitude of the microwave power as a fitting parameter, the authors of [14] eventually achieved good enough agreement between the calculated and measured values of $I_c^P(T)$.

Obviously, such a comparison of the experimental data with the Eliashberg theory should be considered as a qualitative approximation which cannot be used to obtain quantitative results. First, equations (10) and (11) involve the gap value at zero current ($j_s = 0$) which is different from that with the current applied. Second, the pair-breaking curve $j_s(\Delta)$ in the equilibrium state, which is implicitly assumed in (11), differs significantly from that in the microwave field [15]. Of course, such model assumptions might nevertheless give a relatively good numerical approximation to an appropriate formula (actually, that is what has been used in [14]), yet the basic inconsistency of such an approximation is due to the qualitatively different behaviour of $I_c^P(T)$ and $\Delta_P(T)$ in the vicinity of the critical temperature. Indeed, as the temperature approaches T_c^P, the enhanced order parameter $\Delta_P(T)$ approaches a finite (though small) value, $\Delta_P(T_c^P - 0) = (1/2)\hbar\omega$, and vanishes through a jump at $T > T_c^P$ [13–15], whereas the critical current vanishes continuously, without any jump. Thus, the temperature dependence of the critical current cannot, in principle, be described adequately by an equation of type (11); incidentally, this can be seen from a pronounced deviation of the formula (11) from the experimental points in the close vicinity of T_c. In the present paper, the experimental data will be analysed by means of the exact formula (6), in which the numerical solution of equation (5) for the quantity Δ_m is used.

Table 1. Parameters of the film samples. Here L is the length, w the width, d the thickness of the sample, and l_i is the electron mean free path.

Sample	L (μm)	w (μm)	d (nm)	$R_{4.2}$ (Ω)	$R^{\square}$ (Ω)	T_c (K)	l_i (nm)	R_{300} (Ω)
SnW8	84	25	136	0.206	0.061	3.816	148	3.425
SnW10	88	7.3	181	0.487	0.040	3.809	169	9.156
SnW13	90	18	332	0.038	0.008	3.836	466	1.880

3. Experimental results

We investigate superconducting Sn thin films fabricated by a novel technique [6] which ensures minimum defects both at the film edge and in its bulk. The critical current of such samples approaches the maximum possible theoretical value [2]. This implies that the current density at the film edges approaches a value of the order of j_c^{GL} and thereby indicates the absence of edge defects which might produce a local reduction of the edge barrier to the vortex entry and a corresponding decrease in I_c. While measuring the I–V curves (IVC), the samples were placed in a double screen of annealed permalloy. The I–V curves were measured by a four-probe method. The external irradiation was applied to the sample, which was placed inside a rectangular waveguide. The electric component of the microwave field in the waveguide was directed parallel to the transport current in the sample. The parameters of some measured films are listed in table 1; the conventional values $v_F = 6.5 \times 10^7$ cm s^{-1} and $\tau_\varepsilon = 4 \times 10^{-10}$ s were used for the Fermi velocity and inelastic relaxation time of Sn.

The IVC of one of the samples is shown in figure 1. The film resistivity caused by motion of the Abrikosov vortices occurs within the current region $I_c < I < I_m$ (the vortex portion of the IVC), where I_m is the maximum current of the existence of the vortex state [2, 6]. When the current exceeds I_m, the IVC shows voltage steps, indicating the appearance of phase-slip lines.

Figure 2 shows experimental temperature dependences of the critical current for the sample SnW10. First, we consider the behaviour of $I_c(T)$ with no electromagnetic field applied (squares). The film width is rather small ($w = 7.3$ μm), therefore the sample behaves as a narrow channel within a relatively wide temperature range, $T_{cros1} < T < T_c =$

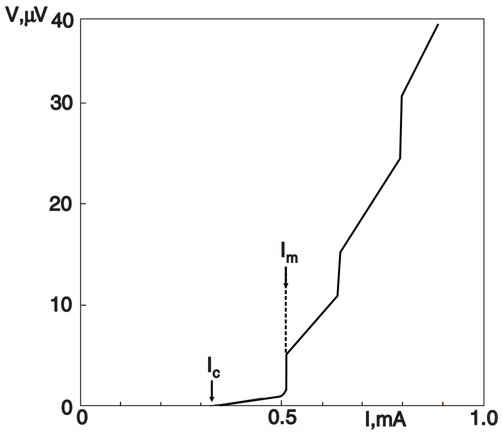

Figure 1. Typical I–V characteristic of a wide [$w \gg \xi(T), \lambda_\perp(T)$] superconducting film (sample SnW13) at the temperature $T = 3.798$ K. I_m is the maximum current of the existence of the vortex state. I_c is the critical current of the wide film.

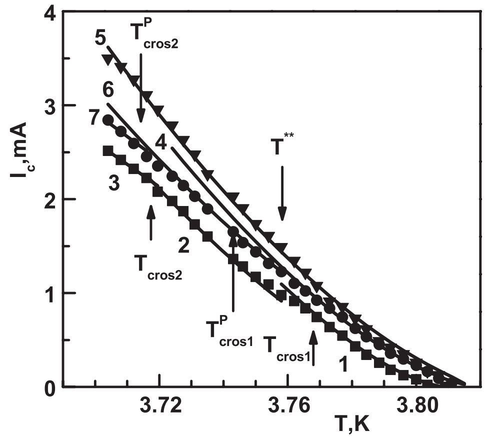

Figure 2. Experimental temperature dependences of the critical current for the sample SnW10 shown by symbols: $I_c(P = 0)$—■; $I_c(f = 9.2$ GHz)—▲; and $I_c(f = 12.9$ GHz)—▼. Theoretical and approximating dependences are shown by curves: 1—theoretical dependence $I_c^{GL}(T) = 7.07 \times 10^2(1 - T/T_c)^{3/2}$ mA [see [7]]; 2—calculated dependence $I_c(T) = 5.9 \times 10^2(1 - T/T_c)^{3/2}$ mA; 3—linear theoretical dependence $I_c^{AL}(T) = 9.12 \times 10^1$ $(1 - T/T_c)$ mA [2]; 4—theoretical dependence $I_c(f = 9.2$ GHz) calculated from (6); this curve can be approximated by dependence $I_c(T) = 6.5 \times 10^2(1 - T/3.818)^{3/2}$ mA; 5—theoretical dependence $I_c(f = 12.9$ GHz) calculated from (6); this curve can be approximated by dependence $I_c(T) = 6.7 \times 10^2$ $(1 - T/3.822)^{3/2}$ mA; 6—theoretical dependence $I_c(f = 9.2$ GHz) calculated from (6) and normalized to curve 2; curve 6 can be approximated by dependence $I_c(T) = 5.9 \times 10^2$ $(1 - T/3.818)^{3/2}$ mA; 7—linear approximating dependence $I_c(T) = 9.4 \times 10^1(1 - T/3.818)$ mA.

3.809 K, in which the critical current is equal to the GL pair-breaking current $I_c^{GL}(T) \propto (1 - T/T_c)^{3/2}$. A crossover temperature, $T_{cros1} = 3.769$ K, corresponds to the transition to the wide film regime: at $T < T_{cros1}$, the vortex portion in the IVC occurs. The temperature dependence $I_c(T)$ at $T < T_{cros1}$ initially holds the form $(1 - T/T_c)^{3/2}$; then the value of $I_c(T)$ becomes smaller than $I_c^{GL}(T)$ below a certain characteristic temperature, $T^{**} \approx 3.76$ K, which is due to formation of an inhomogeneous distribution of the current density and its decrease away from the film edges. Finally, at $T < T_{cros2} = 3.717$ K, the temperature dependence of the critical current becomes linear, $I_c(T) = I_c^{AL}(T) = 9.12 \times 10^1(1 - T/T_c)$ mA, in accordance with the AL theory [2].

In our measurements in a microwave field, the irradiation power was selected to achieve a maximum critical current $I_c^P(T)$. First we discuss the behaviour of $I_c^P(T)$ for the sample SnW10 in the microwave field of the frequency $f = 9.2$ GHz (figure 2, triangles). In the temperature range $T_{cros1}^P(9.2\ \text{GHz}) = 3.744\ \text{K} < T < T_c^P(9.2\ \text{GHz}) = 3.818$ K, no vortex portion in the IVC was observed similar to the narrow channel case. We see that, under optimum enhancement, the narrow channel regime holds down to the temperature $T_{cros1}^P(9.2$ GHz), lower than that in equilibrium state, T_{cros1}. Furthermore, within the region $T^{**} = 3.760\ \text{K} < T < T_c^P$, the experimental values of I_c^P are in good agreement with those calculated from equation (6) (curve 4 in figure 2), in which the microwave power (quantity α) was a fitting parameter. Below the temperature T^{**}, the experimental values of I_c^P descend below the theoretical curve 4, similar to the values of the equilibrium critical current discussed above. Nevertheless, the experimental points can be fitted well by equation (6) (figure 2, curve 6) normalized with a supplementary numerical factor which provides agreement of this equation at zero microwave field with the measured equilibrium critical current $I_c(T)$. We interpret this factor as the form-factor which takes qualitative account of inhomogeneity of the current distribution across the film width. Eventually, at $T < T_{cros2}^P(9.2\ \text{GHz}) = 3.717$ K, the temperature dependence of the critical current becomes linear (figure 2, straight line 7).

The temperature dependence of the enhanced critical current measured at higher frequency, $f = 12.9$ GHz (figure 2, turned triangles), shows that both the critical current and the superconducting transition temperature, $T_c^P(12.9\ \text{GHz}) = 3.822\ \text{K} > T_c^P(9.2$ GHz), increase with frequency, as in narrow channels. Furthermore, at this frequency, the IVC shows no vortex portion in the temperature range studied, down to 3.700 K and even somewhat below. Thus the temperature T_{cros1}^P of the transition to the wide film regime (not shown in figure 2) decreases, $T_{cros1}^P(12.9\ \text{GHz}) < 3.700\ \text{K} < T_{cros1}^P(9.2$ GHz), which considerably extends the region of the narrow channel regime. It is interesting to note that the experimental dependence $I_c^P(T)$ is in good agreement with the equation (6) (curve 5) without any additional normalization in the whole temperature range studied. This would mean that, in moderately wide films, the temperature T^{**} of deviation of the experimental dependence $I_c^P(T)$ from the Eliashberg theory is likely to decrease at high enough frequency, similar to the crossover temperature T_{cros1}^P.

The temperature dependences of the critical current for the sample SnW8 are shown in figure 3. We begin with an analysis of the behaviour of $I_c(T)$ with no electromagnetic field applied. The film width is relatively large, $w = 25\ \mu$m, therefore this sample can be treated as a narrow channel only very close to $T_c = 3.816$ K; at $T < T_{cros1} = 3.808$ K it behaves as a wide film, with the vortex portion in the IVC. The temperature dependence of the critical current holds the form $(1 - T/T_c)^{3/2}$ from T_c down to $T_{cros2} = 3.740$ K, although the

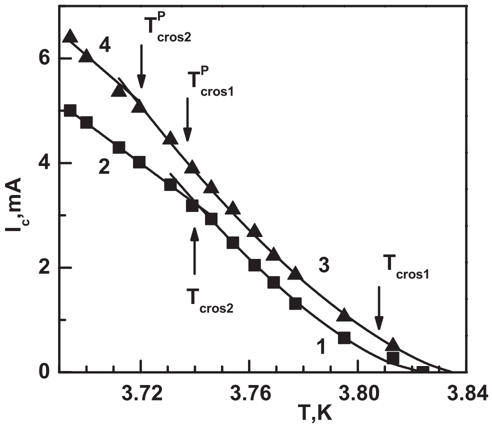

Figure 3. Experimental temperature dependences of the critical current $I_c(P=0)$—■ and $I_c(f=15.2\text{ GHz})$—▲, for the sample SnW8. Theoretical and approximating dependences are shown by curves: 1—calculated dependence $I_c(T)=1.0\times10^3(1-T/T_c)^{3/2}$ mA; 2—linear theoretical dependence $I_c^{AL}(T)=1.47\times10^2(1-T/T_c)$ mA [2]; 3—theoretical dependence $I_c(f=15.2\text{ GHz})$ calculated by (6) and normalized to curve 1, curve 3 can be approximated by dependence $I_c^P(T)=1.0\times10^3(1-T/3.835)^{3/2}$ mA; 4—linear calculated dependence $I_c^P(T)=1.72\times10^2(1-T/3.835)$ mA.

value of I_c is smaller than I_c^{GL} within this temperature range. This means that substantial current inhomogeneity develops very close to T_c as well (the difference between T_c and the characteristic temperature T^{**} cannot be resolved reliably). At $T<T_{cros2}$, the temperature dependence of the critical current becomes linear, according to the AL theory [2]: $I_c(T)=I_c^{AL}(T)=1.47\times10^2(1-T/T_c)$ mA.

In the microwave field of frequency $f=15.2$ GHz, the superconducting transition temperature increases to $T_c^P=3.835$ K, whereas the temperatures of both the crossover to the linear AL dependence, $T^P_{cros2}=3.720$ K, and the transition to the wide film regime, $T^P_{cros1}=3.738$ K, exhibit a noticeable decrease. At the same time, in order to achieve good agreement between the experimental dependence $I_c^P(T)$ and equation (6), we must apply the normalization of this equation on the measured equilibrium critical current $I_c(T)=1.0\times10^3(1-T/T_c)^{3/2}$ mA within the whole temperature range $T>T^P_{cros2}$ (figure 3, curve 3) including the close vicinity of the critical temperature (the temperature T^{**} is still indistinguishable from T_c^P). Thus, in rather wide films, even the frequency $f=15.2$ GHz is not high enough to transform the absolute magnitude of $I_c^P(T)$ to the bare dependence (6) calculated for a narrow channel, unlike the case of the relatively narrow sample SnW10 at $f=12.9$ GHz discussed above.

4. Discussion of the results

We begin this section with a discussion of the superconductivity enhancement effect, which manifests itself as an increase in the superconducting transition temperature, T_c^P, and in the magnitude of the critical current, I_c^P, compared with their equilibrium values. A qualitative similarity of the results with those obtained for the narrow channels [8] and the possibility to describe the temperature dependence I_c^P quantitatively by the equations of the Eliashberg theory convince us that the mechanism of the enhancement effect is the same for the wide films and the narrow channels, i.e. it consists of an enhancement of the energy gap caused by a redistribution of the microwave excited nonequilibrium quasiparticles to higher energies [11].

This conclusion seems to be not quite evident for the wide films with inhomogeneous current distribution across the sample, because it is the current inhomogeneity that may be suggested to suppress the superconductivity stimulation completely in bulk superconductors. Indeed, in the latter case, the concentration of the transport current and the microwave field within the Meissner layer near the metal surface gives rise to an extra mechanism of quasiparticle relaxation, namely spatial diffusion of the microwave excited nonequilibrium quasiparticles from the surface to the equilibrium bulk. The efficiency of this mechanism is determined by the time of quasiparticle escape, $\tau_D(T)=\lambda^2(T)/D$, from the Meissner layer, which is three-to-four orders of magnitude smaller than the typical inelastic relaxation time. Such a high efficiency of the diffusion relaxation mechanism is likely to result in the suppression of the enhancement effect in bulk superconductors. However, relying on a qualitative difference between the current states in bulk and thin-film superconductors outlined in the introduction, one can argue that moderate nonuniformity of the current distribution in wide films (with no concentration of an exciting factor at short distances) causes no fatal consequences for the enhancement effect, and that the diffusion of nonequilibrium quasiparticles excited within the whole bulk of the film introduces only insignificant quantitative deviations from the Eliashberg theory.

In our consideration, we used a model approach to take these deviations into account by introducing the numerical form-factor of the current distribution into the formula (6) for the enhanced critical current. We evaluate this form-factor by fitting the limit form of equation (6) at zero microwave power, i.e. equation (7), to the measured values of the equilibrium critical current. Then we apply the obtained values of the form-factor (0.83 for SnW10 and 0.57 for SnW8) to equation (6) at $P\neq0$, which results in a considerably good fit to the experimental data, as demonstrated in the previous section.

There is a question worth discussing of how the Eliashberg mechanism works in the regime of a wide film, i.e. when superconductivity breaking is due to the vortex nucleation rather than due to exceeding the maximum value allowed by the pair-breaking curve by the transport current. We believe that, in this case, the enhancement of the energy gap results in corresponding growth of the edge barrier for the vortex entry, and this is what enhances the critical current in the wide film regime. It is interesting to note that no essential features appear in the curves $I_c(T)$ when the films enter the vortex resistivity regime. From this, we conclude that the transition between the regimes of the uniform pair-breaking and vortex resistivity has no effect on both the magnitude and the temperature dependence of the critical current.

To complete the discussion of the superconductivity enhancement effect, we draw one's attention to the empirical fact that all the fitting curves for $I_c^P(T)$ obtained by the equations of the Eliashberg theory are excellently approximated by the power law $(1-T/T_c^P)^{3/2}$. This law is quite similar to the temperature dependence of the GL pair-breaking current, in which the critical temperature T_c is replaced by its enhanced value T_c^P. Explicit expressions for such approximating dependences, with numerical coefficients, are presented in the captions to figures 2 and 3.

The next important result of our studies is the essential extension of the temperature range of the narrow channel regime of wide films on the enhancement of superconductivity: in the microwave field, the temperature of the crossover to the wide film regime, T_{cros1}^P, decreases considerably compared with its equilibrium value, T_{cros1}. At first glance, this result somewhat contradicts the criterion of the transition between the different regimes mentioned in the introduction, $w = 4\lambda_\perp(T_{cros1})$, because an increasing energy gap under irradiation implies a decreasing magnitude of $\lambda_\perp$ and, correspondingly, a decreasing characteristic size of the vortices. This obviously facilitates the conditions of vortex entry to the film, thus the crossover temperature should *increase* in the microwave field. We believe, however, that there exists a more powerful stabilizing effect of irradiation on the vortices. The role of this effect is to delay the vortex nucleation and/or motion and to maintain the existence of the narrow channel regime down to low enough temperatures at which the current inhomogeneity eventually becomes well developed. Indeed, for the sample SnW10 at zero microwave power, the transition to the wide film regime occurs at the temperature $T_{cros1}^P(T)$ *higher* than the temperature T^{**} at which the deviation of $I_c^P(T)$ from the GL uniform pair-breaking current begins to be observed. This means that in equilibrium case the vortex nucleation begins at relatively small inhomogeneity which still weakly affects the magnitude of the critical current. In contrast to this, under microwave irradiation of the frequency $f = 9.2$ GHz, the vortex resistivity occurs at the temperature T_{cros1}^P *lower* than T^{**}, i.e. the inhomogeneous current state shows enhanced stability with respect to the vortex nucleation in the presence of a microwave power. A similar conclusion can be drawn regarding the behaviour of the critical current in the wider film SnW8.

We suggest the following qualitative explanation of the stabilization effect. Since the dimensions of the samples are small compared to the electromagnetic wavelength (the sample length is $\sim 10^{-4}$ m and the minimum wavelength is $\sim 10^{-2}$ m), we deal, in fact, with an alternating high-frequency current, $I_f \propto \sqrt{P}$, flowing through the sample. The relative power $P/P_c \sim 0.1$–0.2, corresponding to the maximum enhanced current $I_c^P(T)$, is rather high, therefore the amplitude of I_f may be comparable with the magnitude of the critical transport current. This results in a considerable modulation of the net current flow through the sample, which presumably enhances the stability of the current flow with respect to the nucleation and motion of the vortices. A possible analogy of such a phenomenon is given by stabilization of the current state in narrow channels at supercritical currents caused by the self-radiation of phase-slip centres [16].

As noted in the previous section, the stabilizing effect becomes more pronounced with increasing irradiation frequency f: the region of the narrow channel regime considerably extends as the frequency grows. In the framework of our assumption about the stabilizing role of the high-frequency modulation of the current flow, such an effect can be explained as follows. The relative microwave power of optimum enhancement, P/P_c, was found to increase with f [8], while the critical power P_c changes with f only at small enough frequencies, $f < f_\Delta$ [17, 18], where $f_\Delta \approx (1 - T/T_c)^{1/2}/2.4\tau_\varepsilon$ is the inverse of the gap relaxation time and does not exceed 0.1 GHz for our samples and temperatures. At larger frequencies used in our experiments, P_c holds a constant value, which enables us to attribute the variations in P/P_c to variations in the absolute magnitude of the irradiation power P, i.e. in the amplitude of the modulating high-frequency current I_f. Thus the increase in the irradiation frequency causes the microwave power of optimum enhancement of superconductivity to increase and, correspondingly, the stabilizing effect of the electromagnetic field on the vortices to be enhanced.

5. Conclusions

The results of our investigation enable us to conclude that the mechanism of superconductivity enhancement by a microwave field is the same for both the narrow channels and wide films—this is the Eliashberg mechanism of enhancement of the superconducting energy gap caused by the excitation of nonequilibrium quasiparticles to a high-energy region within the whole bulk of a superconductor. In the vicinity of T_c, where any film can be treated as a narrow channel, the Eliashberg mechanism enhances the critical current in a usual way, through modification of the pair-breaking curve for the superconducting current. At lower temperatures, when the wide film enters the regime of the vortex resistivity, the enhancement of the critical current is likely associated with the growth of the edge barrier to the vortex entry, nevertheless giving a magnitude and temperature dependence of the critical current quite similar to those in the previous case. Such a similarity extends down to low enough temperatures, at which a linear temperature dependence predicted for extremely wide films is observed.

Another important effect of the microwave irradiation is to stabilize the current state of the film with respect to the entry of the vortices, presumably by deep modulation of the transport current by the induced high-frequency current. This results in the extension of the temperature range in which the film behaves as a narrow channel, showing no vortex resistivity in the current–voltage characteristic. The stabilizing effect grows with frequency, which is explained by a simultaneous increase in the pumping power and, correspondingly, in the amplitude of the high-frequency current.

We achieve a good accord of the experimental data with Eliashberg theory by introducing a numerical form-factor into equations initially derived for homogeneous distribution. Despite the simplicity and high enough quality of such an approximation, this problem requires a more consistent approach, involving the solution of diffusive equations of nonequilibrium superconductivity in a spatially inhomogeneous case.

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References

[1] Larkin A I and Ovchinnikov Yu N 1972 *Sov. Phys.—JETP* **34** 651

[2] Aslamazov L G and Lempitskiy S V 1983 *Sov. Phys.—JETP* **57** 1291

[3] Shmidt V V 1969 *Sov. Phys.—JETP* **30** 1137

[4] Likharev K K 1971 *Izv. Vyssh. Uchebn. Zaved. Radiofizika* **14** 909

[5] Andratskiy V P, Grundel' L M, Gubankov V N and Pavlov N B 1974 *Sov. Phys.—JETP* **38** 794

[6] Dmitriev V M and Zolochevskii I V 2006 *Supercond. Sci. Technol.* **19** 342

[7] Agafonov A B, Dmitriev V M, Zolochevskii I V and Khristenko E V 2001 *Low Temp. Phys.* **27** 686

[8] Dmitriev V M, Zolochevskii I V, Salenkova T V and Khristenko E V 2005 *Low Temp. Phys.* **31** 957

[9] Wyatt A F G, Dmitriev V M, Moore W S and Sheard F W 1966 *Phys. Rev. Lett.* **16** 1166

[10] Dmitriev V M and Khristenko E V 1978 *Sov. J. Low Temp. Phys.* **4** 387

[11] Eliashberg G M 1970 *Sov. Phys.—JETP Lett.* **11** 114

[12] Eliashberg G M 1972 *Sov. Phys.—JETP* **34** 668

[13] Ivlev B I, Lisitsyn S G and Eliashberg G M 1973 *J. Low Temp. Phys.* **10** 449

[14] Klapwijk T M, van den Bergh J N and Moij J E 1977 *J. Low Temp. Phys.* **26** 385

[15] Dmitriev V M and Khristenko E V 1979 *Le J. Phys.-Lett.* **40** L85

[16] Dmitriev V M, Zolochevskii I V and Khristenko E V 1999 *J. Low Temp. Phys.* **115** 173

[17] Pals J A and Ramekers J J 1982 *Phys. Lett.* A **87** 186

[18] Bezuglyi E V, Dmitriev V M, Svetlov V N and Churilov G E 1987 *Low Temp. Phys.* **13** 517