B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine

AMPH 2014

II International Conference

ANALYSIS AND MATHEMATICAL PHYSICS

June 16-20, 2014

Book of Abstracts

Kharkiv 2014

II International Conference "ANALYSIS AND MATHEMATICAL PHYSICS II": Book of abstracts, Kharkiv: B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 2014, 47 p.

Program Committee

V. Marchenko, L. Pastur, E. Khruslov.

Organizing Committee:

G. Feldman, L. Fardigola, V. Vasilchuk.

CONTENTS

SENIOR PARTICIPANTS

Bratteli diagrams: structures, measures, orderings

Sergey Bezuglyi, *Kharkiv, Ukraine*

Bratteli diagrams are widely used in the theory of operator algebras and in Cantor dynamics. Our goal is to give a survey of the most recent results about invariant measures on Bratteli diagrams. In other words, this is equivalent to the study of measures (finite and infinite) of an arbitrary homeomorphism of a Cantor set.

The following topics are planned to discuss:

- the structure of simple and non-simple Bratteli diagrams;
- the tail equivalence relation R on the path space of a Bratteli diagrams and existence of continuous dynamics on Bratteli diagrams;
- ergodic R-invariant measures and their supports;
- examples.

All necessary definitions will be given.

Subdiagrams and invariant measures of Bratteli diagrams

Sergey Bezuglyi, *Kharkiv, Ukraine*
Olena Karpel, *Kharkiv, Ukraine*
Jan Kwiatkowski, *Olsztyn, Poland*

We study ergodic measures on the path space X_B of a Bratteli diagram B invariant with respect to the tail equivalence relation $\mathcal{E}$. Our aim is to characterize those subdiagrams that support an ergodic finite invariant measure.

The interest and motivation for this work arise from the following result proved by Bezuglyi, Kwiatkowski, Medynets and Solomyak (2013): for any ergodic probability measure μ on a finite rank diagram B, there exists a subdiagram $\overline{B}$ of B defined by a sequence of vertices $W = (W_n)$, where $W_n \subset V_n$, such that $\mu(X_w^{(n)})$ is bounded from zero for all $w \in W_n$ and n. Here V_n is the set of all vertices of B on level n and $X_v^{(n)}$ is the set of all paths that go through the vertex $v \in V_n$. It was also shown that μ can be obtained as an extension of an ergodic measure on the subdiagram $\overline{B}$, in other words, $\overline{B}$ supports μ.

We look for analogous result in case of a general Bratteli diagram. Given a subdiagram $\overline{B}$ of a Bratteli diagram B, consider an ergodic probability measure ν on $X_{\overline{B}}$. This measure can be naturally extended (by $\mathcal{E}$-invariance) to a measure $\widehat{\nu}$ defined on the $\mathcal{E}$-saturation $\widehat{X}_{\overline{B}}$ of the path space $X_{\overline{B}}$. We give criteria and sufficient conditions for the finiteness of the extended measure.

Suppose now that a probability measure μ is given on a diagram B. We answer the question when the path space of the subdiagram $\overline{B}$ has positive measure.

Theorem 1 *Let B be a simple Bratteli diagram and μ a probability ergodic measure on X_B. Suppose $\overline{B}$ is a vertex subdiagram of B defined by a sequence (W_n) of vertices subsets. Then $\mu(X_{\overline{B}}) = 0$ if and only if for every $\varepsilon > 0$ there exists $n = n(\varepsilon)$ such that for every $w \in W_n$*

$$\frac{\overline{h}_w^{(n)}}{h_w^{(n)}} < \varepsilon,$$

where $h_w^{(n)}$ is a height of the tower $X_w^{(n)}$ and $\overline{h}_w^{(n)}$ is the height of the corresponding tower in the subdiagram.

Corollary 1 *Let B, μ, $\overline{B}$ be as in Theorem above. Suppose $\mu(X_{\overline{B}}) = 0$. Then for any probability invariant $\overline{\mu}$ on $\overline{B}$ we have $\widehat{\overline{\mu}}(\widehat{X}_{\overline{B}}) = \infty$.*

Combinatorial models for spaces of cubic polynomials

Alexander Blokh, *Birmingham, AL, USA and Bonn, Germany*
Lex Oversteegen, *Birmingham, AL, USA*
Vladlen Timorin, *Moscow, Russia*
Ross Ptacek, *Moscow, Russia*

To construct a model for a connectedness locus of polynomials of degree $d \geq 3$ (cf with Thurston's model of the Mandelbrot set), we define *linked* geolaminations $\mathcal{L}_1$ and $\mathcal{L}_2$. An *accordion* is defined as the union of a leaf ℓ of $\mathcal{L}_1$ and leaves of $\mathcal{L}_2$ crossing ℓ (or vice versa). We show that any accordion behaves like a gap of one lamination and prove that the maximal *perfect* (without isolated leaves) sublaminations of $\mathcal{L}_1$ and $\mathcal{L}_2$ coincide.

In the cubic case let $\mathcal{D}_3 \subset \mathcal{M}_3$ be the set of all *dendritic* (with only repelling cycles) polynomials. Let $\mathcal{MD}_3$ be the space of all *marked* polynomials (P, c, w), where $P \in \mathcal{D}_3$ and c, w are critical points of P (perhaps, $c = w$). Let c^* be the *co-critical point* of c (i.e., $P(c^*) = P(c)$ and, if possible, $c^* \neq c$). By Kiwi, to

$P \in \mathcal{D}_3$ one associates its lamination $\sim_P$ so that each $x \in J(P)$ corresponds to a convex polygon G_x with vertices in $\mathbb{S}$. We relate to $(P, c, w) \in \mathcal{MD}_3$ its *mixed tag* $\mathrm{Tag}(P, c, w) = G_{c^*} \times G_{P(w)}$ and show that mixed tags of distinct marked polynomials from $\mathcal{MD}_3$ are disjoint or coincide. Let $\mathrm{Tag}(\mathcal{MD}_3)^+ = \bigcup_{\mathcal{D}_3} \mathrm{Tag}(P, c, w)$. The sets $\mathrm{Tag}(P, c, w)$ partition $\mathrm{Tag}(\mathcal{MD}_3)^+$ and generate the corresponding quotient space MT_3 of $\mathrm{Tag}(\mathcal{MD}_3)^+$. We prove that $\mathrm{Tag} : \mathcal{MD}_3 \to \mathrm{MT}_3$ is continuous so that MT_3 serves as a model space for $\mathcal{MD}_3$.

Inessentiality of PSC-manifolds

Dmitry Bolotov, *Kharkiv, Ukraine*

We represent the our join work with A. Dranishnikov (Florida, USA).

The following definition was introduced by M.Gromov in [1].

Definition 1 *An n-manifold M with the fundamental group π is called essential if its classifying map $u : M \to B\pi$ cannot be deformed into the $(n-1)$-skeleton $B\pi^{(n-1)}$ and it is called inessential if u can be deformed into $B\pi^{(n-1)}$. M is called rationally inessential if the immage $u_* : H_n(M; \mathbb{Q}) \to H_n(B\pi; \mathbb{Q})$ is zero.*

We say that group π satisfies Rosenberg–Stolz conditions (RS-conditions) if

1. The homomorphism $ko_n(B\pi) \to KO_n(B\pi)$ induced by the transformation of spectra $ko \to KO$ is a monomorphism;
2. The Strong Novikov Conjecture holds for π: The analytic assembly map $\alpha : KO(B\pi) \to KO(C^*(\pi))$ is a monomorphism, where $C^*(\pi)$ is reduced C^* - algebra of π.

We prove the following theorem.

Theorem 1 *Let M be a closed orientable n-manifold, $n \geq 5$, with positive scalar curvature (PSC-manifold) whose fundamental group π is an RS - group. Then M is rationally inessential.*

Remark 1 *The case of spin-manifolds was proved us in [2]*

Remark 2 *The Theorem above allows us to draw important conclusions about macroscopic dimension of PSC-manifolds with special fundamental groups.*

[1] M. Gromov, *Positive curvature, macroscopic dimension, spectral gaps and higher signatures,*Functional analysis on the eve of the 21st century. Vol. II, Birkhauser, Boston, MA, 1996.

[2] D. Bolotov and A. Dranishnikov, *On Gromov's scalar curvature conjecture*, Proc. AMS 138 (2010), N 3,4, 1517–1524

Inverse isoperimetric inequality

Alexander Borisenko, *Sumy, Ukraine*

The classical isoperimetric property of a circle in the two-dimensional space of constant curvature equal to c claims that among all simple closed curves of a fixed length, the maximal area is enclosed only by a circle.

At the same time, there exist simple closed curves of fixed perimeter that bound domains whose areas are arbitrary close to zero.

It is true the following

Theorem 1 *Let G be a domain homeomorphic to the disk in two-dimensional Alexandrov space of the curvature $\geq c$.*

If specific rotation of boundary curve γ greater or equal $\lambda > 0$ and perimeter γ is equal L than the area F of the domain G satisfies the inequality

1.
$$F \geq \frac{L}{2\lambda} - \frac{1}{\lambda^2} \sin\left(\frac{L\lambda}{2}\right), \tag{1}$$
for $c = 0$;

2.
$$F \geq \frac{4}{k^2} \arctan\left(\frac{\lambda}{\sqrt{\lambda^2 + k^2}} \tan\left(\frac{\sqrt{\lambda^2 + k^2}}{4} L\right)\right) - \frac{\lambda}{k^2} L, \tag{2}$$
for $c = k^2$;

3. (a)
$$F \geq \frac{\lambda}{k^2} L - \frac{4}{k^2} \arctan\left(\frac{\lambda}{\lambda^2 - k^2} \tan\left(\frac{\sqrt{\lambda^2 - k^2}}{4} L\right)\right), \tag{3}$$
for $c = -k^2, \lambda > k$;

(b)
$$F \geq \frac{1}{k} L - \frac{4}{k^2} \arctan\left(\frac{k}{4} L\right), \tag{4}$$
for $c = -k^2, \lambda = k$;

(c)
$$F \geq \frac{\lambda}{k^2} L - \frac{4}{k^2} \arctan\left(\frac{\lambda}{\sqrt{k^2 - \lambda^2}} \tanh\left(\frac{\sqrt{k^2 - \lambda^2}}{4} L\right)\right), \tag{5}$$
for $c = -k^2, \lambda < k$.

In inequalities (1-5) the equality case holds only for λ-lunes on two-dimensional spaces of constant curvature c.

Dynamics in 3D viscous primitive equations

Igor Chueshov, *Kharkiv, Ukraine*

We deal with the 3D viscous primitive equations which arise in the study of oceanic motions. We concentrate on the so-called Ladyzhenskaya squeezing property and its consequences related to long-time dynamics in this system.

Heisenberg Odometers

Alexandre Danilenko, *Kharkiv, Ukraine*
Mariusz Lemańczyk, *Torun, Poland*

Let $H_3(\mathbb{R})$ denote the 3-dimensional real Heisenberg group. Given a family of lattices $\Gamma_1 \supset \Gamma_2 \supset \cdots$ in it, let T stand for the associated uniquely ergodic $H_3(\mathbb{R})$-$odometer$, i.e. the inverse limit of the $H_3(\mathbb{R})$-actions by rotations on the homogeneous spaces $H_3(\mathbb{R})/\Gamma_j$, $j \in \mathbb{N}$. The decomposition of the underlying Koopman unitary representation of $H_3(\mathbb{R})$ into a countable direct sum of irreducible components is explicitly described. The ergodic 2-fold self-joinings of T are found. It is shown that in general, the $H_3(\mathbb{R})$-odometers are neither isospectral nor spectrally determined.

Long-time asymptotics for the Toda shock problem

Iryna Egorova, *Kharkiv, Ukraine*

We derive the long-time asymptotics for the Toda shock problem using the nonlinear steepest descent analysis for oscillatory Riemann–Hilbert factorization problems. We show that the half plane of space/time variables splits into five main regions: The two regions far outside where the solution is close to free backgrounds. The middle region, where the solution can be asymptotically described by a two band solution, and two regions separating them, where the solution is asymptotically given by a slowly modulated two band solution. In particular, the form of this solution in the separating regions verifies a conjecture by Venakides, Deift, and Oba [1].

The work is done in collaboration with J. Michor and G. Teschl.

[1] S. Venakides, P. Deift, and R. Oba, *The Toda shock problem*, Comm. Pure Appl. Math. 44 (1991), no.8-9, 1171–1242.

On transformation operators in controllability problems for the wave equations with variable coefficients on a half-axis controlled by the Dirichlet boundary condition

Larissa Fardigola, *Kharkiv, Ukraine*

In this paper necessary and sufficient conditions of L^∞-controllability and approximate L^∞-controllability are obtained for the control system

$$w_{tt} = \frac{1}{\rho}\left(k w_x\right)_x + \gamma w, \qquad x > 0, \quad t \in (0,T), \tag{1}$$

$$w(0,t) = u(t), \qquad t \in (0,T). \tag{2}$$

Here ρ, k, and γ are given functions of x on $[0,+\infty)$; $u \in L^\infty(0,T)$ is a control; $T > 0$ is a constant. These problems are considered in special modified spaces of the Sobolev type that are introduced and studied in the paper. The growth of distributions from these spaces is associated with the equation data ρ and k. Using some transformation operator introduced and studied in the paper, we see that control system (1), (2) replicates the controllability properties of the auxiliary system

$$z_{tt} = z_{xx} - q^2 z, \qquad x > 0, \quad t \in (0,T) \tag{3}$$

$$z(0,t) = v(t), \qquad t \in (0,T), \tag{4}$$

and vise versa. Here $q \geq 0$ is a constant and $v \in L^\infty(0,T)$ is a control. Control problem (3), (4) has been investigated in [1]. Necessary and sufficient conditions of controllability for control system (1), (2) are obtained from the ones for auxiliary control system (3), (4) (see [2]).

[1] L.V. Fardigola, *Controllability problems for the 1-d wave equation on a half-axis with the Dirichlet boundary control*, ESAIM: Control, Optim. Calc. Var., 18 (2012), 748–773.

[2] L.V. Fardigola, *Transformation operators in controllability problems for the wave equations with variable coefficients on a half-axis controlled by the Dirichlet boundary condition*, MCRF, submitted.

Long-time behavior of a one-dimensional nonlinear system of thermoelasticity.

Tamara Fastovska, *Kharkiv, Ukraine*

We study oscillations of beams consisting of a special class of nonlinear thermoelastic materials (for details, see e.g. [1]). The beam, then in equilibrium, occupies the interval $\Omega = (0, l)$.

The system has the form

$$\alpha u_{tt} + \kappa u_{xxxx} - f_x(u_x) - \theta_x = 0, \tag{1}$$

$$\gamma\theta_t - \beta\theta_{xx} - \theta u_{tx} = 0, \qquad t > 0, \quad x \in \Omega \tag{2}$$

and is supplemented with boundary conditions

$$\begin{array}{l} u(0,t) = 0, \;\; u_x(0,t) = 0, \;\; \theta_x(0,t) = 0, \\ u(l,t) = 0, \;\; u_x(l,t) = 0, \;\; \theta_x(l,t) = 0. \end{array} \tag{3}$$

and corresponding initial conditions, $\alpha, \kappa, \gamma, \beta$ are positive constants. The variable $u(x,t)$ stands for the vertical displacement while $\theta(t,x)$ denotes the difference between the absolute temperature and its mean value.

The relevant result is recorded below.

Theorem 1 *Let the nonlinearity satisfies the conditions*

$$F(v) \geq -C, \qquad F(v) = \int_0^v f(\xi)d\xi,$$

$$|f''(v)| \leq C(1 + |v|^p), \qquad v \in \mathbb{R}, 0 \leq p < \infty$$

Then the dynamical system generated by (1)-(3) has a unique weak solution in the space

$$H = H_0^2(\Omega) \times L_2(\Omega) \times L_2(\Omega). \tag{4}$$

The dynamical system generated by (1)-(3) possesses a compact global attractor whose fractal dimension is finite.

[1] William J. Hrusa and Salim A. Messaoudi, *On formation of singularities in one-dimensional nonlinear thermoelasticity*, Arch. Rat. Mech. Anal. 111 (1990), 135–151.

On the Skitovich–Darmois theorem for the group of p-adic numbers

Gennadiy Feldman, *Kharkiv, Ukraine*

Let p be a prime number. We need some properties of the group of p-adic numbers Ω_p. As a set Ω_p coincides with the set of sequences of integers of the form $x = (\dots, x_{-n}, x_{-n+1}, \dots, x_{-1}, x_0, x_1, \dots, x_n, \dots)$, where $x_n \in \{0, 1, \dots, p-1\}$, such that $x_n = 0$ for $n < n_0$, where the number n_0 depends on x. Correspond to each element $x \in \Omega_p$ the series $\sum\limits_{k=-\infty}^{\infty} x_k p^k$. Addition and multiplication of series are defined in a natural way and define the operations of addition and multiplication in Ω_p. With respect to these operations Ω_p is a field. Denote by Δ_p a subgroup of Ω_p consisting of $x \in \Omega_p$ such that $x_n = 0$ for $n < 0$. Elements of the group Δ_p we write in the form $x = (x_0, x_1, \dots, x_n, \dots)$. The family of subgroups $\{p^m \Delta_p\}_{m=-\infty}^{\infty}$ can be considered as an open basis at zero of the group Ω_p and defines a topology on Ω_p. Each topological automorphism $\alpha \in \mathrm{Aut}(\Omega_p)$ is the multiplication by an element $x_\alpha \in \Omega_p$, $x_\alpha \neq 0$, i.e. $\alpha g = x_\alpha g$, $g \in \Omega_p$. We will identify the automorphism α with the corresponding element x_α, i.e. when we write αg, we will suppose that $\alpha \in \Omega_p$. Denote by Δ_p^0 the subset of Δ_p, consisting of all invertible elements of Δ_p, $\Delta_p^0 = \{x = (x_0, x_1, \dots, x_n, \dots) \in \Delta_p : x_0 \neq 0\}$. Each element $g \in \Omega_p$ is represented in the form $g = p^k c$, where k is an integer, and $c \in \Delta_p^0$. Denote by $I(\Omega_p)$ the set of shifts of the Haar distributions m_K of the compact subgroups K of Ω_p.

Theorem 1 *Let $\alpha \in \mathrm{Aut}(\Omega_p)$, $\alpha = p^k c$, $c \in \Delta_p^0$. Then the following statements hold.*

1. *Assume that either $k = 0$ or $|k| = 1$. Let ξ_1 and ξ_2 be independent random variables with values in Ω_p and distributions μ_1 and μ_2. Assume that the linear forms $L_1 = \xi_1 + \xi_2$ and $L_2 = \xi_1 + \alpha\xi_2$ are independent. Then*

$1(i)$ *If $k = 0$, then $\mu_1, \mu_2 \in I(X)$; moreover if $c = (1, c_1, \dots)$, then μ_1 and μ_2 are degenerate distributions;*

$1(ii)$ *If $|k| = 1$, then either $\mu_1 \in I(\Omega_p)$ or $\mu_2 \in I(\Omega_p)$.*

2. *If $|k| \geq 2$, then there exist independent random variables ξ_1 and ξ_2 with values in Ω_p and distributions μ_1 and μ_2 such that the linear forms $L_1 = \xi_1 + \xi_2$ and $L_2 = \xi_1 + \alpha\xi_2$ are independent whereas $\mu_1, \mu_2 \notin I(\Omega_p)$.*

This theorem is an analogue for the group Ω_p of the well-known Skitovich–Darmois theorem, where a Gaussian distribution on the real line is characterized by the independence of two linear forms.

Schoenberg matrices and Riesz sequences of translates in $L^2(\mathbb{R}^n)$

Leonid Golinskii, *Kharkiv, Ukraine*

Given a function f on the positive half-line $\mathbb{R}_+$ and a sequence (finite or infinite) of points $X = \{x_k\}_{k=1}^{\omega}$ in $\mathbb{R}^n$, we define and study matrices $\mathcal{S}_X(f) = \|f(|x_i - x_j|)\|_{i,j=1}^{\omega}$ called Schoenberg's matrices. We are primarily interested in those matrices which generate bounded and invertible linear operators $S_X(f)$

on $\ell^2(\mathbb{N})$. We provide conditions on X and f for the latter to hold. If f is an ℓ^2-positive definite function, such conditions are given in terms of the Schoenberg measure $\sigma(f)$. Examples of Schoenberg's operators with various spectral properties are presented.

We also approach Schoenberg's matrices from the viewpoint of harmonic analysis on $\mathbb{R}^n$, wherein the notion of the strong X-positive definiteness plays a key role. In particular, we prove that each radial ℓ^2-positive definite function is strongly X-positive definite whenever X is a separated set. We implement a "grammization" procedure for certain positive definite Schoenberg's matrices. This leads to Riesz–Fischer and Riesz sequences (Riesz bases in their linear span) of the form $\mathcal{F}_X(f) = \{f(x - x_j)\}_{x_j \in X}$ for certain radial functions $f \in L^2(\mathbb{R}^n)$.

[1] I. Schoenberg, *Metric spaces and completely monotone functions*, Ann. Math. 39 (1938), 811–841.

[2] M.M. Malamud, K. Schmüdgen, *Spectral theory of Schrödinger operators with infinitely many point interactions and radial positive definite functions*, J. Funct. Theory 263 (2012), 3144–3194.

Spectral problems in a domain with "trap"-like geometry of the boundary

Andrii Khrabustovskyi, *Karlsruhe, Germany*

It is well known that under smooth perturbation of a domain, the eigenvalues of the Neumann Laplacian vary continuously. If the perturbation is only C^0, then, in general, this is not true. The following example demonstrating this was considered in the classical book [1]. Let $\varepsilon > 0$ be a small parameter. Let Ω^ε be a domain consisting of a fixed domain Ω and a small domain (in what follows we will call such small domains as "traps"), which is a union a small square B^ε with a side length b^ε and a thin rectangle T^ε of the width d^ε and height h^ε. Here $d^\varepsilon = \varepsilon^4$, $h^\varepsilon = b^\varepsilon = \varepsilon$. The domain Ω^ε can be viewed as a C^0 perturbation of Ω. It was shown in [1] that the first principal eigenvalue of the Neumann Laplacian $-\Delta_{\Omega^\varepsilon}$ in Ω^ε goes to zero as $\varepsilon \to 0$, although the first principal eigenvalue of the Neumann Laplacian in Ω is positive.

In the present talk we consider the domain Ω^ε obtained by attaching to Ω many "traps". Their number is finite for a fixed ε and goes to ∞ as $\varepsilon \to 0$. The traps are attached along a flat part of $\partial\Omega$ (we denote it Γ). We consider the operator

$$\mathcal{A}^\varepsilon = -\frac{1}{\rho^\varepsilon}\Delta_{\Omega^\varepsilon},$$

where the weight $\rho^\varepsilon(x)$ is positive and equal to 1 in Ω. Our goal is to study the behaviour of its spectrum as $\varepsilon \to 0$.

For a wide range of values of d^ε, b^ε, h^ε and $\rho^\varepsilon|_{\Omega^\varepsilon\setminus\Omega}$ we prove that the spectrum of the operator $\mathcal{A}^\varepsilon$ converges as $\varepsilon\to 0$ to the spectrum of some operator $\mathcal{A}$ acting either in $L_2(\Omega)$ or in $L_2(\Omega)\oplus L_2(\Gamma)$. The form of the operator $\mathcal{A}$ depends on some relations between d^ε, b^ε, h^ε and $\rho^\varepsilon|_{\Omega^\varepsilon\setminus\Omega}$. In particular, in some cases $\mathcal{A}$ may have nonempty essential spectrum.

This is a joint work with Giuseppe Cardone (University of Sannio, Benevento, Italy). The work is supported by DFG via GRK 1294.

[1] R. Courant, D. Hilbert, *Methoden der mathematischen Physik*, Springer, Berlin, 1931.

A model Riemann–Hilbert problem for unimodular Baker–Akhiezer function

Vladimir Kotlyarov, *Kharkiv, Ukraine*

Let E_j and $\hat{E}_j$ $(j=1,2,...,n+1)$ be different complex numbers such that intervals $(E_1,\hat{E}_1),(E_2,\hat{E}_2),......,(E_{n+1},\hat{E}_{n+1})$ and $(\hat{E}_1,E_2),(\hat{E}_2,E_3),......,(\hat{E}_n,E_{n+1})$ are mutually disjoint on the complex plane $\mathbb{C}$. Let Σ be a peace-wise contour such that $\Sigma:=\bigcup\limits_{j=1}^{n+1}(E_j,\hat{E}_j)\cup\bigcup\limits_{j=1}^{n}(\hat{E}_j,E_{j+1})$. The orientation on Σ is choosing from the E_1 to $\hat{E}_{n+1}$. Let $\mathcal{X}$ be hyperelliptic curve generated by E_j and $\hat{E}_j$ as the branching points. Let $f(z)$, $g(z)$ be normilized Abelian integrals of the second kind with poles at the marked points of $\mathcal{X}$ and prescribed principal parts.

Model Riemann–Hilbert problem.
Let $M(z,x,t)$ be the 2×2 matrix solution of the Riemann–Hilbert problem:

- $M(z,x,t)$ is analytic in $\mathbb{C}\setminus\Sigma$;
- non-tangential boundary values $M_\pm(z,x,t)$ is continuous on Σ with exception of the points E_j and $\hat{E}_j$, where it has fourth root singularities and they satisfy the jump condition:

$$\begin{aligned} M_-(z,x,t) &= M_+(z,x,t)J(z,x,t), \qquad z\in\Sigma,\\ J(z,x,t) &= \begin{pmatrix} 0 & \mathrm{i}\\ \mathrm{i} & 0\end{pmatrix}, \qquad z\in(E_j,\hat{E}_j), \quad j=1,2,...,n+1;\\ &= \begin{pmatrix} e^{\mathrm{i}(xB_j^f+tB_j^g+B_j)} & 0\\ 0 & e^{-\mathrm{i}(xB_j^f+tB_j^g+B_j)}\end{pmatrix}, \quad z\in(\hat{E}_j,E_{j+1}), \end{aligned}$$

where B_j^f, B_j^g are nonzero periods of $f(z)$ and $g(z)$, and B_j $(j=1,2,...,n)$ are arbitrary complex number.

- $M(z,x,t) = I + O(z^{-1}) \qquad z \to \infty.$

Then **unimodular Baker–Akhiezer function** takes the form

$$\Phi(z,x,t) = e^{\mathrm{i}(xf_0+tg_0)\sigma_3} M(z,x,t) e^{-\mathrm{i}(xf(z)+tg(z))\sigma_3}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

where f_0, g_0 are the constants defined by $f(z)$ and $g(z)$.

The matrix $M(z,x,t)$ is expressed in an explicit form through the Riemann theta functions. The matrix $\Phi(z,x,t)$ is a compatible solution of some over-determined system of differential in x and t matrix equations. It generates a quasi-periodic solution of the corresponding nonlinear equations.

A linear interpolation problem for vector polynomials

Mikhail Kudryavtsev, *Kharkov, Ukraine*
Luis Silva, *Mexico, Mexico*
Sergio Palafox, *Mexico, Mexico*

Let us denote by $\mathbb{P}$ the space of n-dimensional vector polynomials, viz.,

$$\mathbb{P} := \{\boldsymbol{p}(z) = (P_1(z), P_2(z), \dots, P_n(z))^*;\ P_k \text{ is a scalar polynomial}\}.$$

Consider the following interpolation problem. Given a collection of complex numbers $z_1, \dots, z_N$, which are called interpolation nodes, and other collections of complex numbers $\alpha_k(1), \dots, \alpha_k(N)$, $k = 1, \dots, n$, such that $\sum_{k=1}^n |\alpha_k(j)| > 0$ for every $j \in \{1, 2, \dots, N\}$, find all the vector polynomials P_k, $k = 1, \dots, n$, which satisfy

$$\sum_{k=1}^{n} \alpha_k(j) P_k(z_j) = 0 \qquad \forall j \in \{1, 2, \dots, N\}.$$

Definition 1 *Let the function $h : \mathbb{P} \to \mathbb{N} \cup \{0, -\infty\}$ be defined by*

$$h(\boldsymbol{p}) := \begin{cases} \max_{j\in\{1,\dots,n\}} \{n \deg P_j(z) + j - 1\}, & \boldsymbol{p} \neq 0, \\ -\infty, & \boldsymbol{p} = 0. \end{cases}$$

The number $h(\boldsymbol{p})$ is called the height of the vector polynomial $\boldsymbol{p}$.

Theorem 1 *There exist n vector-polynomials $\boldsymbol{r}_j$, $j = 1, \dots, n$, which we call generators of the interpolation problem, such that*

$$h(\boldsymbol{r}_1) \le n, \qquad \sum_{j=1}^{n} h(\boldsymbol{r}_j) = Nn + \frac{n(n-1)}{2},$$

and any solution $\boldsymbol{p}$ of the interpolation problem can be uniquely represented in the form

$$\boldsymbol{p} = \sum_{j=1}^{n} S_j \boldsymbol{r}_j \,,$$

where S_j is a scalar polynomial. Conversely, any vector polynomial of this form with arbitrary polynomials S_j is a solution of the interpolation problem.

[1] M. Kudryavtsev, L. Silva, and S. Palafox, *On a linear interpolation problem for n-dimensional vector polynomials*, submitted to Linear Algebra and Applications.

Complex methods in Gabor Analysis

Yurii Lyubarskii, *Trondheim, Norway*

Given a function $g \in L^2(\mathbb{R})$ and numbers $a, b > 0$ we consider the corresponding *Gabor system* $\mathcal{G}(g; a, b) = \{e^{2ibnt} g(t - am)\}_{m,n\in\mathbb{Z}}$. Such systems apear naturally in Signal Processing, Pseudo-Differential Operators and some other areas. The Gabor Analysis studies expansions of arbitrary functions in $L^2(\mathbb{R})$ in elements of $\mathcal{G}(g; a, b)$.

There are various methods for dealing with such expansions. We present some techniques and results which are motivated by methods and techniques of Complex Analysis.

Trace formulas for pairs of resolvent comparable operators

Mark Malamud, *Donetsk, Ukraine*

Generalized shaddocks

Anatoly Milka, *Kharkiv, Ukraine*
Vasyl Gorkavyy, *Kharkiv, Ukraine*
Dmitro Kalinin, *Kharkiv, Ukraine*

We will discuss a series of *generalized shaddocks* discovered earlier by A.D. Milka. Every generalized shaddock $P(n, \beta)$ is a closed symmetric non-convex polyhedron with the following set of faces: two regular n-gonal bases, $2n$ regular triangular faces and $2n$ "petals" congruent to a rhombus broken along a diagonal of length L with dihedral angle β, all the edges have the length 1. The series of generalized shaddocks includes a shaddock of A. Douady $P(n, \frac{\pi}{2})$ [1] and a family of shaddocks $\{P(3,\beta)\}_{\beta\in S^1}$ studied by A.D. Milka in [2]. For any $n > 3$ there is also a continuous family of generalized shaddocks $\{S(n,\beta)\}_{\beta\in S^1}$.

It is proved that for any $n \geq 3$ the family $\{S(n,\beta)\}_{\beta\in S^1}$ contains a unique $S(n,\beta_0)$ with locally maximal value of L. Besides, it is shown that for any $n \geq 3$ the family $\{S(n,\beta)\}_{\beta\in S^1}$ contains a unique generalized shaddock, which is non-rigid of first order, if the velocity vectors of corresponding infinitesimal bendings are supposed to inherit the symmetry properties of $S(n,\beta)$. This non-rigid generalized shaddock turns out to coincide with $S(n,\beta_0)$. Finally, for a sufficiently small deformation of $S(n,\beta_0)$ in the family $\{S(n,\beta)\}_{\beta\in S^1}$ we construct an effective approximation by a continuous linear bending of $S(n,\beta_0)$.

It was suggested in [3], [4], that the shaddock of Douady is an example of a *model flexor*, i.e. a theoretically non-flexible polyhedron whose physical model admits essential continuous deformations without visible distortions of material like physical models of theoretical flexors. A similar phenomenon turns out to be observed for the generalized shaddocks $S(n,\beta_0)$. Like other known examples of model flexors, the star-like pyramids of Alexandrov-Vladimirova, the model flexibility of $S(n,\beta_0)$ may be explained apparently by the infinitesimal non-rigidity as well as by the continuous linear bendings mentioned above.

[1] M. Berger, *Geometry I*, Springer, Berlin, 1987.

[2] A. Milka, *Linear bendings of regular convex polyhedra*, Math. physics, Analysis, Geometry 1 (1994), 116–130 (in Russian).

[3] A. Milka, *Linear isometric deformations of polyhedra*, ICM-98, Berlin, Short Communications, 1998.

[4] A. Milka, *Linear bendings of convex polyhedra*, Int. conf. on geometry in large, Cherkassy, 1995. Abstracts of short communications, 61–62 (in Russian).

Random walks on discrete abelian groups

Margaryta Myronyuk, *Kharkiv, Ukraine*

Let $(\Omega,\mathfrak{A},P)$ be a probability space, X be a countable discrete abelian group, μ be a distribution on X. A random walk on X generated by μ is a sequence

$$S_n = \xi_1 + \dots + \xi_n, \quad n = 1, 2, \dots,$$

where ξ_j are independent identically distributed random variables with distribution μ defined on $(\Omega,\mathfrak{A},P)$ and with values in X.

The random walk on X is recurrent if

$$P\{\omega\in\Omega : S_n(\omega) = x \text{ for an infinite number of indices } n\in\mathbf{N}\} = 1$$

for all $x\in X$.

R. Dudley ([1]) proved that a countable discrete abelian group X has a recurrent random walk iff X contains no subgroup isomorphic to $\mathbb{Z}^3$.

Let X be a second countable locally compact abelian group, Y be its character group. Let (x, y) be the value of a character $y \in Y$ at an element $x \in X$, let m_Y be a Haar measure on Y. Let $\widehat{\mu}(y) = \int_X (x, y) d\mu(x)$ be the characteristic function of a distribution μ on X. S. Kesten and F. Spitzer proved the following criterion of recurrence of random walks ([2]): the random walk on a countable discrete abelian group X generated by a distribution μ is recurrent iff

$$\int_Y Re \frac{1}{1 - \hat{\mu}(y)} dm_Y = \infty.$$

We use this criterion to find necessary and sufficient conditions for the recurrence of random walks on arbitrary subgroups of rational numbers $\mathbb{Q}$ not isomorphic to $\mathbb{Z}$.

Note that in [3] Nabil Freig and S.A. Molchanov obtained necessary and sufficient conditions for the recurrence of random walks on the subgroup of rational numbers $H_p = \left\{\frac{m}{p^n}, m, n \in \mathbb{Z}\right\}$ (p is fixed). We generalize and strengthen results of [3]. Our proof is completely different from the one given in [3].

[1] R. Dudley, *Random walks on abelian groups*, Proc. Amer. Math. Soc., 13 (1962), 447–450.

[2] S. Kesten, F. Spitzer, *Random walk on countably infinite abelian groups*, Acta. Math., 114 (1965), 237–265.

[3] Nabil Freig, S.A. Molchanov, *On random walks on Abelian groups with infinite number of generators*, Vestn. Mosk. Univ., Ser. I (1978), No. 5, 22–29 (in Russian).

Schur—Weyl duality for the unitary groups of II_1-factors

Nikolay Nessonov, *Kharkiv, Ukraine*

We obtain the analogue of Schur–Weyl duality for the unitary group of an arbitrary II_1-factor

Let $\mathcal{M}$ be a separable II_1-factor , let $U(\mathcal{M})$ be its unitary group and let tr be a unique normalized normal trace on M. Denote by $\mathcal{M}'$ commutant of $\mathcal{M}$. Assume that $\mathcal{M}$ acts on $L^2(\mathcal{M}, \mathrm{tr})$ by left multiplication: $\mathfrak{L}(a)\eta = a\eta$, where $a \in \mathcal{M}$, $\eta \in L^2(\mathcal{M}, \mathrm{tr})$. Then $\mathcal{M}'$ coincides with the set of the operators that act on $L^2(\mathcal{M}, \mathrm{tr})$ by right multiplication: $\mathfrak{R}(a)\eta = \eta a$, where $\eta \in L^2(\mathcal{M}, \mathrm{tr})$, $a \in \mathcal{M}$. Let $\mathfrak{S}_p$ be the symmetric group of the n symbols $1, 2, \ldots, p$. Take $u \in U(\mathcal{M})$ and define the operators $\mathfrak{L}^{\otimes p}(u)$ and $\mathfrak{R}^{\otimes p}(u)$ on $L^2(\mathcal{M}, \mathrm{tr})^{\otimes p}$ as follows

$$\mathfrak{L}^{\otimes p}(u)\left(x_1 \otimes x_2 \otimes \cdots \otimes x_p\right) = ux_1 \otimes ux_2 \otimes \cdots \otimes ux_p,$$
$$\mathfrak{R}^{\otimes p}(u)\left(x_1 \otimes x_2 \otimes \cdots \otimes x_p\right) = x_1u^* \otimes x_2u^* \otimes \cdots \otimes x_pu^*,$$
$$\text{where} \quad x_1, x_2, \ldots, x_p \in L^2\left(\mathcal{M}, \mathrm{tr}\right).$$

Obviously the operators $\mathfrak{L}^{\otimes p}(u)$ and $\mathfrak{R}^{\otimes p}(u)$, where $u \in U(\mathcal{M})$, form the unitary representations of the group $U(\mathcal{M})$. Also, we define the representation $\mathcal{P}_p$ of $\mathfrak{S}_p$ that acts on $L^2\left(\mathcal{M}, \mathrm{tr}\right)^{\otimes p}$ by

$$\mathcal{P}_p(s)\left(x_1 \otimes x_2 \otimes \cdots \otimes x_p\right) = x_{s^{-1}(1)} \otimes x_{s^{-1}(2)} \otimes \cdots \otimes x_{s^{-1}(p)}, s \in \mathfrak{S}_p. \quad (1)$$

Denote by $\mathrm{Aut}\,\mathcal{M}$ the automorphism group of factor $\mathcal{M}$. Let θ_p^s be the automorphism of factor $\mathcal{M}^{\otimes p}$ that acts as follows

$$\theta_p^s(a) = \mathcal{P}_p(s)a\mathcal{P}_p(s^{-1}), \text{ where } s \in \mathfrak{S}_p, a \in \mathcal{M}^{\otimes p} \cup \mathcal{M}'^{\otimes p}. \quad (2)$$

Let $\mathcal{A}$ be the set of the operators on Hilbert space H, let $\mathcal{N}_{\mathcal{A}}$ be the smallest von Neumann algebra containing $\mathcal{A}$, and let $\mathcal{A}'$ be a commutant of $\mathcal{A}$. By von Neumann's bicommutant theorem $\mathcal{N}_{\mathcal{A}} = \{\mathcal{A}'\}' = \mathcal{A}''$.

Set $\left(\mathcal{M}^{\otimes p}\right)^{\mathfrak{S}_p} = \left\{a \in \mathcal{M}^{\otimes p} : \theta_p^s(a) = a \text{ for all } s \in \mathfrak{S}_p\right\}$.

The irreducible representations of $\mathfrak{S}_p$ are indexed by the partitions of p ($\lambda \vdash p$). Let χ^λ be the character of the corresponding irreducible representation T^λ. If $\dim \lambda$ is the dimension of T^λ, then operator $P_p^\lambda = \frac{\dim \lambda}{p!} \sum\limits_{s \in \mathfrak{S}} \chi^\lambda(s)\mathcal{P}_p(s)$ is the orthogonal projection on $L^2\left(\mathcal{M}, \mathrm{tr}\right)^{\otimes p}$. Denote by Υ_p the set of all partitions of p. The following statement is an analogue of the Schur-Weil duality.

Theorem 1 *Fix the nonnegative integer numbers p and q. Let λ and μ be the partitions from Υ_p and Υ_q, respectively, and let $\Pi_{\lambda\mu}$ be the restriction of representation $\mathfrak{L}^{\otimes p} \otimes \mathfrak{R}^{\otimes q}$ to the subspace $\mathcal{H}_{\lambda\mu} = P_p^\lambda \otimes P_q^\mu\left(L^2\left(\mathcal{M}, \mathrm{tr}\right)^{\otimes p} \otimes L^2\left(\mathcal{M}, \mathrm{tr}\right)^{\otimes q}\right)$. The following properties are true.*

- *(1) $\left\{\mathfrak{L}^{\otimes p} \otimes \mathfrak{R}^{\otimes q}\left(U(\mathcal{M})\right)\right\}'' = \left(\mathcal{M}^{\otimes p}\right)^{\mathfrak{S}_p} \otimes \left(\mathcal{M}'^{\otimes q}\right)^{\mathfrak{S}_q}$. In particular, the algebra $\left(\mathcal{M}^{\otimes p}\right)^{\mathfrak{S}_p} \otimes \left(\mathcal{M}'^{\otimes q}\right)^{\mathfrak{S}_q}$ is the finite factor.*

- *(2) For any λ and μ the representation $\Pi_{\lambda\mu}$ is quasi-equivalent to $\mathfrak{L}^{\otimes p} \otimes \mathfrak{R}^{\otimes q}$.*

- *(3) Let $\gamma \vdash p$ and $\delta \vdash q$. The representations $\Pi_{\lambda\mu}$ and $\Pi_{\gamma\delta}$ are unitary equivalent if and only if $\dim \lambda \cdot \dim \mu = \dim \gamma \cdot \dim \delta$.*

[1] T. Enomoto, M. Izumi, *Indecomposable characters on infinite dimensional groops associated with operator algebras*, arXiv:1308.6329v1 [math.OA] 28 Aug 2013.

[2] N. I. Nessonov, *Schur–Weyl duality for the unitary groups of* II_1*-factors*, arXiv:1312.0824 [math.RT] 13 Dec 2013.

Annihilating random walks as extended Pfaffian point process

Mihail Poplavskyi, *Coventry, United Kingdom*

In this talk we discuss two well-known stochastic models, connections between them and the eigenvalue dynamics for real Ginibre ensemble.

The first model is Glauber dynamics for 1D Ising model. We assume that evolution of spins $\sigma_t(x) = \pm 1$ with $x \in \mathbb{Z}$ is given by flipping rates

$$\omega_t(x) = 1/2\left(1 - 1/2\sigma_t(x)\left(\gamma^{(-)}(x)\,\sigma_t(x-1) + \gamma^{(+)}(x)\,\sigma_t(x+1)\right)\right),$$

where $\left|\gamma^{(-)}(x)\right| + \left|\gamma^{(+)}(x)\right| \le 2, \forall x \in \mathbb{Z}$. We also consider domain walls which separate $+1$ and -1 regions and defined by

$$\eta_t(x+1/2) = (1 - \sigma_t(x)\,\sigma_t(x+1))/2.$$

They form a point process on the dual lattice $\mathbb{Z} + 1/2$ that is the main object of our study. Using properties of the Glauber dynamics we prove

Theorem 1 *Fix $K, m \in \mathbb{Z}_+$, times $0 < t_1 < \ldots t_K < t$, particle positions $x_1^{(k)} < \ldots < x_{n_k}^{(k)}$ for $1 \le k \le K$, and even number of spin positions $y_1 \le \ldots \le y_{2m}$. Then under assumption of deterministic initial spin distribution the mixed particle-spin correlations satisfy*

$$\mathbb{E}\left[\prod_{k=1}^{K}\prod_{i=1}^{n_k}\eta_{t_k}\left(x_i^{(k)} + 1/2\right)\prod_{j=1}^{2m}\sigma_t(y_j)\right] = \mathrm{Pf}(K),$$

where kernel K is given in terms of a difference equation solution.

In the case of $\gamma^{(-)}(x) = \gamma^{(+)}(x) \equiv 1$ one can see that domain walls behave as Annihilating Random Walks (ARW). We present explicit kernels for different initial spin distributions. For independent $\sigma_0(x) = 2B_{1/2} - 1$ we show the convergence of the kernel to one, obtained in [1] for Annihilating Brownian Motions. One-sided initial distribution is also studied and connection to the eigenvalue dynamics of Ginibre ensemble is discussed. The talk is based on a joint work with Barnaby Garrod [2].

[1] R.Tribe, S.K. Yip, and O.Zaboronski, *One dimensional Annihilating and Coalescing particle systems as Extended Pfaffian Process*, Electron. Commun. Probab., 17, 40, 1–7.

[2] B. Garrod and M. Poplavskyi, *Annihilating Random Walks as Extended Pfaffian Point Process*, in prep.

Critical points of the Ginzburg–Landau functional with semi-stiff boundary conditions

Vladimir Rybalko, *Kharkiv, Ukraine*

We consider a variational problem for Ginzburg–Landau functional when the unknown complex-valued function u (order parameter) is constrained to take boundary values on the unit circle. This constraint results in the so-called semi-stiff boundary conditions for critical points: the Dirichlet condition $|u| = 1$ and the Neumann condition for phase. To find a nontrivial solutions one can prescribe winding numbers on connected components on the boundary, but this might lead to a noncompact variational problem. The corresponding existence/nonexistence results obtained in the last decade will be discussed.

TBA

Maria Shcherbina, *Kharkiv, Ukraine*

Universality of the local regime for the block band matrices with a finite number of blocks

Tatyana Shcherbina, *St. Petersburg, Russia*

Set $\Lambda = [1, m]^d \cap \mathbb{Z}^d$ and consider Hermitian matrices H_N, $N = |\Lambda|W$ with elements $H_{jk,\alpha\beta}$, where $j, k \in \Lambda$ (they parameterize the position of the block) and $\alpha, \beta = 1, \dots, W$ (they parameterize the entries inside the block). The entries $H_{jk,\alpha\beta}$ are random Gaussian variables with mean zero such that

$$\langle H_{j_1k_1,\alpha_1\beta_1} H_{j_2k_2,\alpha_2\beta_2} \rangle = \delta_{j_1k_2}\delta_{j_2k_1}\delta_{\alpha_1\beta_2}\delta_{\beta_1\alpha_2} J_{j_1k_1}. \tag{1}$$

Here $J = 1/W + \alpha\Delta/W, \quad \alpha < 1/4d$, where Δ is the discrete Laplacian on Λ. Such models were first introduced and studied by Wegner and can be considered as one of the possible realizations of the Gaussian random band matrices (RBM).

There is a physical conjecture (see e.g. [1]), which states that for 1D RBM there is a crossover: for $W \gg \sqrt{N}$ the local behavior of the eigenvalues is the same as for GUE (delocalized states), and for $W \ll \sqrt{N}$ we get another behavior, which determines by the Poisson statistics (localized states).

The first part of this conjecture for the second mixed moment of the characteristic polynomials was proved in [2] using the supersymmetry approach (SUSY). The approach is widely used in physics and is potentially very powerful but the rigorous control of the integral representations, which can be obtained by this method, is difficult and so far it has been performed only for the density of states, but not for the higher correlation functions. From the SUSY point of view characteristic polynomials correspond to the so-called fermionic sector of the SUSY full model, which describes the higher correlation functions. Here we present the rigorous SUSY result about the second correlation function of block RBM (i.e. about the SUSY full model) although with finite number of blocks:

Theorem 1 *Let $|\Lambda|$ be fixed and R_2 be the second correlation function of (1), and $\rho(\lambda_0) = (2\pi)^{-1}\sqrt{4-\lambda_0^2}$. Then for any $|\lambda_0| < \sqrt{2}$ and $\xi_1, \xi_2 \in [-M, M]$ we have, as $W \to \infty$:*

$$(N\rho(\lambda_0))^{-2} R_2\left(\lambda_0 + \frac{\xi_1}{\rho(\lambda_0)\,N}, \lambda_0 + \frac{\xi_2}{\rho(\lambda_0)\,N}\right) \xrightarrow{w} 1 - \frac{\sin^2(\pi(\xi_1-\xi_2))}{\pi^2(\xi_1-\xi_2)^2}.$$

[1] Y.V. Fyodorov, A,D. Mirlin, *Scaling properties of localization in random band matrices: a σ-model approach*, Phys. Rev. Lett., 67 (1991), 2405–2409.

[2] T. Shcherbina, *On the second mixed moment of the characteristic polynomials of the 1D band matrices*, Commun. Math. Phys., 328 (2014), 45–82.

The Ostrovsky–Vakhnenko equation by a Riemann–Hilbert approach

Dmitry Shepelsky, *Kharkiv, Ukraine*

We present an inverse scattering transform approach for the equation

$$u_{txx} - 3u_x + 3u_x u_{xx} + u u_{xxx} = 0. \tag{1}$$

This equation can be viewed as the short wave model for the Degasperis–Procesi equation [1], which is a model of wave propagation in shallow water. On the other hand, it arises (and is known as the "Vakhnenko equation") in the context of propagation of high-frequency waves in a relaxing medium [2]. Yet another domain where it arises is the study of weakly nonlinear surface and internal waves in a rotating ocean influenced by Earth rotation [3], where it is known as the "reduced Ostrovsky equation" or the "Rotation-Modified KdV equation".

Our approach is based on using the Lax pair representation of (1), which allows formulating an associated Riemann–Hilbert problem giving a representation for the classical (smooth) solution of the Cauchy problem for the Ostrovsky–Vakhnenko equation in terms of a solution to this problem, to get the principal term of its long time asymptotics, and also to describe loop (generalized, multi-valued) soliton solutions. A specific feature of the Lax pair equations for (1) is that they are 3×3 matrix ODEs.

[1] A. Degasperis and M.Procesi, *Asymptotic integrability*, Symmetry and perturbation theory, World Sci. Publ., River Edge, NJ, 1998, 23–37.

[2] V.O. Vakhnenko, *The existence of loop-like solutions of a model evolution equation*, Ukr. Journ. Phys., 42 (1997), 104–110.

[3] L.A. Ostrovsky, *Nonlinear internal waves in a rotating ocean*, Oceanology 18, (1978), no. 2, 181–191.

Generic symmetries on the Laurent extension of quantum plane

Sergey Sinel'shchikov, *Kharkiv, Ukraine*

The standard quantum plane is a unital algebra with the two generators x, y and a single relation $yx = qxy$. Our subject is the Laurent extension $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$ of this algebra, which is derived by letting the generators x, y to be invertible. The problem we consider is that of describing the $U_q(\mathfrak{sl}_2)$-symmetries (in other terminology, the structures of $U_q(\mathfrak{sl}_2)$-module algebra [1]) on $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$. Here $U_q(\mathfrak{sl}_2)$ is the quantum universal enveloping algebra of $\mathfrak{sl}_2$, defined by its generators k, k^{-1}, e, f, together with the well known relations [1]. By now there exists a complete classification of $U_q(\mathfrak{sl}_2)$-symmetries on the standard quantum plane [2]. It turns out that $\mathbb{C}_q\left[x^{\pm 1}, y^{\pm 1}\right]$ is much more symmetric.

Theorem 1 *There exists a two-parameter ($\alpha, \beta \in (\mathbb{C}^*)^2$) family of $U_q(\mathfrak{sl}_2)$-symmetries of $\mathbb{C}_q[x^{\pm 1}, y^{\pm 1}]$, in which the action of the Cartan generator* k *on monomials does not reduce to multiplying by constants as in [2]:*

$$\begin{aligned} \pi(\mathsf{k})x &= \alpha^{-1}x^{-1} \qquad & \pi(\mathsf{k})y &= \beta^{-1}y^{-1} \\ \pi(\mathsf{e})x &= 0 & \pi(\mathsf{e})y &= 0 \\ \pi(\mathsf{f})x &= 0 & \pi(\mathsf{f})y &= 0. \end{aligned}$$

This is a complete list of symmetries with the above property. All these symmetries are isomorphic, in particular, to that with $\alpha = \beta = 1$.

Theorem 2 *Let $\alpha, \beta \in (\mathbb{C}^*)^2$ be such that $\alpha^u \beta^v = q^2$ for some $u, v \in \mathbb{Z}$ and $\alpha^m \neq \beta^n$ for non-zero integers m, n. Then there exists a one-parameter*

($a \in \mathbb{C}^*$) family of generic $U_q(\mathfrak{sl}_2)$-symmetries of $\mathbb{C}_q[x^{\pm 1}, y^{\pm 1}]$:

$$\pi(\mathsf{k})x = \alpha x \qquad \pi(\mathsf{k})y = \beta y$$
$$\pi(\mathsf{e})x = -a\frac{1-\alpha q^v}{(1-q^2)^2}x^{u+1}y^v \qquad \pi(\mathsf{e})y = -a\frac{q^u-\beta}{(1-q^2)^2}x^u y^{v+1}$$
$$\pi(\mathsf{f})x = \frac{q^{uv+3}(\alpha^{-1}-q^{-v})}{a}x^{-u+1}y^{-v} \quad \pi(\mathsf{f})y = \frac{q^{uv+3}(\beta^{-1}q^{-u}-1)}{a}x^{-u}y^{-v+1}$$

[1] C. Kassel, *Quantum Groups*, Springer–Verlag, New York, 1995.

[2] S. Duplij and S. Sinel'shchikov, *Classification of $U_q(\mathfrak{sl}_2)$-module algebra structures on the quantum plane*, J. Math. Phys., Anal., and Geom., 6 (2010), No. 4, 406–430.

TBA

Mikhail Sodin, *Tel-Aviv, Israel*

On the fluctuations of entries of deformed unitary invariant ensembles

Vladimir Vasilchuk, *Kharkiv, Ukraine*

We consider first the additive ensemble of $n \times n$ random matrices $H_n = A_n + U_n^* B_n U_n$, where A_n and B_n are Hermitian (real symmetric), having the limiting Normalized Counting Measure of eigenvalues, and U_n is unitary, uniformly distributed over $U(n)$. We find the leading term of asymptotic expansion for the covariance of elements of resolvent of H_n and establish the Central Limit Theorem for elements of sufficiently smooth statistic function of H_n as $n \to \infty$. Then analogously we study the multiplicative ensemble $W_n = S_n U_n^* T_n U_n$.

Eigenvalue distribution of large weighted bipartite random graphs

Valentin Vengerovsky, *Kharkiv, Ukraine*

We study eigenvalue distribution of the weighted adjacency matrix $A^{(N,p,\alpha)}$ of random bipartite graphs $\Gamma = \Gamma_{N,p,\alpha}$. We assume that the graphs have N vertices, the ratio of parts is $\frac{\alpha}{1-\alpha}$ and the average number of edges attached to one vertex is $\alpha \cdot p$ or $(1-\alpha) \cdot p$. To each edge of the graph e_{ij} we assign a weight given by a random variable a_{ij}.

The weak convergence in probability of normalized eigenvalue counting measures is proved. We derive closed system of equations that uniquely determine the limiting measure.

Constant slope hypersurfaces in the Euclidean space

Alexander Yampolsky, *Kharkiv, Ukraine*

Let X be a unit vector field on the Riemannian manifold (M^{n+1}, g). We say that F^n is a constant angle hypersurface if $g(X, n) = \cos\theta = const$, where n is a unit normal vector field on F^n. If M^{n+1} is Euclidean and X is a constant vector field, then the constant angle surface generalizes the concept of helix. If X is a radial vector field on $E^{n+1} \setminus \{0\}$, then the constant angle surface generalizes the concept of logarithmic spiral. In the latter case the hypersurface is called *constant slope* hypersurface. M. Munteanu [1] described all constant slope hypersurfaces in E^3. He proved particularly that in general case when $\theta \in \left(0, \frac{\pi}{2}\right)$ the position vector of a surface is of the form

$$\vec{r}(u,v) = e^{u\tan\theta}\sin\theta\big(\cos u\vec{f}(v) + \sin u\vec{f}(v)\times\vec{f}'(v)\big),$$

where $f(v)$ is a unit speed curve on the unit sphere S^2 centered at the origin.

We generalize the result of Munteanu and give local description of constant slope hypersurfaces in the following way.

Theorem 1 *Let $F^n \subset E^{n+1} \setminus \{0\}$ be constant slope regular hypersurface with $\theta \in \left(0, \frac{\pi}{2}\right)$. Then its position vector is of the form*

$$\vec{r}(u,v) = e^{u\tan\theta}\sin\theta\big(\cos u\vec{a}(v) + \sin u\vec{b}(v)\big),$$

where $\vec{a} : D^n(v^1, \dots, v^n) \to S^{n+1}$ is a local parameterization of a hypersurface in the unit sphere S^{n+1} centered at the origin and $\vec{b}$ is a field of unit normals of the latter hypersurface.

The constant slope hypersurfaces have a nice geometrical description.

Theorem 2 *Let $\vec{r} = |\vec{r}|(\cos\theta\vec{n} + \sin\theta\vec{e}_1)$ be a position vector of constant slope hypersurface $F^n \subset E^{n+1} \setminus \{0\}$ with $\theta \in \left(0, \frac{\pi}{2}\right)$. Then*

- *$\vec{e}_1$ is a principal direction on F^n;*
- *the integral trajectories of $\vec{e}_1$ are geodesic lines on F^n;*
- *the distribution $\vec{e}_1^{\perp}$ is integrable on F^n and the integral submanifolds are sections of the hypersurface by one-parametric family of spheres centered at the origin.*

[1] M. Munteanu, *From golden spirals to constant slope surfaces*, arXiv:0903.1348v1 [math.DG].

Inverse spectral problem for operators with non-local potentials

Vladimir Zolotarev, *Kharkiv, Ukraine*

Operator which is a finite-dimensional perturbation of the operator of second derivative with self-adjoint boundary conditions is being studied. Description of the spectrum of such operator is given and the inverse problem of the recovery of parameters of perturbation by the spectral data is being solved.

YOUNG PARTICIPANTS

On the second correlation function of characteristic polynomials of sparse hermitian random matrices

Ievgenii Afanasiev, *Kharkiv, Ukraine*

We consider the asymptotic behavior of the correlation function of the product of two characteristic polynomials of sparse hermitian $n \times n$ random matrices

$$M_n = (d_{jk} w_{jk})_{j,k=1}^{n}$$

where

$$d_{jk} = p^{-1/2} \begin{cases} 1 \text{ with probability } \frac{p}{n}; \\ 0 \text{ with probability } 1 - \frac{p}{n} \end{cases}$$

and $\Re w_{jk}$, $\Im w_{jk}$ are i.i.d. Gaussian random variables with zero mean such that

$$2\mathbf{E}\{|\Re w_{jk}|^2\} = 2\mathbf{E}\{|\Im w_{jk}|^2\} = \mathbf{E}\{|w_{ll}|^2\} = 1, \quad j \neq k.$$

Let $F(Z) = \mathbf{E}\{\det(M_n - z_1)\det(M_n - z_2)\}$, $Z = (z_1, z_2) \in \mathbb{R}^2$.

The main result is

Theorem 1 *Let* $a_n = \sqrt{\frac{2(n-p)}{pn}}$. *Then for* $z_0^2 < 4 - 4a_n^2$ *we have*

$$F\left(Z_0 + X/n\right) = C_1 n \exp\{n(z_0^2 + a_n^2 - 2)/2 + z_0(x_1 + x_2)/2\} \times \frac{\sin((x_1 - x_2)\sqrt{4 - 4a_n^2 - z_0^2}/2)}{(x_1 - x_2)}(1 + o(1)),\ n \to \infty.$$

In the case of $z_0^2 > \max\{4 - 4a_n^2, 0\}$ *we have*

$$F\left(Z_0 + X/n\right) = C_2 \exp\left\{nA/2 + \alpha z_0(x_1 + x_2)\right\} \frac{x_1 + x_2}{x_1 - x_2} \times \frac{\alpha^2}{(2\alpha - 1)^{3/2}(2 - \alpha(1 - \alpha)(3 - 2\alpha)z_0^2)^{1/2}}(1 + o(1)),\ n \to \infty.$$

Here $Z_0 = (z_0, z_0)$, $X = (x_1, x_2)$; $z_0, x_1, x_2 \in \mathbb{R}$,

$$A = 2(1 - \alpha)z_0^2 + \left(\frac{1 - \alpha}{\alpha}\right)^2 a_n^2 - 2 + 2\ln\frac{\alpha}{1 - \alpha},$$

$\alpha \in (1/2,\, 1)$ *such that*

$$\alpha(1 - \alpha)z_0^2 + \left(\frac{1 - \alpha}{\alpha}\right)^2 a_n^2 = 1,$$

and C_1, C_2 *are some absolute constants.*

Regularized integrals of motion of the KdV equation in a class of non-decreasing functions

Kyrylo Andreiev, *Kharkiv, Ukraine*

Synthesis problem for systems with power nonlinearity

Maxim Bebiya, *Kharkiv, Ukraine*

This work is devoted to a solution of the synthesis problem for nonlinear system

$$\begin{cases} \dot{x}_1 = u, \quad |u| \le 1, \\ \dot{x}_i = x_{i-1} + f_{i-1}(x_1, \dots, x_n), \quad i = 2, \dots, n-1, \\ \dot{x}_n = x_{n-1}^{2k+1} + f_{n-1}(x_1, \dots, x_n), \end{cases} \tag{1}$$

which uncontrollable with respect to the first approximation. Using the controllability function method [1], we construct a positional control $u = u(x)$ of the form

$$u(x) = \frac{1}{\theta^m(x)}(a, D(\theta(x))x) + a_{n+1}\frac{x_{n-1}^{2k+1}}{\theta^{m-1}(x)}, \tag{2}$$

where $m = (2k+1)n-2k$, $D(\theta) = \operatorname{diag}\left(\theta^{m-1}, \dots, \theta^{m-n+1}, 1\right)$ is $(n \times n)$-matrix, $a = (a_1, \dots, a_n)^* \in \mathbb{R}^n$, $a_{n+1} \in \mathbb{R}$, and $\theta(x)$ is the unique positive solution of the equation

$$2a_0\theta^{2m} = (FD(\theta)x, D(\theta)x), \quad x \in \mathbb{R}^n \setminus \{0\}, \quad \theta(0) = 0,$$

where F is a solution of singular Lyapunov inequality $A^*F + FA \le 0$ [2],

$$A = \begin{pmatrix} a_1 & a_2 & a_3 & \cdots & \cdots & a_n \\ 1 & 0 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

Theorem 1 *Let $a_i < 0$, $i = 1, \dots, n$ be such that eigenvalues of matrix A has negative real parts, $a_{n+1} = -\frac{fnn}{f_{1n-1}} \cdot \frac{a_{n-1}}{a_n}$ and a_0 is positive solution of*

$$2a_0((2a_0)^{2k}a_{n+1}^2 + 2^{2k+1}a_0^k\lambda_{min}^k\|a\|a_{n+1} + \lambda_{min}^{2k}\|a\|^2) - \lambda_{min} = 0,$$

where λ_{min} is the smallest eigenvalue of matrix F. Then the control $u(x)$ of the form (2) transfers an arbitrary initial point x_0 to the origin along the trajectory of the system (1) in some finite time $T(x_0)$ and satisfies the preassigned constraints $|u(x)| \le 1$, $x \in \mathbb{R}^n$.

[1] V. I. Korobov, *The controllability function method*, R&C Dynamics, M.-Izhevsk, 2007 (in Russian).

[2] V. Korobov and M. Bebiya, *Stabilization of some class of nonlinear systems that are uncontrollable to the first approximation*, Dopividi NAN Ukraine (2014), no. 2, 20–25 (in Russian).

An operator analogue of the Bruwier series

Sergey Gefter, *Kharkiv, Ukraine*
Ann Tanasichuk, *Kharkiv, Ukraine*

We study an operator analogue of the Bruwier series. This series is used to solve simple linear homogeneous differential-difference equations in Banach spaces .

Let E be a complex Banach space, $A : E \to E$ be a bounded linear operator and $h \in C$, $h \neq 0$. We call the next formal operator series

$$\sum_{n=0}^{+\infty} \frac{1}{n!} A^n (z + nh)^n \tag{1}$$

by the Bruwier series.

In the scalar case, this series has been studied in detail in [1, 2, 4] (see also [3] , page 180). If $h = 0$, we obtain the power series for the exponent of a bounded linear operator.

Theorem 1 *Suppose that* $r(A)$ *is the spectral radius of A. If* $r(A) < \frac{1}{e|h|}$, *then the series (1) converges uniformly in any compact, and its sum is an entire operator function of exponential type* $\sigma < \frac{1}{|h|}$.

We now consider the next Cauchy problem

$$\begin{cases} u'(z) = Au(z + h), \\ u(0) = u_0. \end{cases} \tag{2}$$

where $u_0 \in E$.

Theorem 2 *If* $||A|| < \frac{1}{2e|h|}$, *then the Cauchy problem (2) has an entire solution of exponential type for any initial vector* $u_0 \in E$.

[1] L. Bruwier, *Sur l'equation fonctionelle* $y^{(n)}(x)+a_1y^{(n-1)}(x+c)+...+a_{n-1}y'(x+(n-1)c)+a_ny(x+nc) = 0$, Comptes Rendus de Congres National des Science, Brulles, 1930, 1931, 91–97; *Sur une equation aux derives et aux differences melees*, Mathesis, 47 (1933), 96–105.

[2] O. Perron, *Über Bruwiersche Reihen*, Math. Z., 45 (1939), 127–141.

[3] R. Bellman, K.L. Cookg, *Differential-difference equations. Mathematics in science and engineering*, The RAND corporation, Santa Monica, California, Newyork academic press, London, 1963.

[4] F. Vogl, *On the Uniform Approximation of a Class of Analytic Functions by Bruwier Series*, Journal of Approximation Theory, 107 (2000), Issue 2, 281–292.

Interaction between the two "accelerating-packing" flows in a gas of hard spheres

Vyacheslav Gordevskyy, *Kharkiv, Ukraine*
Natalya Lemesheva, *Kharkiv, Ukraine*

We consider the approximate solutions of the non-linear Boltzmann equation [1-2], which have the structure of the linear combination of two Maxwellians, namely $f = \varphi_1 M_1 + \varphi_2 M_2$, where the coefficient functions $\varphi_i = \varphi_i(t,x)$, $i = 1,2$ are nonnegative and smooth, and M_i have the form [3]:

$$M_i = \overline{\rho}_i \cdot \left(\frac{\beta_i}{\pi}\right)^{3/2} \cdot e^{\beta_i(2\overline{u}_i x - v^2 + 2v(\overline{v}_i - \overline{u}_i t))}.$$

The purpose is to find such a form of the functions $\varphi_i(t,x)$, $i = 1,2$ and such behavior of all hydrodynamical parameters so that the integral error

$$\Delta_1 = \int_{R^1} dt \int_{R^3} dx \int_{R^3} |D(f) - Q(f,f)|\, dv$$

tends to zero.

We find some sufficient conditions to minimize this error and built bimodal explicit approximate solutions of the Boltzmann equation, where the functions φ_i, $i = 1,2$ have the form of finite "δ - plateau" [4].

The obtained results form the content of the paper which was sent for publication.

[1] C. Cercignani, *The Boltzmann Equation and its Applications*, Springer, New York, 1988.

[2] M.N. Kogan *The dinamics of a Rarefied Gas*, Nauka, Moscow, 1967.

[3] V.D. Gordevskyy and N.V. Andriyasheva, *Interaction between "Accelerating-Packing" Flows in a Low-Temperature Gas*, Math. Phys., Anal., Geom. 5 (2009), no. 1, 38–53.

[4] V.D. Gordevsky, *Trimodal Approximate Solutions of the Non-linear Boltzmann Equation*, Math. Meth. Appl. Sci. 21 (1998), 1479–1494.

Homogenized model of diffusion in porous media with nonlinear absorption at the boundary

Larysa Khilkova, *Rubizhne, Ukraine*

Let Ω be a bounded domain in R^n $(n \geq 2)$, F^ε be a closed set in Ω depending on a small parameter ε, so that the set F^ε as $\varepsilon \to \infty$ is becoming more porous and is more dense in Ω. We assume that the boundary of F^ε is piecewise smooth.

Consider in $\Omega^\varepsilon = \Omega \setminus F^\varepsilon$ boundary value problem:

$$\begin{cases} -\Delta u^{\varepsilon} = f^{\varepsilon}(x), \quad x \in \Omega^{\varepsilon}, \\ \dfrac{\partial u^{\varepsilon}}{\partial \nu} + \sigma^{\varepsilon}(x, u^{\varepsilon}) = 0, \quad x \in \partial F^{\varepsilon}, \\ u^{\varepsilon} = 0 \quad \text{на} \quad \partial\Omega, \end{cases} \tag{1}$$

where $\Delta = \sum\limits_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}$ is the Laplace operator, ν is the exterior unit normal to the Ω^{ε}; the function $f^{\varepsilon} \in L^q(\Omega^{\varepsilon})$, $\left(q > \frac{2n}{n+2}\right)$ and the function $\sigma^{\varepsilon}(x, s)$ satisfies the certain conditions monotonicity and bounded growth.

The problem (1) describes the process of stationary diffusion in a porous medium with absorption on the walls of the pores F^{ε}.

We study the asymptotic behavior of the generalized solution $u^{\varepsilon}(x)$ of the problem (1) as $\varepsilon \to 0$. We prove that $u^{\varepsilon}(x)$ converges in $L^p(\Omega^{\varepsilon})$ $\left(p \leq \frac{2n}{n-2}\right)$ to a function $u(x)$ that solves the following boundary value problem

$$\begin{cases} -\sum\limits_{i,k=1}^{n} \dfrac{\partial}{\partial x_i}\left(a_{ik}(x)\dfrac{\partial u}{\partial x_k}\right) + \dfrac{1}{2}c_u(x, u) = f(x), \quad x \in \Omega, \\ u(x) = 0, \quad x \in \partial\Omega. \end{cases} \tag{2}$$

Here $\{a_{ik}\}_{i,k=1}^{n}$ is a symmetric positive definite tensor in R^n which describes the conductivity of the porous medium, $c_u(x, u) = \frac{\partial}{\partial u}c(x, u)$ and the function $c(x, u)$ characterizes the absorption properties of the boundary.

On attractors of plate models with strong nonlinear damping

Stanislav Kolbasin, *Kharkiv, Ukraine*

The talk is devoted to long-time behaviour of solutions to the Cauchy problem for the following equation:

$$u_{tt} + \mathcal{D}(u, u_t) + \mathcal{A}u + F(u) = 0. \tag{1}$$

The above equation is considered in an abstract Hilbert space H, with operators $\mathcal{A}$, F, and $\mathcal{D}(u, u_t)$ (for each $u \in H$) densely defined in H. Also, $\mathcal{A}$ here is positive and self-adjoint, and $\mathcal{D}(u, u_t)$ maps $D(\mathcal{A}^{1/2}) \times D(\mathcal{A}^{\theta})$ into $D(\mathcal{A}^{-\theta})$ with $0 < \theta \leq 1/2$.

An example of this abstract model can be the plate oscillation equation

$$u_{tt} - div[\sigma_1(u)\nabla u_t] + g(u, u_t) + \Delta^2 u + F(u) = 0 \tag{2}$$

with locally Lipschitz functions $\sigma_1(s)$ and $g(s_1, s_2)$ and feedback force $F(u)$ corresponding to Kirchhoff, von Karman, or Berger plate models (see discussion in [1]).

The main result in study of (1) is existence of a compact global attractor of finite dimension, as well as an exponential fractal attractor.

The talk is based on the results of the joint article [1] with I.D. Chueshov.

[1] I. Chueshov and S. Kolbasin, *Long-time dynamics in plate models with strong nonlinear damping*, Communications on Pure and Applied Analysis 11, 659–674.

A method for research on polynomial systems

Valery Korobov, *Kharkiv, Ukraine*
Ekaterina Gladkova, *Kharkiv, Ukraine*

Let us consider the polynomial system of the form:

$$\sum_{i=1}^{n} T_i^k = S_k, \quad k \in I \tag{1}$$

where I is some set of indexes.

In the case when $I = \{1, 2, \dots, n\}$ it can be solved by reducing to finding the roots of the nth-order polynomial equation

$$t^n - \sigma_1 t^{n-1} + \cdots + (-1)^n \sigma_n = 0 \tag{2}$$

where the coefficients $\sigma_1, \dots, \sigma_n$ can be expressed via $S_1, \dots, S_n$ by using Newton's identities.

In the work [1] the polynomial systems with even power gaps that alternate in sign are discussed. Such systems arise in optimal control problems.

The authors investigated the system (1) in the case when

$$I = \{1, \dots, m, m + k_1, m + k_2, \dots, m + k_{n-m}\}$$

where $m, k_1, \dots, k_{n-m} \in \mathbb{N}$, $m < n$, $1 \leqslant k_1 < k_2 < \cdots < k_{n-m} < m + 2$. The method for solving of this system that also consists in its reducing to the polynomial equation (2) is suggested,with the coefficients $\sigma_1, \dots, \sigma_n$ being expressed via S_k, $k \in \{1, \dots, m, m + k_1, m + k_2, \dots, m + k_{n-m}\}$. The necessary and sufficient conditions on right-hand sides under which the system has a unique (up to permutations) solution, infinite number of solutions and does not have any solution are given.

The system of equations of the form (1) appears, for instance, in modified Le Verrier's problem of finding the eigenvalues of a matrix A by knowing the traces S_k of matrices A^k, $k \in I$.This is connected with the fact that the trace S_k equals $\sum_{i=1}^{n} T_i^k$ where T_i are the eigenvalues of the matrix A.

[1] V.I. Korobov, A.N. Bugayevskaya, *The solution of one time-optimal problem on the basis of the Markov moment min-problem with even gaps*, Mathematicheskaya fizika, analiz, geometriya 10 (2003), no. 4, 505–523 (in Russian).

[2] Y. Wu, C.N. Hadjicostis, *On solving composite power polynomial equations*, Mathematics of computation 74 (2004), no. 250, 853–868.

[3] D.K. Faddeev, *Lectures on Algebra*, Nauka, Moscow, 1984 (in Russian).

Approximation properties of the generalized Fup-functions

Victor Makarichev, *Kharkiv, Ukraine*

Consider the function $f(x) \in L_2(\mathbb{R})$ such that $supp\ f(x) = [-1, 1]$, $f(x)$ is an even function, $f(x) \geq 0$ for any $x \in [-1, 1]$ and $\int\limits_{-\infty}^{\infty} f(x)dx = 1$. By $F(t)$ denote the Fourier transform of this function.

Let

$$f_{N,m}(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} e^{itx} \left(\frac{\sin\left(\frac{t}{N}\right)}{\frac{t}{N}} \right)^{m+1} F\left(\frac{t}{N}\right) dt,$$

where $N \neq 0$ and $m = 2, 3, 4, \ldots$. The function $f_{N,m}(x)$ generalizes the functions $Fup_n(x)$ and $Fmup_{s,n}(x)$, which were introduced in $[1-3]$. Therefore we shall say that the function $f_{N,m}(x)$ is a generalized Fup-function.

In this paper we consider approximation properties of spaces of linear combinations of the functions $f_{N,m}(x)$.

Let $V_{N,m}$ be the space of 2π-periodic functions

$$f(x) = \sum_k c_k \cdot f_{N,m}\left(\frac{x}{\pi} - \frac{2k}{N} + 1 + \frac{m+2}{N} \right), x \in [\pi, \pi].$$

Denote by $\widetilde{W_2^r}$ the class of functions $f \in C^{r-1}_{[-\pi,\pi]}$ such that $f^{(k)}(-\pi) = f^{(k)}(\pi)$ for any $k = 0, 1, \ldots, r-1$, $f^{(r-1)}(x)$ is absolutely continuous and $\left\| f^{(r)} \right\|_{L_2[-\pi,\pi]} \leq 1$. Let $E_x(A, L) = \sup\limits_{\phi \in A} \inf\limits_{\psi \in L} \|\phi - \psi\|_X$ and also let $d_N(K, X) = \inf\limits_{\dim\ L = N} E_X(K, L)$ be the Kolmogorov width.

Theorem 1 *If $m \geq r-1$ and $m \leq N-2$ then there exists such $C = C(r, m, N)$ that $E_{L_2[-\pi,\pi]}\left(\widetilde{W_2^r}, V_{N,m}\right) \leq C \cdot d_N\left(\widetilde{W_2^r}, L_2[-\pi,\pi]\right)$.*

[1] V.L. Rvachev and V.A. Rvachev, *Nonclassical methods of approxima-tion theory in boundary value problem*, Naukova Dumka, Kiev, 1979 (in Russian).

[2] V.A. Rvachev, *Compactly supported solutions of functional - diffe-rential equations and their applications*, Uspekhi Mat. Nauk 45 (1990), no. 1, 77–103 (in Russian); English transl.: Russian Math. Surveys 45 (1990), no. 1, 87–120.

[3] V.A. Makarichev, *Approximation of periodic functions by* $mup_s(x)$, Math. Zametki 93 (2013), no. 6, 878–901 (in Russian); English transl.: Math. Notes 93 (2013), no. 6, 858–880.

Correlation intercepts and virial expansion of the equation of state for $\tilde{\mu}, q$-deformed Bose gas models

Yuriy Mishchenko, *Kiev, Ukraine*
Alexandre Gavrilik, *Kiev, Ukraine*

Deformed Bose gas models are nonlinear extensions of the standard Bose gas model and thus have a potential to incorporate in an effective way different factors of nonideality inherent to real gases or other situations. Let us also mention that *deformed oscillators* can be utilized [1] to model composite bosons. Another type of deformed Bose gas models were applied [2] for thermodynamic description of real gases. To this end, certain thermodynamic or statistical relations of ideal gas were initially deformed (e.g. by means of Jackson derivative), the others being deduced as consequences. The trial $\tilde{\mu}, q$-deformation, used in the present work, combines the quadratically nonlinear $\tilde{\mu}$-deformation from [1] and the Arik-Coon exponential q-deformation, applied to incorporate [2] the interparticle interaction. This $\tilde{\mu}, q$-deformation is characterized by the deformation structure function $\varphi_{\tilde{\mu},q}(n) = (1+\tilde{\mu})[n]_q - \tilde{\mu}([n]_q)^2$ where $[n]_q \equiv (1-q^n)/(1-q)$.

For the $\tilde{\mu}, q$-deformed Bose gas model, we calculate the deformed analogs of one- and two-particle distributions $\langle (a^\dagger_{\mathbf{k}})^r (a_{\mathbf{k}})^r \rangle$, $r = 1, 2$, and also the 2*nd* order correlation function intercept $\lambda^{(2)}(\mathbf{k}) = \frac{\langle (a^\dagger_{\mathbf{k}})^2 (a_{\mathbf{k}})^2 \rangle}{\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle^2} - 1$, where $a^\dagger_{\mathbf{k}}$, $a_{\mathbf{k}}$ denote the creation resp. annihilation operators of $\tilde{\mu}, q$-deformed oscillator ($\tilde{\mu}, q$-boson). Let us observe that for $q > (1+\tilde{\mu})^{-1}$ and for $-1 < q < (1+\tilde{\mu})^{-1}$ we have two qualitatively different types of behavior. The obtained momentum-dependencies for the intercepts are compared with experimental data for π-meson intercepts extracted in relativistic heavy ion collisions. Besides, slightly different $\tilde{\mu}, q$-deformed Bose gas model based on the deformed relation for the particle number $N_{(\varphi)} = \varphi\big(z\frac{d}{dz}\big) \ln Z$ (note here the φ-deformed analog $\varphi\big(z\frac{d}{dz}\big)$ of derivative) was considered [3]. Some arguments were given that the respective deformed virial expansion $\frac{Pv}{k_{\mathrm{B}}T} = \sum_{k=1}^{\infty} V_k(\tilde{\mu}, q)\big(\frac{\lambda^3}{v}\big)^{k-1}$ effectively accounts for (some) interparticle interaction and the composite structure of particles of a gas.

[1] A.M. Gavrilik, I.I. Kachurik, Yu. A. Mishchenko, *J. Phys. A: Math. Theor.*, 44 (2011), 475303.

[2] A. Scarfone, P. Narayana Swamy, *J. Stat. Mech.*, P02055 (2009).

[3] A.M. Gavrilik, Yu.A. Mishchenko, *Ukr. J. Phys.*, 58 (2013), 1171–1177.

Various types of convergence of sequences of subharmonic functions

Nguyen Van Quynh, *Kharkiv, Ukraine*

We study the question about connection between the convergence of sequences of subharmonic functions in the sense of the theory of generalized functions and other types of convergence.

The convergence of a sequence $\upsilon_n(x)$ to $\upsilon(x)$ in the space $L_p(\gamma)$ (γ is some positive measure in the space $\mathbb{R}^m$) means the following

$$\int |\upsilon_n(x) - \upsilon(x)|^p d\gamma(x) \to 0 \quad (n \to \infty).$$

We consider kernel $h_m(x-y) = \|x-y\|^{2-m}$ as a map $h_m(x-y) : \mathbb{R}^m_y \to L_p(\gamma)$.

Theorem 1 *Suppose that $\upsilon_n(x)$ is a sequence of subharmonic functions in a domain $G \subset \mathbb{R}^m$ that converges as a sequence of generalized functions to a generalized function* w. *Then*

1) *the generalized function* w *is a regular generalized function that is represented by a subharmonic function* w(x) *in the domain G;*

2) *if μ is the Riesz measure of* w, *and μ_n the Riesz measure of υ_n, then $\mu = \lim \mu_n$ $(n \to \infty)$ (convergence in the sense of the theory of generalized functions);*

3) *if β is a positive Borel measure with compact support in G such that the function $b(y) = \int h_m(x-y)d\beta(x)$ is continuous and the function $\int h_m(x-y)d|\beta|(x)$ is locally bounded, then*

$$\lim_{n\to\infty} \int \upsilon_n(x)d\beta(x) = \int \mathsf{w}(x)d\beta(x);$$

4) *if γ is a positive finite Borel measure with compact support in G such that the function $h_m(x-y) : \mathbb{R}^m \to L_p(\gamma)$ is uniformly continuous, then*

$$\int |\upsilon_n(x) - \mathsf{w}(x)|^p d\gamma(x) \to 0 \quad (n \to \infty).$$

[1] L. Hörmander, *The analysis of linear partial differential operators. II: Differential operators with constant coefficients,* Springer–Verlag, Berlin–Heidelberg–New York–Tokyo, 1983.

[2] A.F. Grishin, A. Chouigui, *Various types of convergence of sequences of δ-subharmonic functions, Math. sbornyk*, 199, (2008), 27–48 (Russian).

Estimates for Riesz measures of unbounded subharmonic functions

Liudmyla Radchenko, *Kharkiv, Ukraine*
Sergey Favorov, *Kharkiv, Ukraine*

It is well known (see [1]) that if $v(z)$ is bounded subharmonic function in the unit disk then its Riesz measure $\mu = \dfrac{1}{2\pi}\triangle v$ satisfies the following inequality

$$\int_{|\lambda|<1} (1-|\lambda|)\mu(d\lambda) < \infty. \tag{1}$$

It is a subharmonic analog of the classical Blaschke condition for zeros of bounded analytic functions.

There are a lot of generalizations of estimate (1) for analytic and subharmonic functions growing near the boundary of the unit disk (see [2], [3]) or its part (see [4], [5]). Particulary, in [5] the polynomial growth was considered.

We investigate the case of subharmonic functions in the unit disk $\mathbb{D}$ growing near $E \subset \partial\mathbb{D}$ as an arbitrary function φ. Instead of (1) we obtained the inequality

$$\int \psi(\rho(\lambda))(1-|\lambda|)\mu(d\lambda) < \infty \tag{2}$$

under some condition on functions ψ, φ and the set E. We also proved that this conditions are optimal, in a sense.

Further, we extend our results to subharmonic functions in the unit ball in $\mathbb{R}^n$. Moreover, under additional condition of so-called generalized convexity of a compact subset in $\mathbb{R}^n$ we prove integral conditions on Riesz measure of the subharmonic function in the complement of this compact.

[1] W.K. Hayman, P.B. Kennedy, *Subharmonic functions,* Academic Press Inc. (London) LTD, 1976.

[2] J. Garnett, *Bounded analytic functions. Graduate Texts in Mathematics,* vol. 236, Springer, New York, 2007.

[3] F.A. Shamoyan, *On zeros of analytic in the disc functions growing near its boundary,* Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences), 18, No. 1 (1983).

[4] S. Favorov, L. Golinskii, *A Blaschke-type condition gor analytic and subharmonic functions and application to contraction operators,* Amer. Math. Soc. Transl. (2), 226 (2009), 37–47.

[5] A. Borichev, L. Golinskii, and S. Kupin, *A Blaschke-type condition and its application to complex Jacobi matrices,* Bull. London Math. Soc. 41 (2009), 117–123.

Robust feedback synthesis problem for a system with two perturbations

Tetyana Revina, *Kharkiv, Ukraine*

The paper deals with the robust feedback synthesis of a bounded control for a system with two unknown perturbations. Namely, we consider the system

$$\dot{x}_1 = (1+p(t,x))x_2,\ \dot{x}_2 = (1+r_1 p(t,x))x_3,\ \dot{x}_3 = x_4, \dots,\ \dot{x}_{n-1} = x_n,\ \dot{x}_n = u,$$

where unknown bounded perturbation $p(t,x)$ is continuous with respect to all arguments. Our approach is based on the controllability function method created by V.I. Korobov in 1979 [1]. The *robust feedback synthesis problem* is to construct a control of the form $u = u(x)$, $x \in \mathbb{R}^n$, such that:
1) $|u(x)| \leq 1$;
2) the trajectory $x(t)$ of the closed system, starting at an arbitrary initial point $x(0) = x_0 \in \mathbb{R}^n$, ends at the origin at a finite time $T(x_0, p) < \infty$ for any admissible perturbation $d_1 \leq p(t,x) \leq d_2$;
3) the control is independent of $p(t,x)$.

Put

$$F^{-1} = \left(\frac{(-1)^{2n-i-j}}{(n-i)!(n-j)!(2n-i-j+1)(2n-i-j+2)} \right)_{i,j=1}^{n},$$

$$D(\Theta) = \operatorname{diag}\left(\Theta^{-\frac{2n-2i+1}{2}}\right)_{i=1}^{n}, \quad F^1 = ((2n-i-j+2)f_{ij})_{i,j=1}^{n},$$

$$S = \Theta(FD(\Theta)RD^{-1}(\Theta) + D^{-1}(\Theta)R^* D(\Theta)F).$$

Theorem 1 *Suppose that* $[d_1; d_2] \subset (d_1^0; d_2^0)$, *where*

$$d_1^0 = \max\{(1-\gamma_1)\tilde{d}_1^0;\ (1-\gamma_2)\tilde{d}_2^0\}, \quad d_2^0 = \min\{(1-\gamma_1)\tilde{d}_2^0;\ (1-\gamma_2)\tilde{d}_1^0\},$$

$$\tilde{d}_1^0 = 1/\lambda_{\min}((F^1)^{-1}S),\ \tilde{d}_2^0 = 1/\lambda_{\max}((F^1)^{-1}S),\ 0 < \gamma_1 < 1,\ \gamma_2 > 1;$$

and the controllability function $\Theta(x)$ *is the unique positive solution of equation*

$$2a_0\Theta = (D(\Theta)FD(\Theta)x, x), \quad x \neq 0, \quad \Theta(0) = 0, \quad 0 < a_0 \leq \frac{2}{f_{nn}},$$

and the control is given by $u(x) = -\frac{1}{2}b_0^* D(\Theta(x))FD(\Theta(x))x$.

Then the trajectory of the closed system, starting at any initial point $x(0) \in \mathbb{R}^n$, *ends at the point* $x(T) = 0$ *at some finite time* $T = T(x_0, d_1, d_2)$ *such that*

$$\Theta(x_0)/\gamma_2 \leq T(x_0, d_1, d_2) \leq \Theta(x_0)/\gamma_1.$$

[1] V.I. Korobov, *The method of controllability function (Russian)*, R&C Dynamics, M.-Izhevsk, 2007.

Strong solutions for interactive system of full Karman and linearized Navier–Stokes equations

Iryna Ryzhkova-Gerasymova, *Kharkiv, Ukraine*

We consider a model consisting of linearized Navier-Stokes equations on a bounded 3-D domain and full Karman (Marguerre–Vlasov) equations which describe a nonlinear shell, placed on a flexible part of the boundary of the domain occupied by the fluid. The coupling of the equations takes place on the domain, occupied by the shell. The full Karman model accounts both for transversal and longitudinal displacements of the shell. In this model we do not account for rotational inertia of the filaments of the plate. Since an issue of uniqueness of energy solutions of the system is a challengeing open question (as well as for full Karman (Marguerre–Vlasov) equations without fluid), we prove well-posedness of our system in smooth functional spaces and study asymptotical behavior of strong solutions.

Dissipative magnetic 2D Zakharov system in bounded domain

Alexey Shcherbina, *Kharkiv, Ukraine*

We consider the dissipative magnetic Zakharov system in a smooth (2D) bounded domain $\Omega \subset \mathbb{R}^2$ of the form

$$\begin{cases} iE_t + \Delta E - nE + iE \times B + i\gamma_1 E = g_1(x,t), & x \in \Omega, \\ n_{tt} + \gamma_2 n_t - \Delta\left(n + |E|^2\right) = g_2(x,t), & x \in \Omega, \\ B_{tt} - \gamma_3 \Delta B_t + \Delta^2\left(B + iE \times \overline{E}\right) = g_3(x,t), & x \in \Omega, \end{cases} \tag{1}$$

where $n(x,t)$ and $B(t,x) = (0,0,B_3(t,x))$ are the real functions and $E(x,t) = (E_1(t,x), E_2(t,x), 0)$ is a complex one.

If we omit magnetic field B, then the system (1) reduces to the dissipative Zakharov system. This system has been studied by many authors (see [1] and references therein).

In the case $\Omega = \mathbb{R}^d$ for $d = 2,3$ the Cauchy problem for the system (1) has been considered in ([2]). It was obtained local existence and uniqueness results. Our main result is the global well-posedness of the problem (1) in some Sobolev type classes and existence of a global attractor.

[1] I. Chueshov and A. Shcherbina, *On 2D Zakharov system in a bounded domain*, Differential and Integral Equations, 18 (2005), 781–812.

[2] Boling Guo, Jingjun Zhang, Chunxiao Guo, *On the Cauchy problem for the magnetic Zakharov system*. Monatshefte für Mathematik, 170 (2013), 89–111.

Multidimensional affine umbilical immersions

Olena Shugailo, *Kharkiv, Ukraine*

The umbilical hypersurfaces (proper and improper affine spheres) are well studied (see,e.g. [1,2]). The definition of umbilical codimension 2 immersion can be found in [3]. For immersion of higher codimension we introduce a similar definition.

Affine immersion $f : M^n \to \mathbb{R}^{n+k}$ is called affine umbilical if there is a transversal distribution Q such that the Weingarten mapping satisfies $S : (\xi, X) \mapsto \lambda_\xi \cdot X$ for each $\xi \in Q$, where λ_ξ is a smooth function.

If $S \equiv 0$ then the immersion is called *improper affine umbilical*; otherwise the immersion is called *proper affine umbilical*.

Proposition 1 *The image of a proper affine umbilical immersion $f : M^n \to \mathbb{R}^{n+k}$ lies on affine hypercylinder with $(k-1)$-dimensional rulings based on n-dimensional centro-affine hypersurface.*

Proposition 2 *Let $f : M^n \to \mathbb{R}^{n+k}$ be an affine umbilical immersion with induced flat connection. Then it is affinely equivalent to a graph immersion of a smooth map $F : M^n \to \mathbb{R}^k$.*

Proposition 3 *The image of a proper affine umbilical immersion with locally symmetric connection lies on affine cylinder over central quadric.*

We define a power of the curvature operator $R(X, Y)$ as in [4] and find a locally parametrization of a proper affine umbilical immersion with nilpotent curvature operator.

[1] K. Nomizu, U. Pinkall, *On the geometry of affine immersions*, Mathematisce Zeitschrift, 195 (1987), 165–178.

[2] K. Nomizu, T. Sasaki, *Affine differential geometry*, Cambridge University Press, 1994.

[3] K. Nomizu, T. Sasaki, *Centroaffine immersions of codimension two and projective hypersurface theory*, Nagoya Math. J., 132 (1993), 63–90.

[4] E. Sakharova, A. Yampolsky, *Powers of Curvature Operator of Space Forms and Geodesics of the Tangent Bundle*, Ukr. Math. J., 56 (2004), No. 9, 1231–1243.

On finite element Petrov–Galerkin method for solving convection-diffusion problems

Sergii Siryk, *Kyiv, Ukraine*

At the present time, the Petrov–Galerkin method (PGM) in the form of the finite element method is one of the most successful approaches to the construction of numerical approximations in problems of various physical processes investigation. One of the key role of PGM applying for solving convection-dominated and reaction-dominated problems is the correct choice of the weighting functions which prevents spurious oscillations in the numerical solutions and stabilizes the numerical solution while maintaining acceptable accuracy. The option of the weighting functions selection for the integration of one-and two-dimensional convection-diffusion problems was proposed in [1]. These weight functions and their multidimensional generalizations were later successfully applied for the numerical solution of various unsteady convection-diffusion problems (including cases where the velocity in the convective term changes very quickly, both in magnitude and direction) as well as for nonlinear equations [2, 3]. Some generalizations of weighting functions (and corresponding PGMs) were proposed in [4] as well as a detailed theoretical analysis of the corresponding numerical schemes. Some new estimates of PGM for steady one-dimensional convection-diffusion equations were obtained in [5]. The current report presents an overview and generalization of some results of the papers mentioned above.

[1] N.N. Salnikov, S.V. Siryk, I.A. Tereshchenko, *On construction of finite-dimensional mathematical model of convection-diffusion process with usage of the Petrov–Galerkin method*, Journal of Automation and Information Sciences, 42 (2010), 67–83.

[2] S.V. Siryk, N.N. Salnikov, *Numerical Solution of Burgers' Equation by Petrov-Galerkin Method with Adaptive Weighting Functions*, Journal of Automation and Information Sciences, 44 (2012), 50–67.

[3] O.A. Molchanov, S.V. Siryk, N.N. Salnikov, *A Choice of Weight Functions of Petrov–Galerkin Method for Solving of Two-Dimensional Nonlinear Problems of Burgers' Type*, Mathematical Machines and Systems, No. 2 (2012), 136–144.

[4] S.V. Siryk, N.N. Salnikov, V.K. Beloshapkin, *A Choice of Weighting Functions of Petrov-Galerkin Method for Solving Linear One-Dimensional Convection-Diffusion Problems*, USiM, No. 1 (2014), 38–47.

[5] S.V. Siryk, *Accuracy and Stability of the Petrov-–Galerkin Method for Solving the Stationary Convection-Diffusion Equation*, Cybernetics and Systems Analysis, 50 (2014), 278–287.

Hardy inequality and an example of infinitesimal operator with non-Riesz basis family of eigenvectors

Grigory Sklyar, *Szczecin, Poland*
Vitalii Marchenko, *Kharkiv, Ukraine*

The discrete form of Hardy inequality reads that if $p > 1$ and $\{a_k\}_{k=1}^{\infty}$ is a sequence of nonnegative real numbers, then

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}\sum_{k=1}^{n}a_k\right)^p \leq \left(\frac{p}{p-1}\right)^p \sum_{n=1}^{\infty}a_n^p. \tag{1}$$

Let H be a separable Hilbert space with norm $\|\cdot\|$. One of the remarkable achievements in the spectral theory of C_0-semigroups in H was obtained in [1, 2].

However, one essential question was not considered. Namely, is this possible to construct the generator of a C_0-group with non-Riesz basis family eigenvectors? Assume that $\{e_n\}_{n=1}^{\infty}$ is a Riesz basis of H and consider the space $\ell_2(\Delta) = \{x = \{\alpha_n\}_{n=1}^{\infty} : \Delta x \in \ell_2\}$, where Δ denotes a difference operator. We introduce a Hilbert space

$$H_1(\{e_n\}) = \left\{ x = \sum_{n=1}^{\infty} c_n e_n : \{c_n\}_{n=1}^{\infty} \in \ell_2(\Delta) \cap c_0 \right\}$$

with norm $\|x\|_1 = \left\| \sum\limits_{n=1}^{\infty} c_n e_n \right\|_1 = \left\| \sum\limits_{n=1}^{\infty} (c_n - c_{n-1}) e_n \right\|$. The following theorem gives a positive answer to the question above.

Theorem 1 *Let $\{e_n\}_{n=1}^{\infty}$ be a Riesz basis of H. Then $\{e_n\}_{n=1}^{\infty}$ is a bounded non-Riesz basis of $H_1(\{e_n\})$ and the operator $A : H_1(\{e_n\}) \supset D(A) \to H_1(\{e_n\})$ defined by $Ax = A \sum\limits_{n=1}^{\infty} c_n e_n = \sum\limits_{n=1}^{\infty} i \ln n \cdot c_n e_n$, with domain*

$$D(A) = \left\{ x = \sum_{n=1}^{\infty} c_n e_n \in H_1(\{e_n\}) : \{\ln n \cdot c_n\}_{n=1}^{\infty} \in \ell_2(\Delta) \cap c_0 \right\},$$

generates a C_0-group on $H_1(\{e_n\})$.

The Hardy inequality (1) for $p = 2$ plays a key role in the proof of Theorem 1.

[1] G.Q. Xu, S.P. Yung, *The expansion of a semigroup and a Riesz basis criterion*, J. Differential Equations 210 (2005), 1–24.

[2] H. Zwart, *Riesz basis for strongly continuous groups*, J. Differential Equations 249 (2010), 2397–2408.

A new estimate for the Bishop–Phelps–Bollobás modulus of a Banach space

Mariia Soloviova, *Kharkiv, Ukraine*

Let X be a real Banach space, X^* be the dual of X. Denote

$$\Pi(X) = \{(x, x^*) \in X \times X^* : \|x\| = \|x^*\| = 1,\ x^*(x) = 1\},$$

$$\Pi_\varepsilon(X) = \{(x, x^*) \in X \times X^* : \|x\| = \|x^*\| = 1,\ x^*(x) \geq 1 - \varepsilon\}.$$

Definition 1 *The spherical Bishop–Phelps–Bollobás modulus of X is the function $\Phi_X^S(\varepsilon) : (0, 2) \longrightarrow \mathbb{R}^+$, which is determined by the following formula:*
$\Phi_X^S(\varepsilon) = \inf\{\delta > 0 : \ \forall\ (x, x^*) \in \Pi_\varepsilon(X)$ *there exist* $(y, y^*) \in \Pi(X)$ *such that* $\|x - y\| < \delta, \|x^* - y^*\| < \delta\}$.

The function Φ_X^S measures how close can a pair $(y, y^*) \in \Pi(X)$ be selected to $(x, x^*) \in S_X \times S_{X^*}$ depending on how close is $x^*(x)$ to 1. This function was introduced and studied in [2]. In particular it was proved that $\Phi_X^S(\varepsilon) \le \sqrt{2\varepsilon}$. We obtained, jointly with my scientific advisor Vladimir Kadets, an estimate of the Bishop–Phelps–Bollobás modulus through the parameter of uniform non-squareness of X.

Definition 2 *The parameter of uniform non-squareness is the quantity* $\alpha(X) = 2 - \sup\limits_{x,y \in B_X} \left\{ \frac{1}{2}(\|x+y\| + \|x-y\|) \right\}$.

At first, we obtain a stronger version of Phelps theorem [1, page 6].

Theorem 1 *Let X be a Banach space with $\alpha(X) > \alpha_0$. Then $\forall \varepsilon \in (0,2)$, $\forall (x, x^*) \in \Pi_\varepsilon(X)$ and $\forall k \in (\frac{\varepsilon}{2}, 1)$ there exist $(y, y^*) \in \Pi(X)$ such that $\|x - y\| \le \frac{\varepsilon}{k}$ and $\|x^* - y^*\| \le 2k - \frac{2}{3}k\alpha_0$.*

This implies the following estimate of Φ_X^S.

Theorem 2 *Let X be a Banach space with $\alpha(X) > \alpha_0$. Then*

$$\begin{aligned} \Phi_X^S(\varepsilon) &\le \sqrt{2\varepsilon} \cdot \sqrt{1 - \frac{1}{3}\alpha_0} \quad \textit{for} \quad \varepsilon \in \left(0, 2 - \frac{2}{3}\alpha_0\right), \ \textit{and} \\ \Phi_X^S(\varepsilon) &\le \varepsilon \quad \textit{for} \quad \varepsilon \in \left[2 - \frac{2}{3}\alpha_0, 2\right) \end{aligned}$$

[1] R.R. Phelps, *Support Cones in Banach Spaces and Their Applications*, Adv. Math., 13 (1974), 1–19.

[2] M. Chica, V. Kadets, M. Martín, S. Moreno-Pulido, F. Rambla-Barreno, *Bishop–Phelps–Bollobás moduli of a Banach space*, J. Math. Anal. Appl., 412 (2014), 697–719.

Methodology of Petrov–Galerkin weighting functions choice with usage of neural networks for convection-diffusion problem

Igor Tereshchenko, *Kyiv, Ukraine*

Most Petrov–Galerkin formulations take into account the spatial discretization and the weighting functions developed give satisfactory solutions for steady state problems [1]. Though these schemes can be used for transient problems, there is scope for improvement. Nowadays, research is being carried out for the selection of the weighting functions using neural networks [2]. The best result shows SUPG

[3] discretization with additional terms that provide consistency and improve the phase accuracy for convection dominated flows. The task of finding these parameters is resolved also by neural networks. To train the network optimization techniques used which in turn require the computation of the gradient of the error with respect to the network parameters.

Finite element approximation of Petrov–Galerkin formulations for convection-diffusion problem

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial x^2}, \quad x \in [0; L],\ t \in [0; t_1],$$

can be represented as

$$\begin{aligned}\sum_{i=0}^{n} \dot{a}_i(t) \int_0^L N_i(x)W_j(x)dx = &- v\sum_{i=0}^{n} a_i(t) \int_0^L N_i'(x)W_j(x)dx \\ &- k\sum_{i=0}^{n} a_i(t) \int_0^L N_i'(x)W_j'(x)dx,\end{aligned}$$

where $T(t,x) = \sum\limits_{i=0}^{n} a_i(t)N_i(x)$, $N_i(x)$ is a linear basis function [1], $W_j(x)$ is a weight function of SUPG method [3].

Proposed a neural network in which the neural activation functions correspond to α_j parameters of $W_j(x)$ weight functions, and the weights correspond to the value of the function $T(t,x)$ in points discretization as $W_j(x) = N_j + \alpha_j N_j'$. Such approach adapt the weight function of the solution to the problem characteristics.

[1] O.C. Zienkiewicz, R.L. Taylor, *The Finite Element Method. Fifth edition. Volume 3: Fluid Dynamics,* Butterworth-Heinemann, Oxford, 2000.

[2] H. Lee, I. Kang, *Neural algorithms for solving differential equations,* Journal of Computational Physics, 91 (1990), 110–117.

[3] A.N. Brooks, T.J.R. Hughes, *Streamline upwind Petrov–Galerkin formulations for convection dominated flows with particular emphasis on the incompressible Navier–Stokes equations,* Comput. Methods Appl. Mech. Engrg., 32 (1982), 199–259.

AUTHOR INDEX

Milka, A.	milka@ilt.kharkov.ua
Mishchenko, Yu.	mischenko.yuriy@gmail.com
Myronyuk, M.	m_margarita@bk.ru
Nessonov, N.	n.nessonov@gmail.com
Oversteegen, L.	overstee@math.uab.edu
Palafox, S.	sergiopalafoxd@gmail.com
Poplavskyi, M.	mpoplavskyi@gmail.com
Ptacek, R.	rptacek@uab.edu
Quynh, N.V.	quynhsonla1988@gmail.com
Radchenko, L.	liudmyla.radchenko@gmail.com
Revina, T.	t.revina@karazin.ua
Rybalko, V.	rybalko@ilt.kharkov.ua
Ryzhkova-Gerasymova, I.	iryonok@gmail.com
Shcherbina, A.	shcherbina@mail.ru
Shcherbina, M.	shcherbina@ilt.kharkov.ua
Shcherbina, T.	t_shcherbina@rambler.ru
Shepelsky, D.	shepelsky@yahoo.com
Shugailo, O.	shugailo@karazin.ua
Silva, L.	silva@iimas.unam.mx
Sinel'shchikov, S.	sinelshchikov@ilt.kharkov.ua
Siryk, S.	qwyleqwyle@gmail.com
Sklyar, G.	sklar@univ.szczecin.pl
Sodin, M.	sodin@post.tau.ac.il
Soloviova, M.	5_11_16@mail.ru
Tanasichuk, A.	tan_ann@mail.ru
Tereshchenko, I.	tereshchenko.igor@gmail.com
Timorin, V.	vtimorin@hse.ru
Vasilchuk, V.	vasilchuk@ilt.kharkov.ua
Vengerovsky, V.	vengerovsky@yandex.ua
Yampolsky, A.	a.yampolsky@karazin.ua
Zolotarev, V.	vazolotarev@gmail.com